Quantum-enhanced performance in superconducting Andreev-reflection engines

Gonzalo Manzano and Rosa López

Institute for Cross-Disciplinary Physics and Complex Systems
IFISC (UIB-CSIC), Palma de Mallorca (Spain)

6 June 2023

IFISC

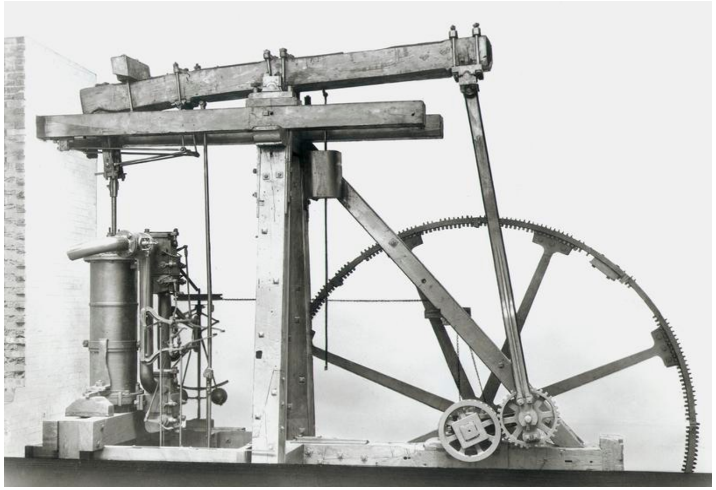
AGENCIA ESTATAL DE INVESTIGACIÓN

UNIT OF EXCELLENCE MARÍA DE MAEZTU
07/2018 - 06/2022

5-6/06 @ IFISC (UIB-CSIC)
Novel trends in topological systems and quantum thermodynamics

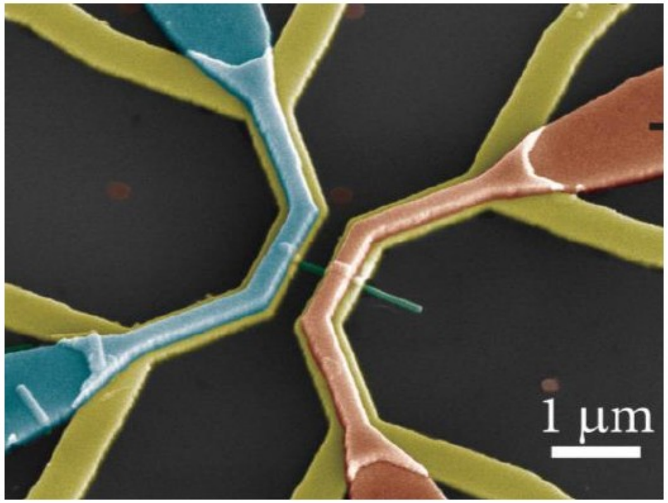
Motivation

Macroscopic (classical) heat engines



(Watt's steam engine, 1769)

Microscopic (quantum) heat engines



(Quantum-dot engine, 2018
H. Linke group, Sweden)

+ Large number of degrees of freedom

+ Fluctuations become negligible

+ Classical thermodynamics

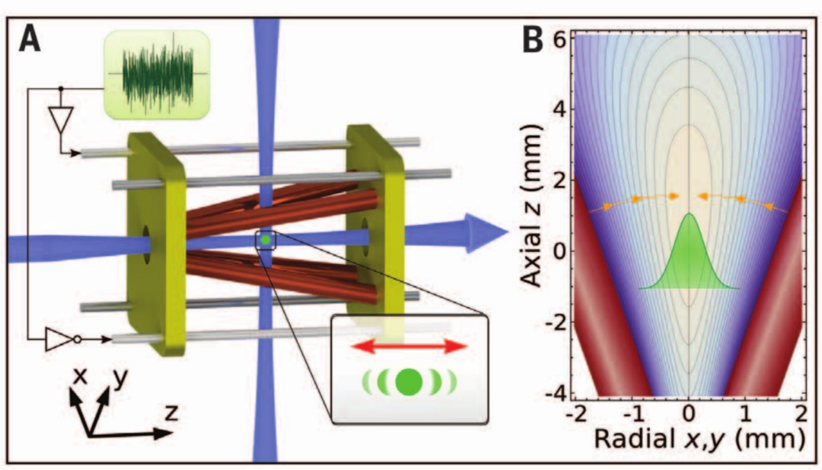
+ Small systems (micro or nanoscale)

+ Fluctuations are important

+ Stochastic and quantum thermodynamics

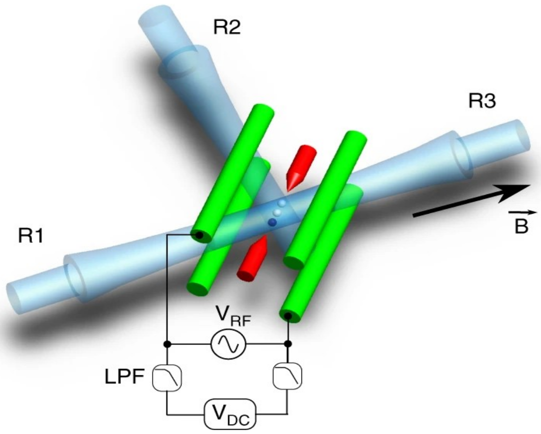
Motivation

Single-ion cyclic quantum engine



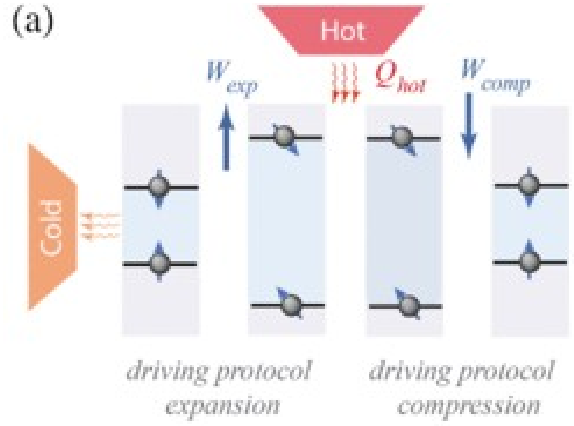
Roßnagel *et al.* Science (2016)

Quantum absorption refrigerator



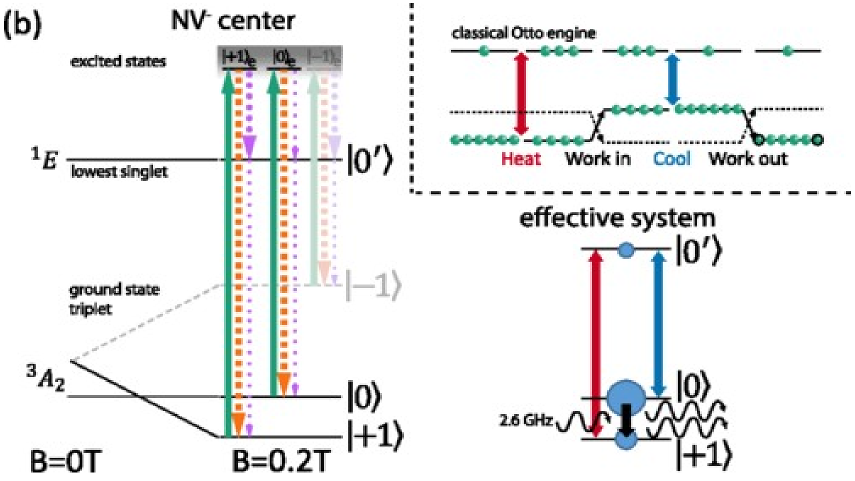
Maslennikov *et al.* Nat Commun. (2019)

Spin Otto cycle with NMR techniques



Peterson *et al.* Phys. Rev. Lett **123** (2019)

Continuous engine with NV centers



Klatzow *et al.* Phys. Rev. Lett **122** (2019)

Motivation

Many models of engines are **based on quantum effects** (e.g. tunneling) or even show an **intrinsic quantum dynamics**, leading e.g. to entanglement in multipartite systems, but...

Quantum-thermodynamic advantage?

+ Define and compare to classical analogs, introduce extra dephasing ...

R. Uzdin, *et al.* PRX (2015), J.O. González, *et al.* PRE (2019), ...

Some problems of classical analogs:

+ model dependent

+ definition of "analog"

+ Breaking of classical nonequilibrium inequalities such as TUR as a witness

→ ***model independent***

Agarwalla & Segal PRB (2018), Ptaszyński PRB (2018), Kalaee et al. PRE (2021), ...

Thermodynamic and Kinetic Uncertainty Relations

Classical Markovian processes in nonequilibrium steady states

TUR

$$\frac{\mathrm{Var}[J(t)]}{\langle J(t)\rangle^2} \geq \frac{2k_B}{\Sigma}$$

Entropy production (EP) rate / dissipation

Arbitrary current

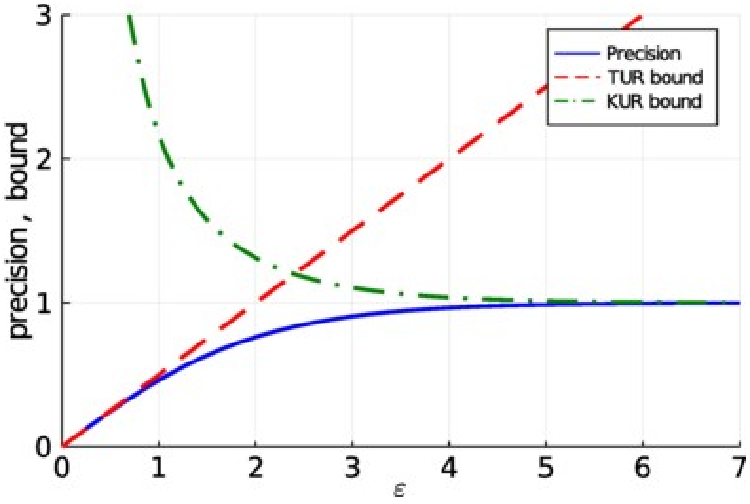
Barato and Seifert, PRL (2015), Gingrich *et al.* PRL (2016), ...

KUR

$$\frac{\mathrm{Var}[J(t)]}{\langle J(t)\rangle^2} \geq \frac{1}{\mathcal{K}}$$

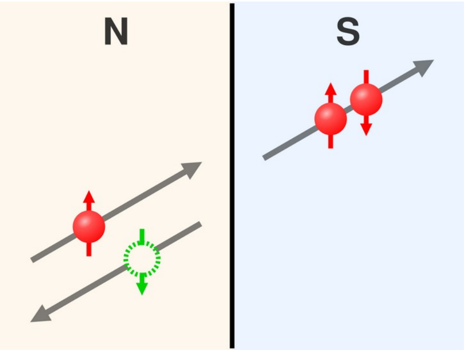
Arbitrary current

Dynamical activity
/ frenesy



Terlizzi and Baisei, JPA (2018), Hiura and Sasa PRE (2021)

Andreev-reflection engine



Andreev reflection: incident electron in N generates a Cooper pair at S and a retro-reflected hole in N of opposite spin.

A. F. Andreev, Soviet Physics-JETP (1964)

Many applications in hybrid NS devices: superconducting transistors, spin-entangled electrons generation, Andreev qubits, topological qubits…

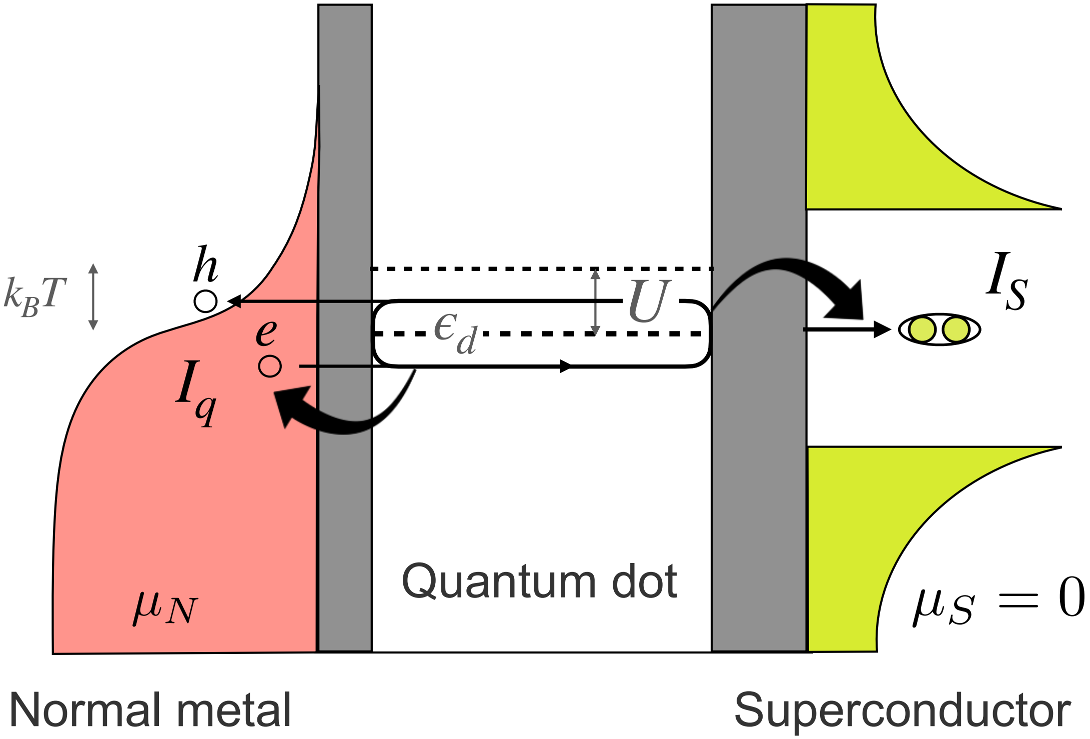
Engine model:

$$H_d = \sum_\sigma \epsilon_\sigma d_\sigma^\dagger d_\sigma + U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow$$

electron's spin $\sigma = \{\uparrow, \downarrow\}$

$\epsilon_{\uparrow,\downarrow} = \epsilon \pm \Delta_Z$ dot level

U Coulomb interaction



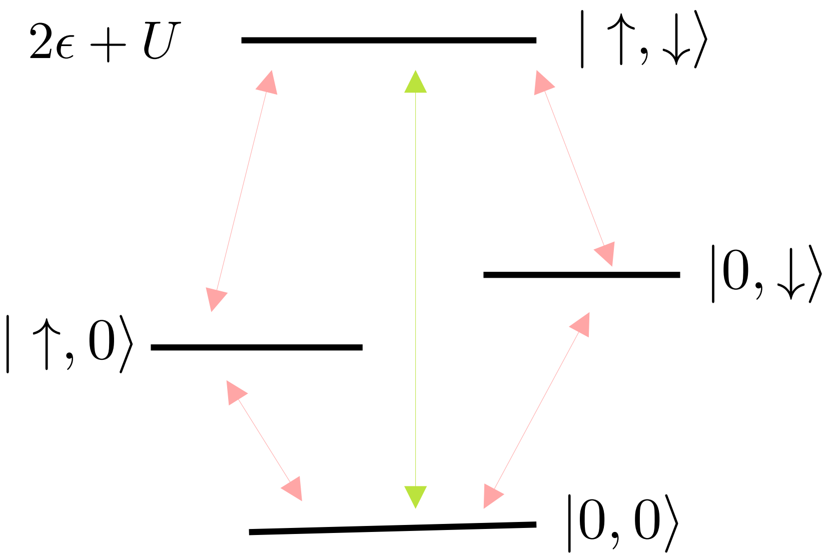
Model

Large superconductor gap limit
(subgap transport)

$$\Delta \to \infty \qquad (\Delta \gg \Gamma_N, k_B T)$$

QD is “proximitized” by superconductor

Rozhkov and Arovas PRB (2000), Lee et al. Nat. Nanotech. (2014), Tabatanei et al. PRB (2022)



superconductor →

$$H_S(t) = \Gamma_S\left(d_\uparrow^\dagger d_\downarrow^\dagger e^{i(2\epsilon+U)t} + d_\downarrow d_\uparrow e^{-i(2\epsilon+U)t}\right)$$

$$\dot{\rho}(t) = -i\Gamma_S[d_\uparrow^\dagger d_\downarrow^\dagger + d_\uparrow d_\downarrow, \rho] \quad + \quad \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k \rho\}$$

coherent contribution
(Cooper pairs)

Dissipative contribution
(normal metal electrons)

Single-electron jumps from normal metal:

$$L_{\sigma,\delta}^{+} = \Gamma_N f(\epsilon_\sigma + \delta U) d_\sigma^\dagger \qquad L_{\sigma,\delta}^{-} = \Gamma_N [1 - f(\epsilon_\sigma + \delta U]) d_\sigma$$

with

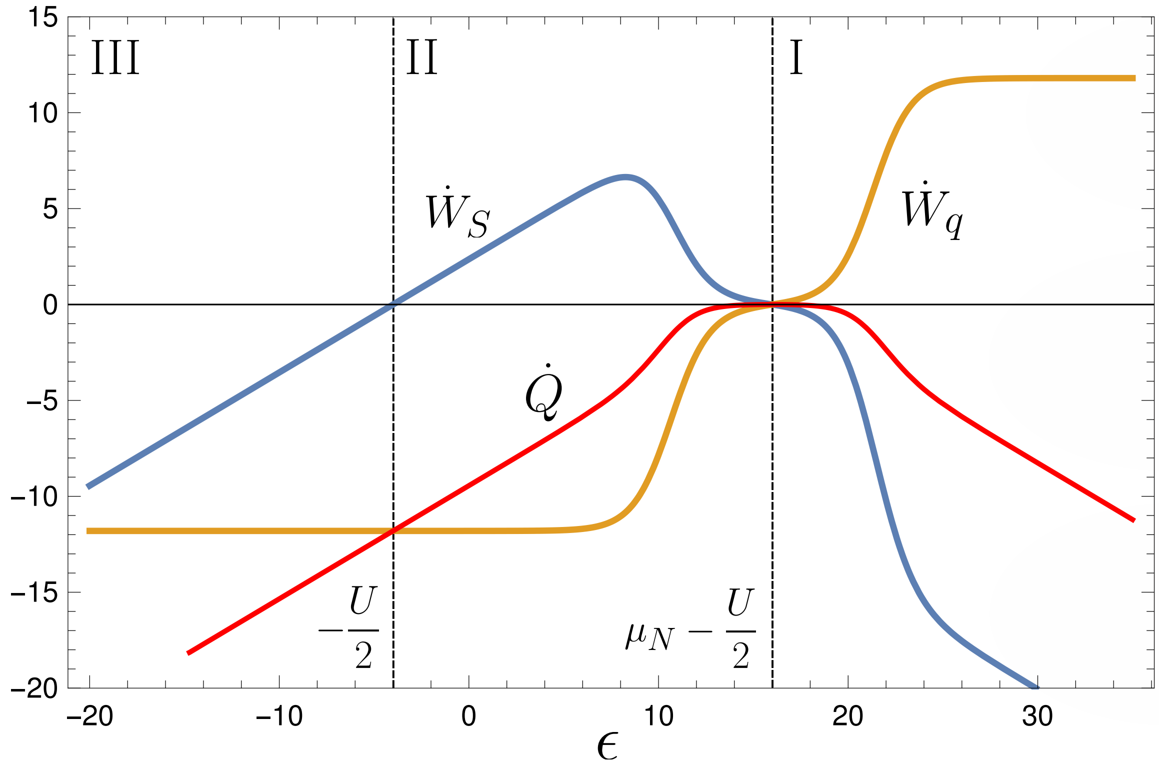
$$\sigma = \{\uparrow,\downarrow\}$$
$$\delta = \{0,1\}$$

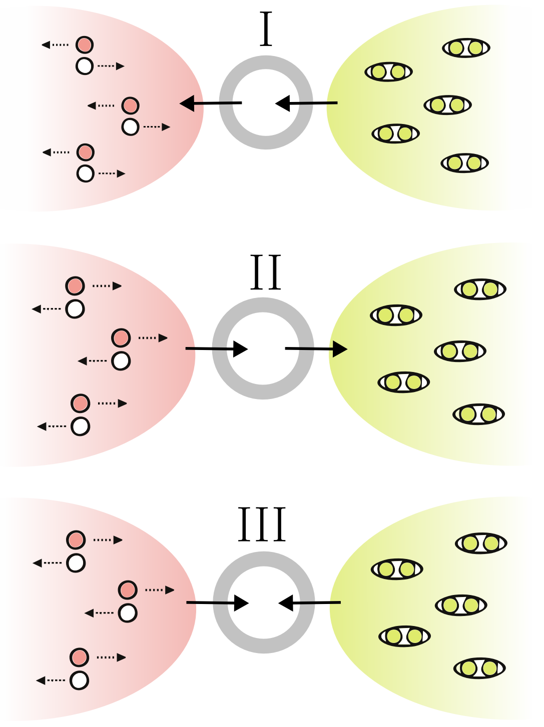
Focus on steady-state operation:

Energy and particle currents from normal metal: $\langle J_E \rangle \;\; \langle J_q \rangle$

$$\dot{Q} = \langle J_E \rangle - \mu_N \langle J_q \rangle \qquad \dot{W}_q = -\mu_N J_{\mathrm{q}}$$ output electrical current

Extra (output) work contribution from superconductor: $\dot{W}_S = -\mathrm{Tr}[\dot{H}_S(t)\rho_s(t)]$





First law: $\dot{W}_q + \dot{W}_S = \dot{Q}$ Second law: $\Sigma = -\dot{Q}/T \geq 0$

Results

Efficiency:

$$\boxed{\eta_I := \frac{\dot{W}_q}{-\dot{W}_S} = \frac{2\mu_N}{2\epsilon + U} \leq 1}$$

→ work-to-work transducer

$$(\eta_{II} := \eta_I^{-1})$$

Stability of the output power: $\mathrm{Var}[\dot{W}_q] = \mu_N^2 \mathrm{Var}[J_q]$ w.r.t. $\dot{W}_q$

Signal-to-noise ratio: $\mathcal{F}_q := \langle J_q \rangle^2 / \mathrm{Var}[J_q]^2$ → Full Counting Statistics methods

Using TUR and KUR we can give classical bounds using normalized “constancy”:

From **TUR**: $$\boxed{\mathcal{C}_T := \frac{2k_B \mathcal{F}_q}{\Sigma} = 2k_B T \frac{\eta_I \mathcal{F}_q}{(1-\eta_I)\dot{W}_q} \leq 1}$$

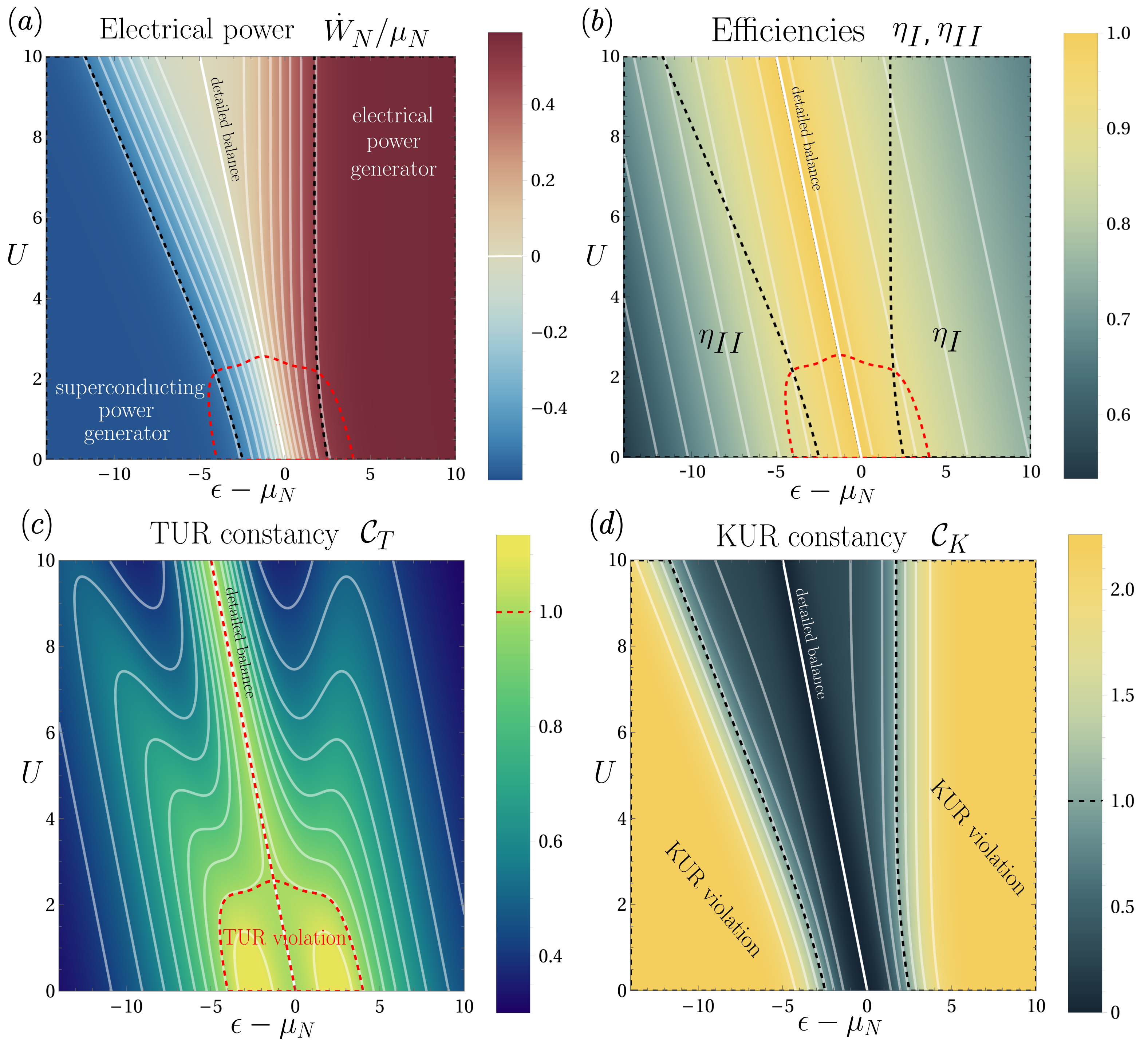
From **KUR**: $$\boxed{\mathcal{C}_K := \frac{\mathcal{F}_q}{\mathcal{K}} \leq 1}$$ → Classical bounds

Then if:

$$\mathcal{C}_T, \mathcal{C}_K > 1$$

enhanced stability !

Results



$$\max \mathcal{C}_T \simeq 1.2$$

$$\max \mathcal{C}_K \simeq 2.2$$

Conclusions

- Superconducting-to-electrical power transducer based on Andreev reflection
- Superconductor acts as a coherent contribution leading to quantum thermodynamic signatures spotted by the violation of TUR and KUR.
- We have higher stability than classically allowed for steady-state engines in relevant regimes with either maximum power or high efficiencies (or a compromise between them).

Possible implementation:

Subgap transport regime $\Delta \sim 2meV$ (Niobium)

Lee *et al.* Nature Nanotech **9** (2014)
PRB **95** (2017)

Weak coupling $\Gamma_N \sim 1\mu eV$

Foxman *et al.* PRB **47** (1993)
Jaliel *et al.* PRL **123** (2019)

$T \sim 0.1K - 1K$ $U \sim 0.2meV$

Thanks page

THANK YOU

for your attention

arXiv: 2302.09414