



# Long-Range Quantum Transfer in Semiconductor Quantum Dots Arrays

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Instituto de Ciencia de Materiales de Madrid (ICMM-CSIC)

“Novel trends in topological systems and quantum thermodynamics”

(i-Link)



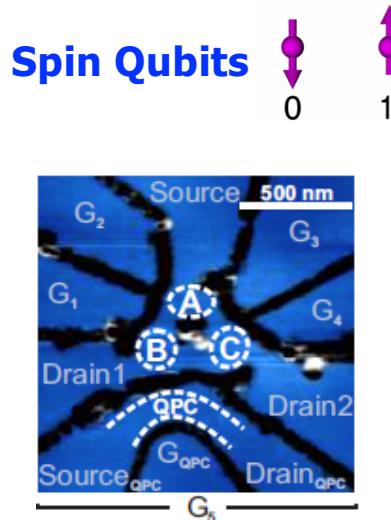
## Outline:

### **Semiconductor quantum dot arrays (artificial molecules) for...**

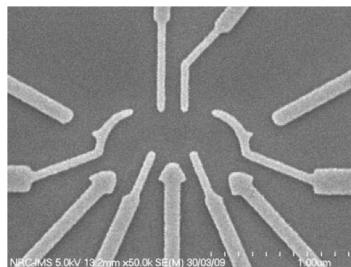
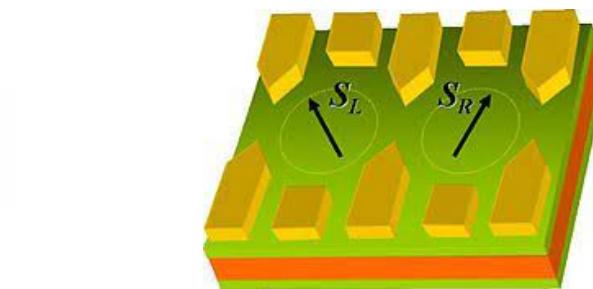
- Long-range transfer of electron and hole spins
  - Quantum simulation of lattices with non trivial topology by Floquet engineering
- 
- Edge states for quantum information transfer

# QDs as a Platform for a Quantum Computer

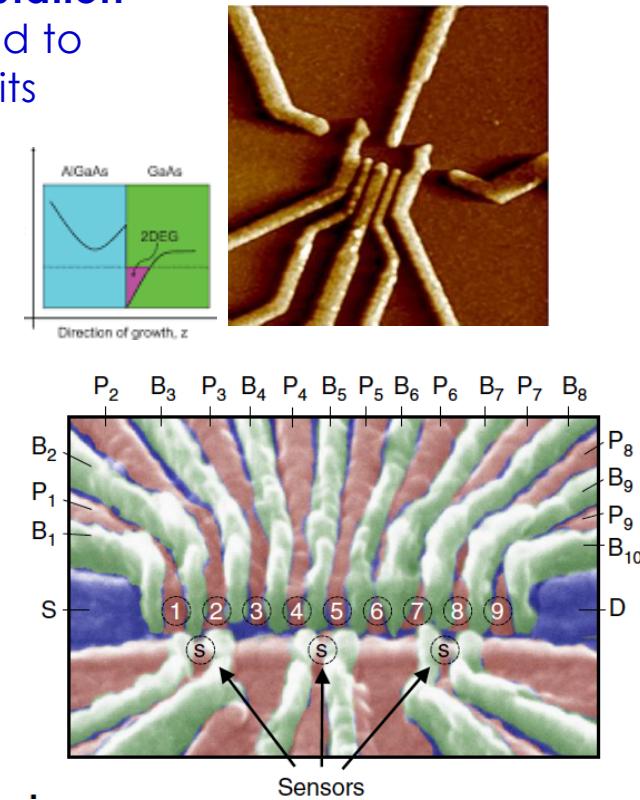
- From the Loss and DiVincenzo proposal: “**Quantum computation with quantum dots**” (PRA 1998), more than 10 years devoted to double quantum dots to implement one qubit and two qubits operations



M.C. Rogge  
et al., PRB  
2008



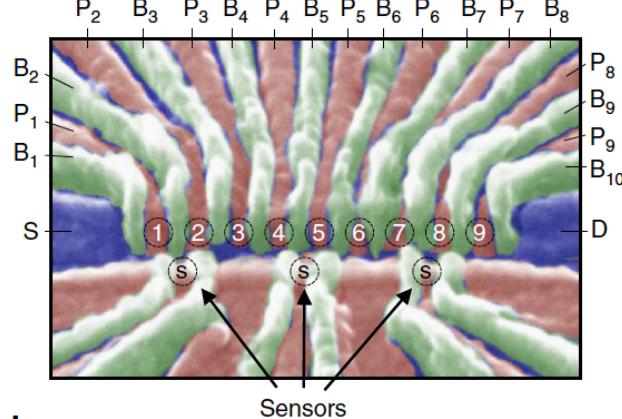
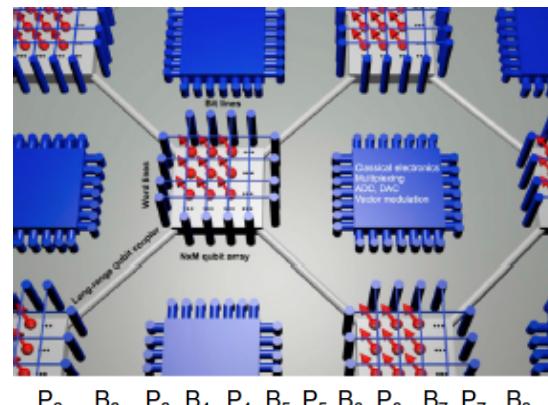
Granger et al.,  
PRB 2010



Zajac et al.,  
Phys. Rev Appl. (2016)

# QDs as a Platform for a Quantum Computer

Vandersypen, npj,  
Quantum Info, 2017



Zajac et al.,  
Phys. Rev Appl. 2016

Communication between distant sites in a quantum chip

Fast and High-fidelity transfer:  
robust against different noise sources

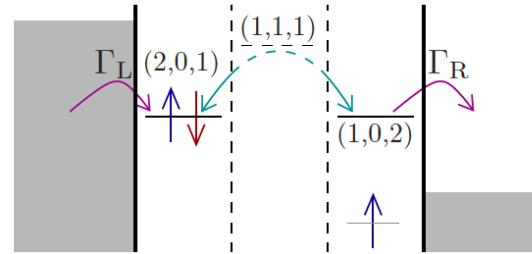
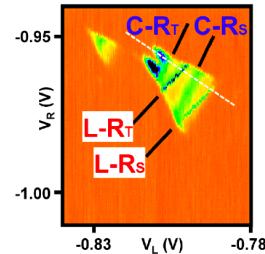
Experimentally feasible driving pulses

Distribution of entangled particles

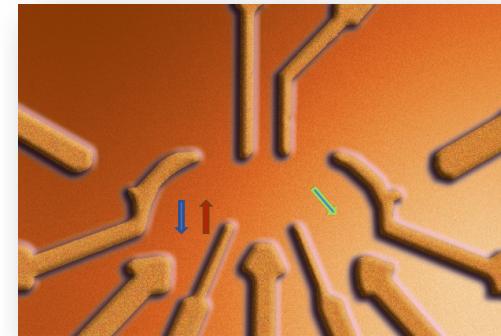
# Quantum State Transfer in QDs

M. Busl et al., Nature Nanotech, 8, 262 (2013)

R. Sánchez et al., PRL 112, 176803 (2014)



$$|LR\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow,0,\uparrow\rangle - |\uparrow,0,\uparrow\downarrow\rangle)$$



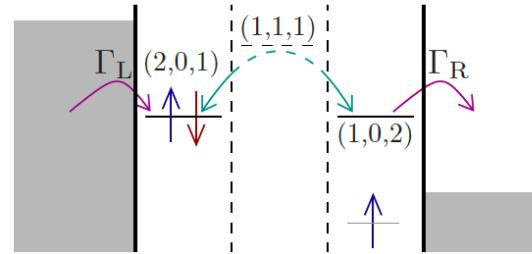
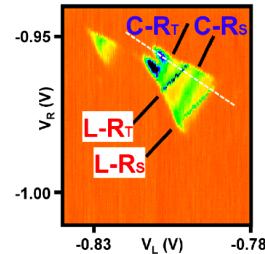
NRC, Ottawa  
A. Sachrajda

As an electron tunnels from one extreme to the other an arbitrary spin state  $\psi$  is transferred in the opposite direction

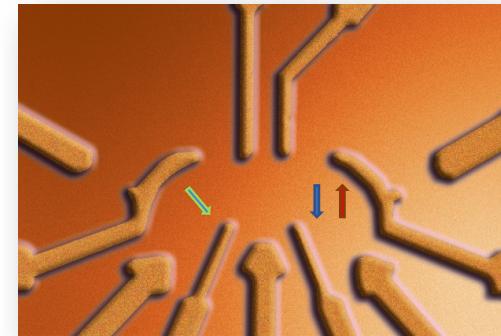
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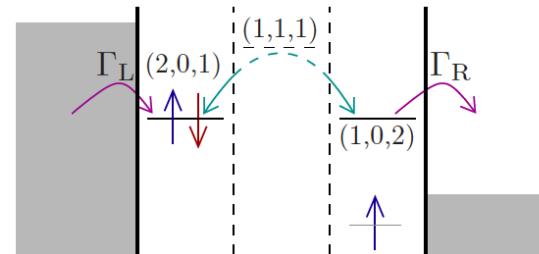
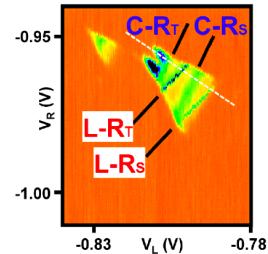
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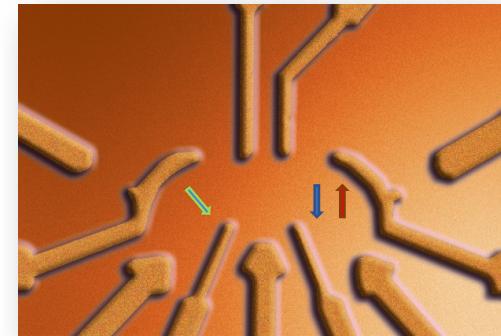
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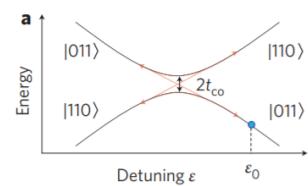
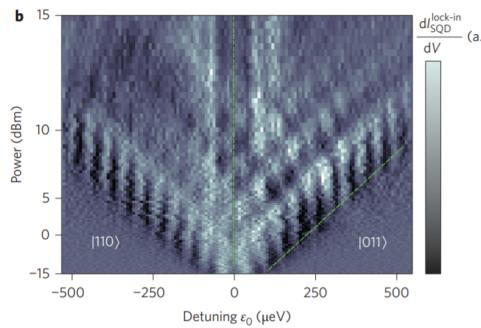
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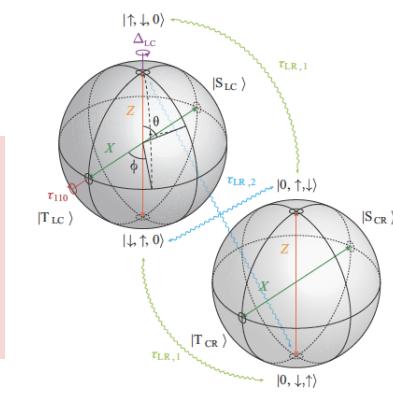
As an electron tunnels from one extreme to the other an arbitrary spin state  $\psi$  is transferred in the opposite direction

## □ Long range photo-assisted tunneling



F. Braakman et al.  
Nature Nanotech  
2013

J. Picó-Cortés, F. Gallego-Marcos  
and G. P., PRB, 99, 155421 (2019)  
P. Stano et al., PRB 2015  
F. Gallego et al., JAP 2015  
F. Gallego et al., PRB 2017



$$|\Psi_L\rangle = \cos(\theta_L/2)|\uparrow, \downarrow, 0\rangle + e^{i\phi_L} \sin(\theta_L/2)|\downarrow, \uparrow, 0\rangle$$

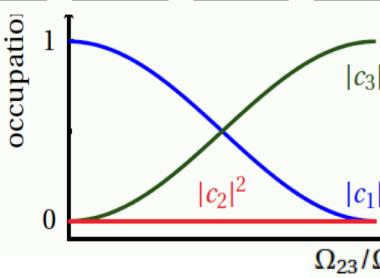
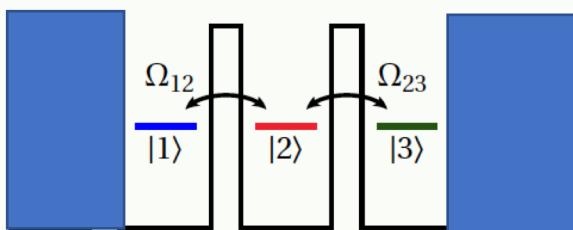
# Quantum State Transfer in QDs

CTAP

A. Greentree et al., PRB, 70, 235317 (2004)

$$\Omega_{12}(t) = \Omega^{\max} \exp \left[ - \left( t - \frac{t_{\max} + \sigma}{2} \right)^2 / (2\sigma^2) \right]$$

$$\Omega_{23}(t) = \Omega^{\max} \exp \left[ - \left( t - \frac{t_{\max} - \sigma}{2} \right)^2 / (2\sigma^2) \right]$$



Dark State

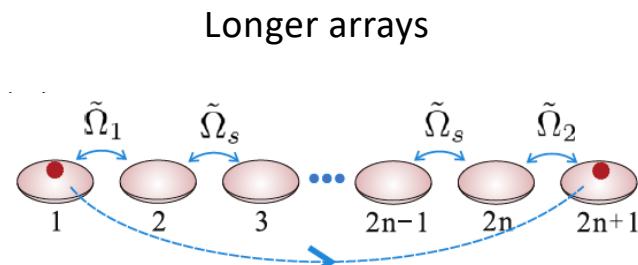
$$\varepsilon = 0$$

$$|\varphi\rangle = |D_0\rangle = \cos\theta|1\rangle - \sin\theta|3\rangle$$

$$\theta = \arctan(\Omega_{12} / \Omega_{23})$$

J. Huneke et al., PRL  
110,036802 (2013)

$$|\mathcal{E}_0 - \mathcal{E}_{\pm}| \gg |\langle \dot{\mathcal{D}}_0 | \mathcal{D}_{\pm} \rangle|$$



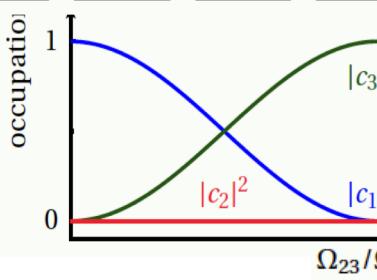
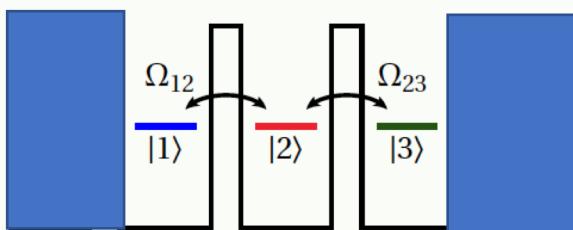
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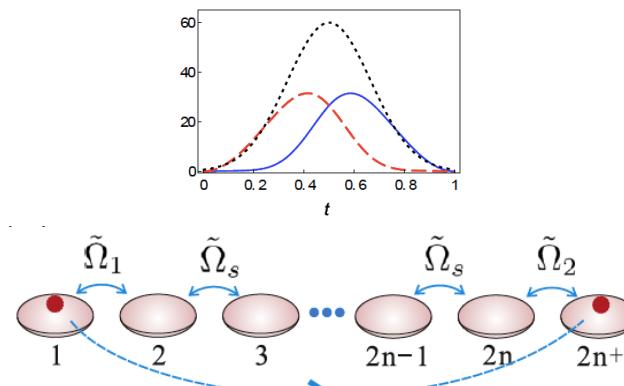
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$$|\phi_0\rangle = \cos\theta|1\rangle - (-1)^n \sin\theta|2n+1\rangle - X \left[ \sum_{j=1}^n (-1)^{j+1}|2j-1\rangle \right], \quad \tan\theta = \Omega_1/\Omega_2$$

In the dark state, dots in the even order remain empty.

Undesirable population in the dots 3<sup>th</sup>, 5<sup>th</sup>, ..., 2n – 1<sup>th</sup> can be effectively limited.

$$X = \frac{\Omega_1 \Omega_2}{\Omega_s \sqrt{\Omega_1^2 + \Omega_2^2}} \ll 1, \quad \Omega_{s0} \gg \Omega_0$$

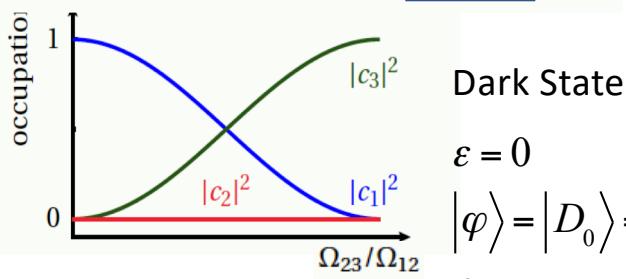
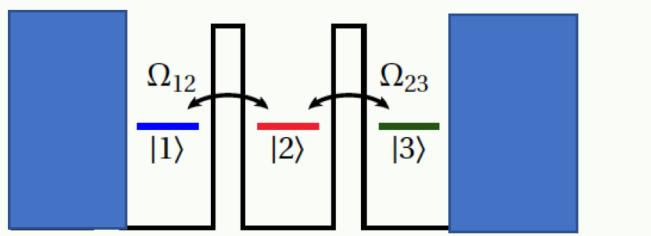
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110,036802 (2013)

Shortcuts to Adiabaticity (STA): versatile ways to speed up adiabatic passages.

(D. Guéry-Odelin et al., Rev. Modern Phys., 91, 045001, 2019)

Inverse Engineering: impose the desired evolution of the occupation and infer from it the time evolution of the parameters.

$$\tilde{H}(t) = \tilde{\Omega}_{12}(t)c_1^+c_2 + \tilde{\Omega}_{23}(t)c_2^+c_3 + h.c$$

$$|\Psi(t)\rangle = \cos\chi \cos\eta|1\rangle - i \sin\eta|2\rangle - \sin\chi \cos\eta|3\rangle$$

$$\text{Boundary conditions } \chi(0) = 0, \chi(t_f) = \pi/2, \eta(0) = 0, \eta(t_f) = 0$$

+ Ansatz for  $\chi, \eta$

$$i\hbar\partial_t \Psi(t) = \tilde{H}(t)\Psi(t) \longrightarrow \tilde{\Omega}_{12}(t), \tilde{\Omega}_{23}(t)$$

Y. Ban, et al., Nanotechnology, 29, 505201 (2018)

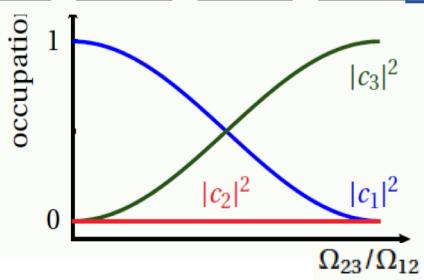
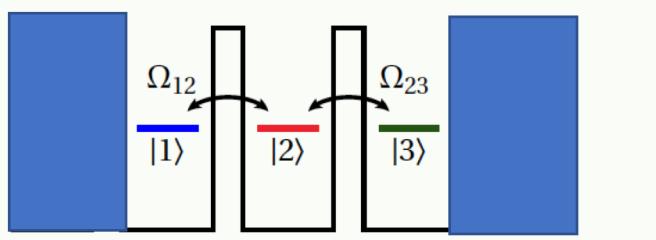
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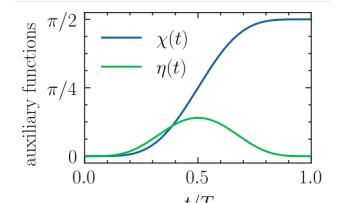
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+ Ansatz for  $\chi, \eta$

$$\begin{aligned} \chi(t) &= \pi \frac{t}{2T} - \frac{1}{3} \sin\left(\frac{2\pi t}{T}\right) + \frac{1}{24} \sin\left(\frac{4\pi t}{T}\right) \\ \eta(t) &= \arctan(\dot{\chi}/\alpha) \end{aligned}$$

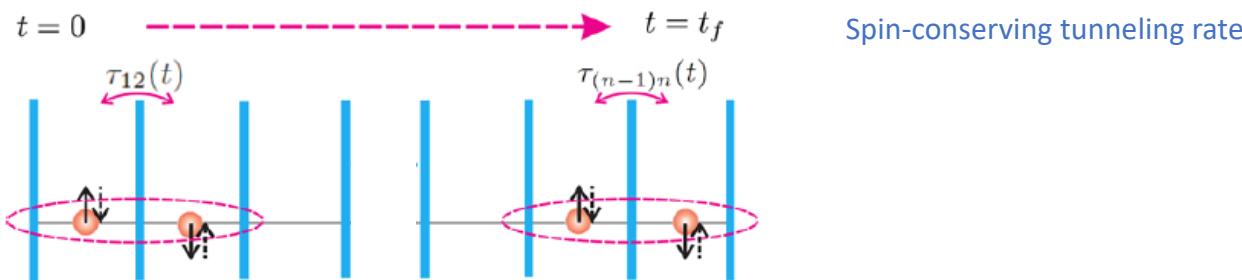
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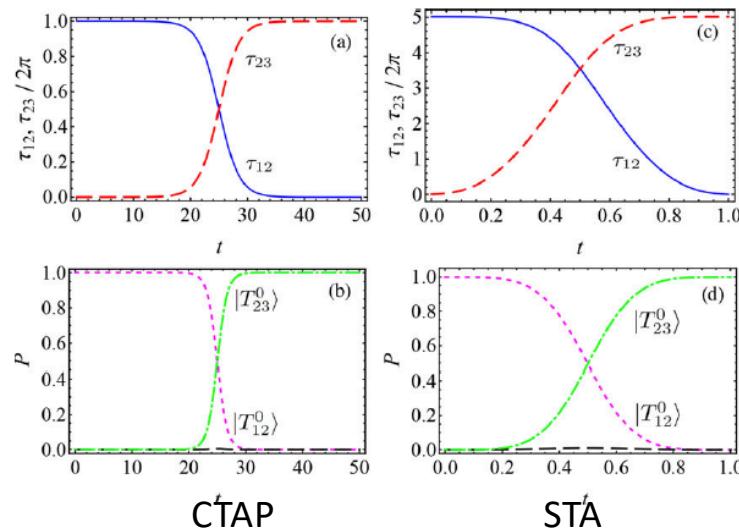
Y. Ban, et al., Nanotechnology, 29, 505201 (2018)

# Quantum State Transfer in QDs

$$H = \sum_i \varepsilon_i n_i + U \sum_i n_{i\uparrow} n_{i\downarrow} + E_Z \sum_i (n_{i\uparrow} - n_{i\downarrow}) - \sum_{i,\sigma} \left( \tau_{N,i} c_{i\sigma}^\dagger c_{i+1\sigma} + \text{h. c.} \right)$$



$$|T_{ij}^0\rangle = (|\uparrow_i\downarrow_j\rangle + |\downarrow_i\uparrow_j\rangle)/\sqrt{2}$$



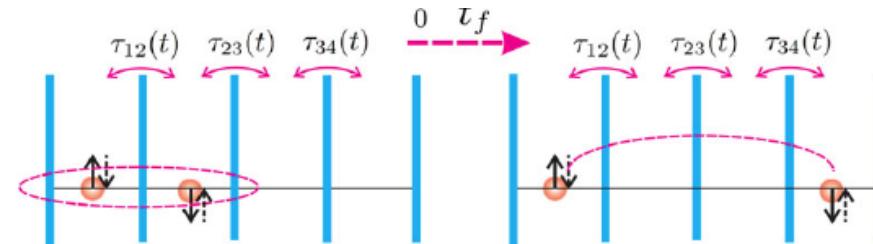
Y. Ban, et al., Advanced Quantum Tech., 1900048 (2019).

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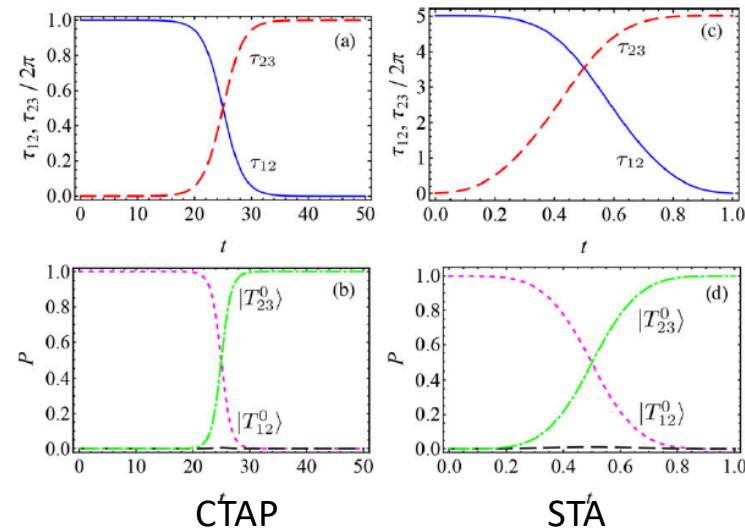
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Spin-conserving tunneling rate

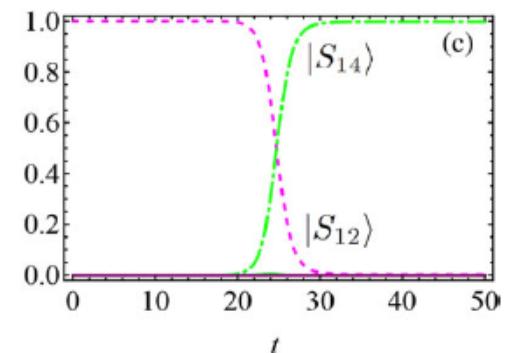
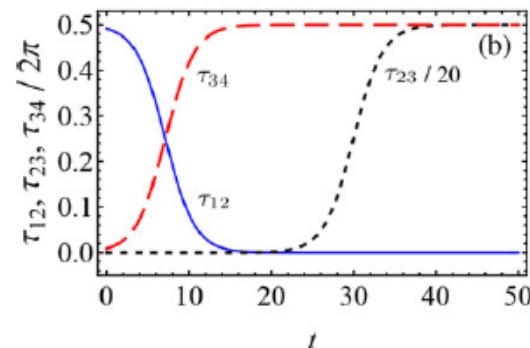


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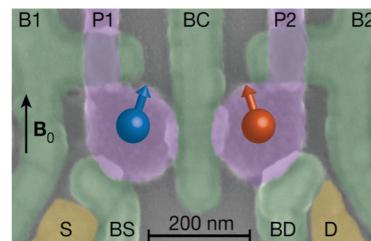
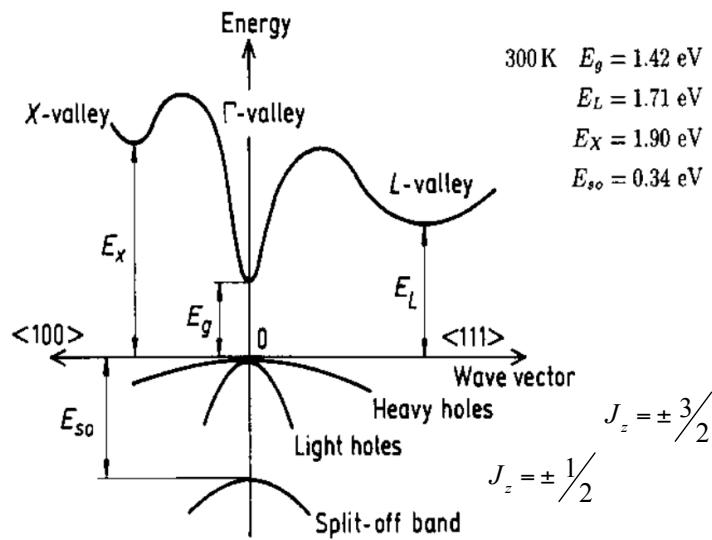
$$|S_{12}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$$

$$|S_{14}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_4\rangle - |\downarrow_1\uparrow_4\rangle)$$



## Hole Spin Qubits

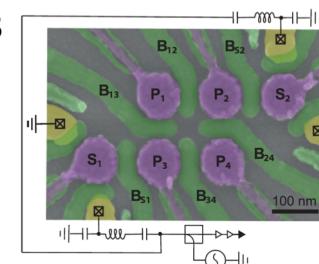
- Weak Hyperfine Interaction
- Long spin decoherence and relaxation times
- Strong Spin-Orbit interaction
- Fast quantum operations (EDSR)
- High fidelity in the one and two quantum bits operations



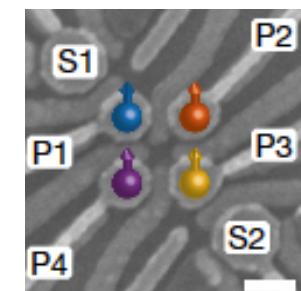
Recent advances in hole-spin qubits

Yinan Fang et al 2023  
Mater. Quantum. Technol.

N.W. Hendrickx et al.,  
Nature, 2020



M. Veldhorst, et. al.  
Appl. Phys. Lett. **118**, 2021



N. W. Hendrickx et al., Nature 2021

# long-range hole spin transfer

$$H = \sum_i \varepsilon_i n_i + U \sum_i n_{i\uparrow} n_{i\downarrow} + E_Z \sum_i (n_{i\uparrow} - n_{i\downarrow}) - \boxed{\sum_{i,\sigma} \left( \tau_{N,i} c_{i\sigma}^\dagger c_{i+1\sigma} + \text{h. c.} \right)} + \boxed{\sum_i \left[ -\tau_{F,i} \left( c_{i\uparrow}^\dagger c_{i+1\downarrow} - c_{i\downarrow}^\dagger c_{i+1\uparrow} \right) + \text{h. c.} \right]}$$

Spin-conserving tunneling rate

Spin-Orbit interaction



Spin-flip tunneling rate

$$H_{\text{SOC}} = \boxed{i\alpha E_\perp (\sigma_+ p_-^3 - \sigma_- p_+^3)} - \boxed{\beta (\sigma_+ p_- p_+ p_- + \sigma_- p_+ p_- p_+)}$$

**Rashba** (structure inversion asymmetry)   **Dresselhaus** (bulk inversion asymmetry)

Burkard, Phys. Rev. Res. 2021

$$H_{\text{SOC}} = \sum_{i \neq j} \sum_{\sigma \neq \sigma'} \left( t_{F,ij} e^{i\vartheta_{ij}} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma'} + \text{H.c.} \right)$$

**Due to spin-orbit coupling there is an effective spin-flip tunneling rate**

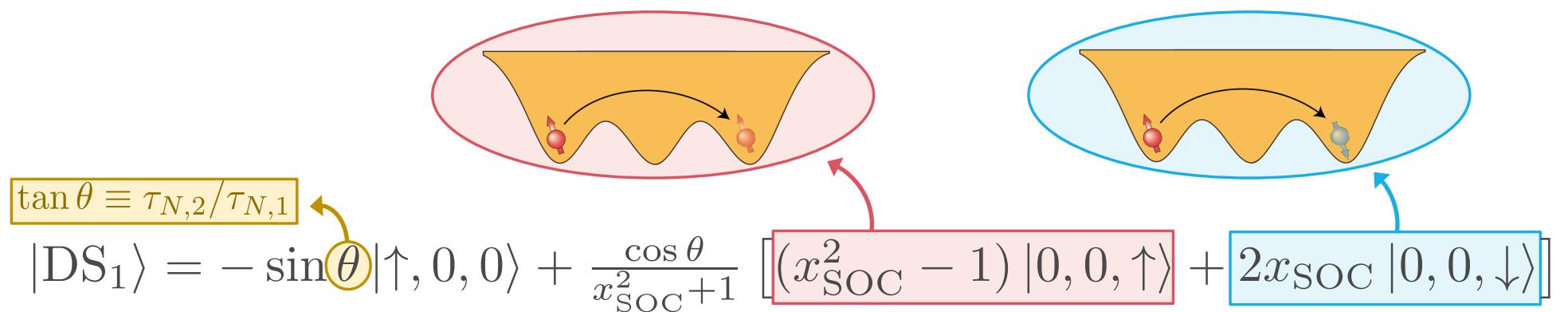
# Dark State TQD

## Triple Quantum Dot

All levels in resonance:  $E_Z = 0$        $\varepsilon_i = 0$

- Proportional **spin-flip** and **spin-conserving** tunneling rates

$$\tau_{F,i}(t)/\tau_{N,i}(t) = x_{\text{SOC}}$$

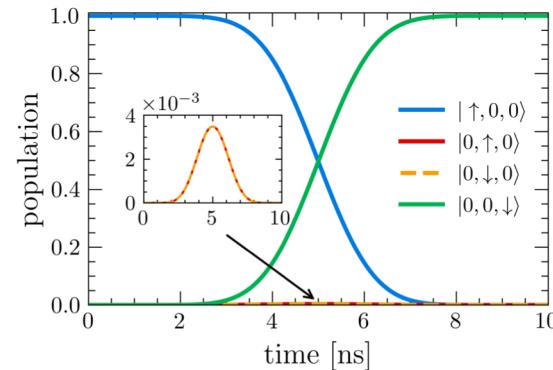
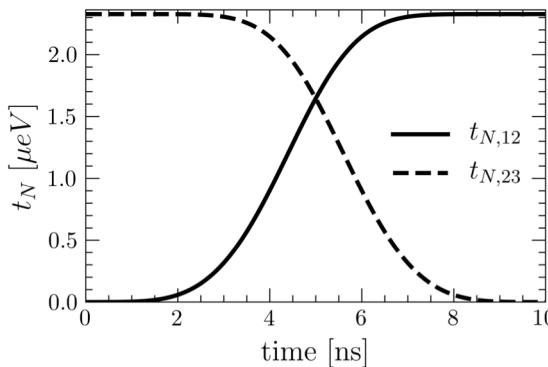


- Using the dark state, we connect **distant sites with minimal population in the middle site**
- The final **spin projection is controlled via the SOC**

# long-range hole spin transfer

STA

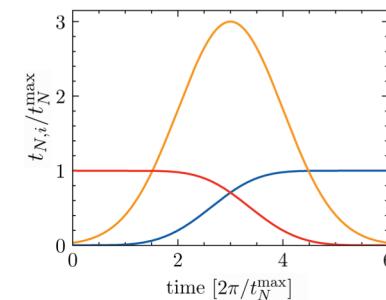
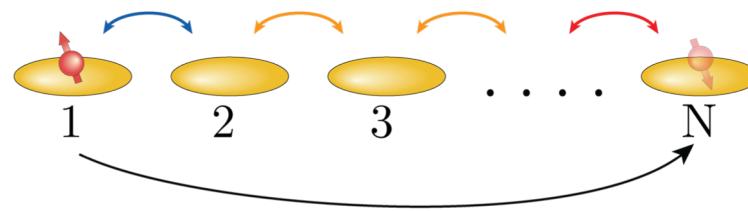
## 1 Hole spin transfer in a TQD



$x_{\text{SOC}}=1$

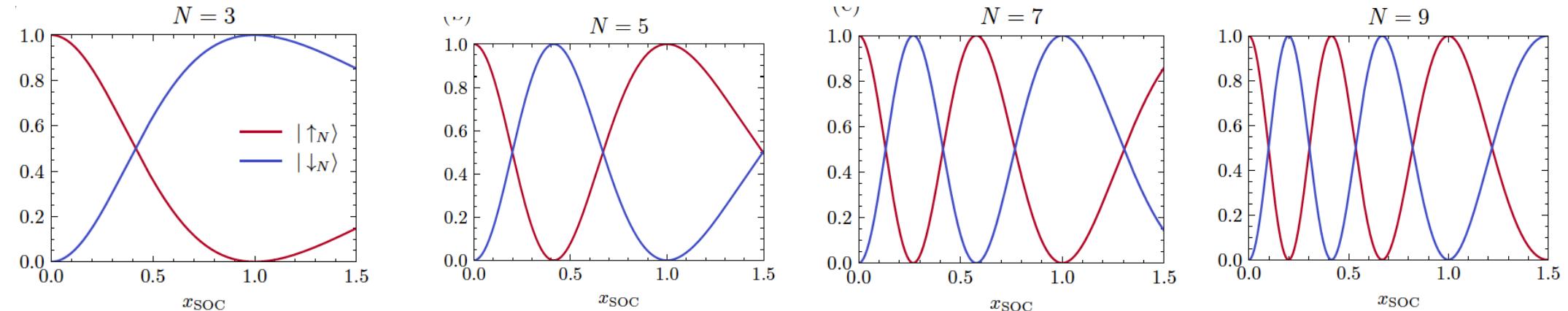
## 1 Hole spin transfer in N QDs Arrays

N-dots arrays ( $N > 3$ )



# long-range hole spin transfer

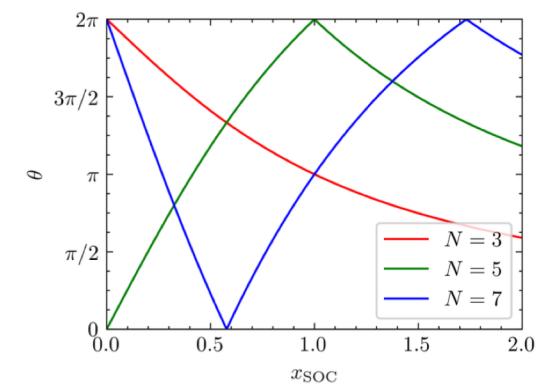
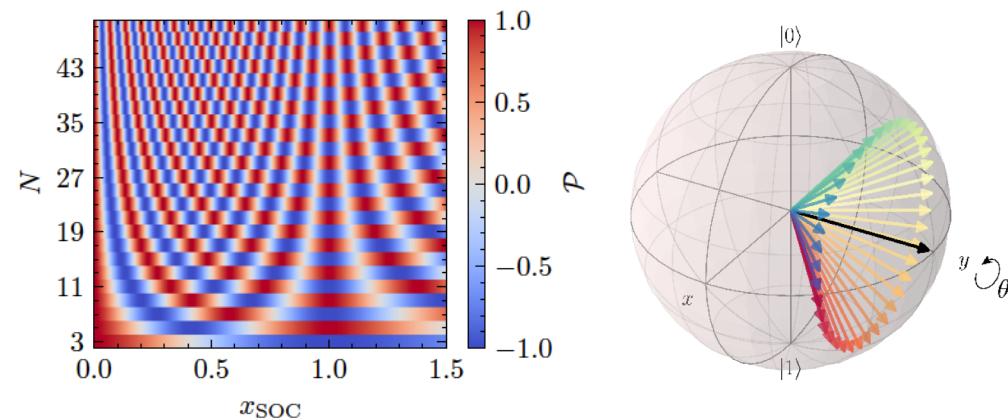
## 1 Hole spin transfer in N QDs Arrays (STA)



Initial state:  $|\uparrow_1\rangle$

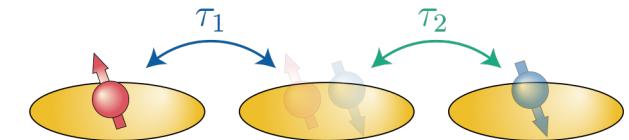
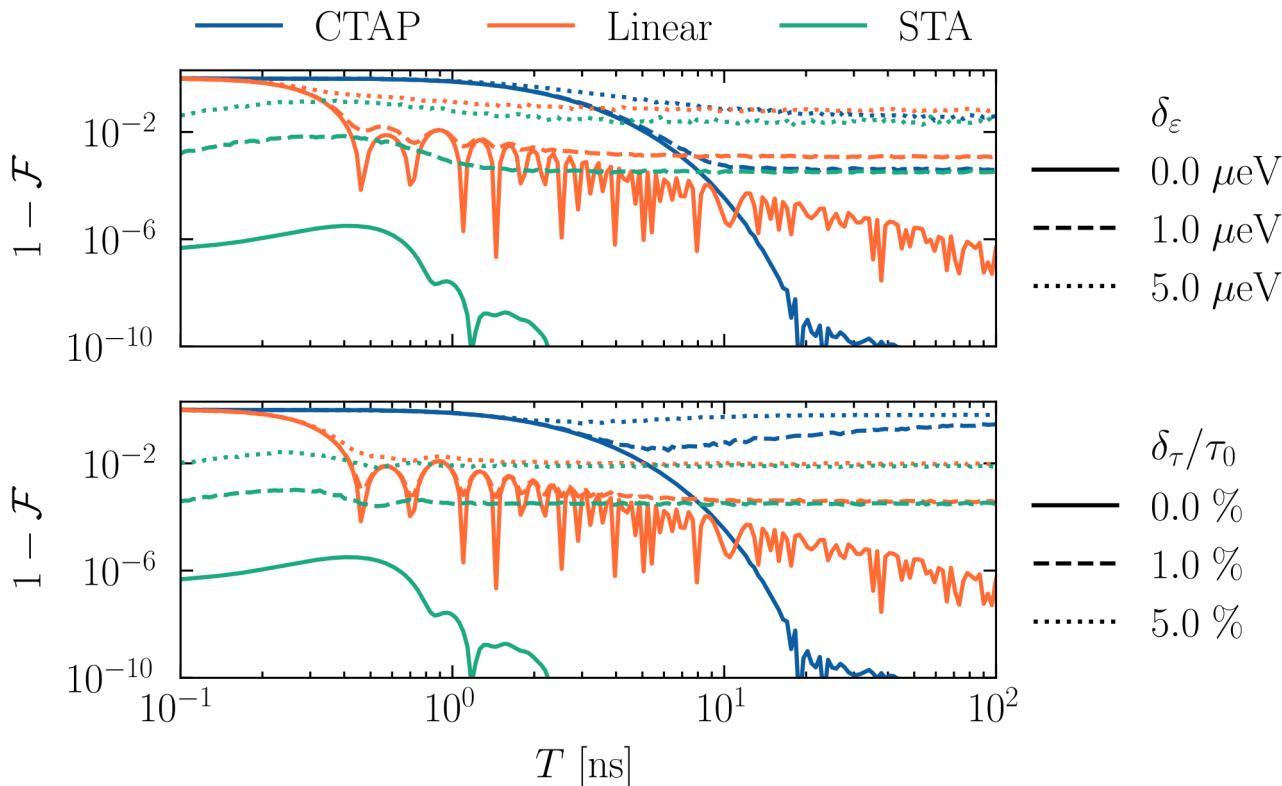
Tuning  $x_{\text{SOC}} = \tau_{F,i}/\tau_{N,i}$

Control of the spin polarization



# Noisy transfer

- Transfer fidelity defined as:  $\mathcal{F} \equiv |\langle 0, 0, \downarrow | \Psi(T) \rangle|^2$



Parameters	
$x_{\text{SOC}}$	= 1
$\sigma$	= $T/6$
$f_{\min}$	= 0.16 mHz
$f_{\max}$	= 0.1 MHz
$\tau_0$	= 10 $\mu\text{eV}$

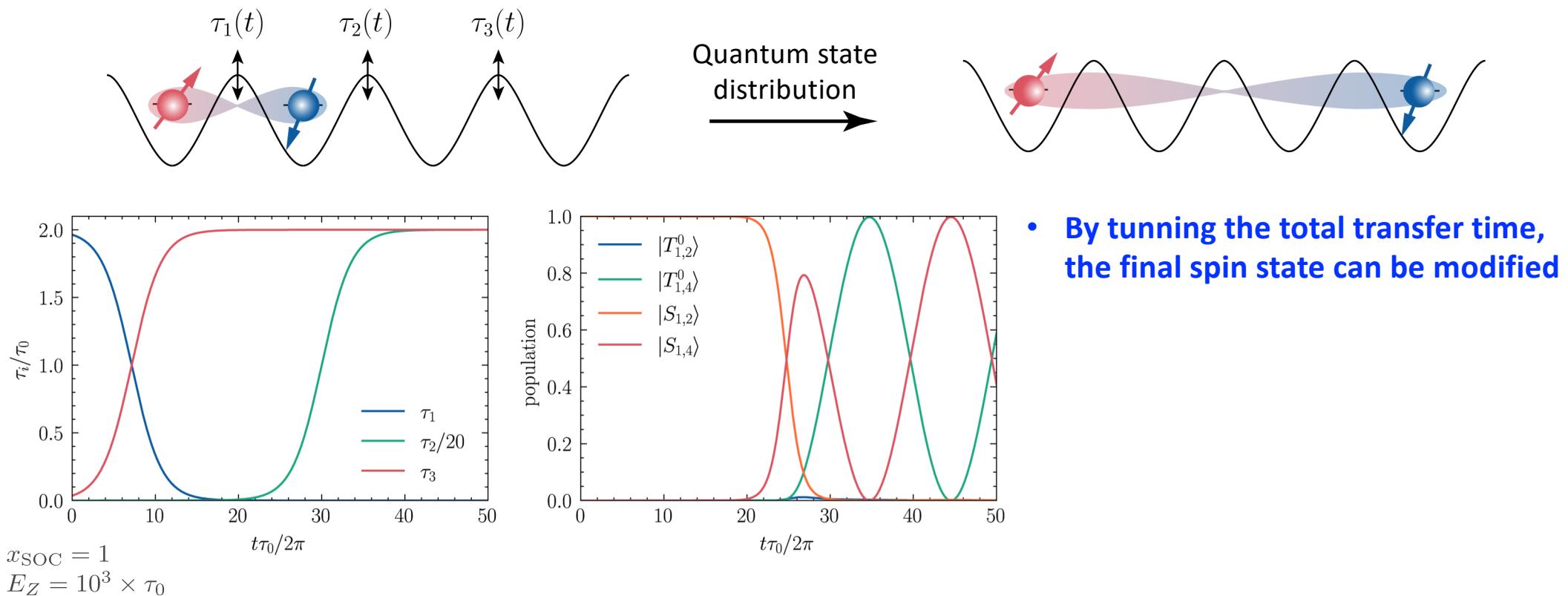
- CTAP** highly sensitive to error in the tunneling rates
- Linear** pulse obtain good results if the transfer time is large enough
- STA** is the best among all the protocols for low transfer times

# Quantum State Distribution

4 dots

- To communicate between distant quantum processors, we must be able to distribute entangled pairs

(Yue Ban, et al., Adv. Quant. Tech. 2, 1900048 (2019))

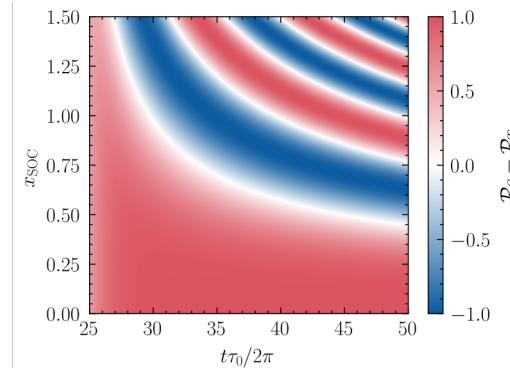
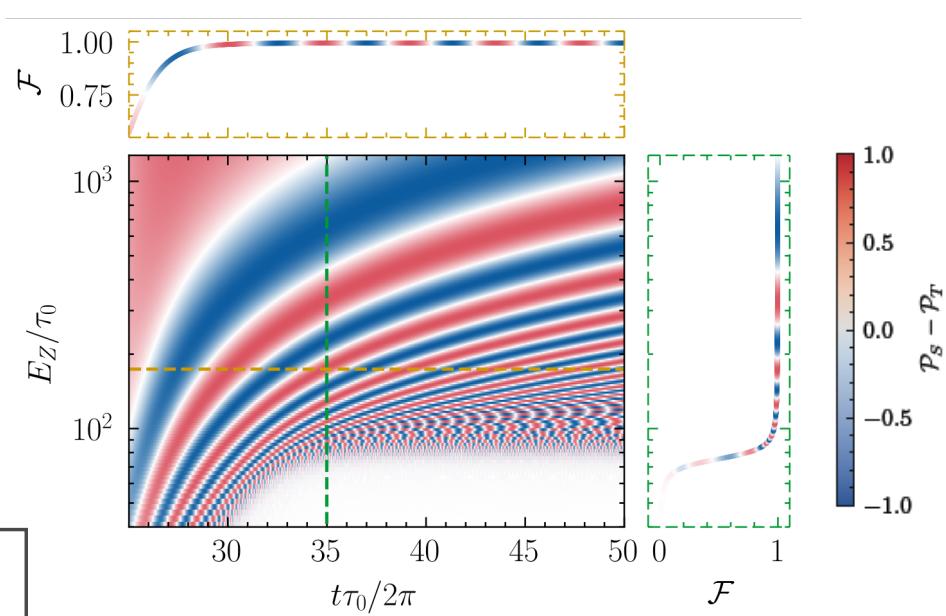


- By tuning the total transfer time, the final spin state can be modified

# Quantum State Distribution

Initial state  $|S_{1,2}\rangle$

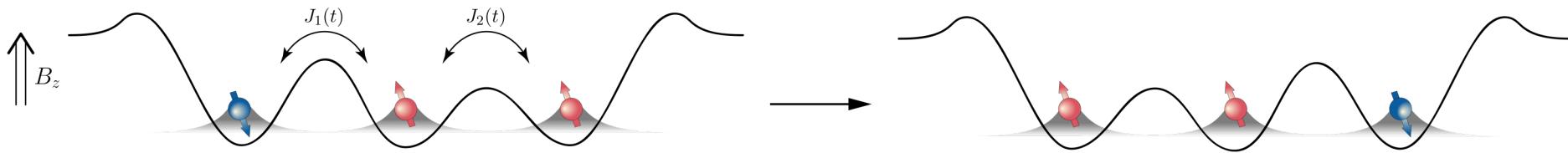
- $\mathcal{P}_S(t) \equiv |\langle S_{1,4} | \Psi(t) \rangle|^2$     ○ Spin polarization between ends of the QD chain
- $\mathcal{P}_T(t) \equiv |\langle T_{1,4}^0 | \Psi(t) \rangle|^2$
- $\mathcal{F} \equiv \mathcal{P}_S(T) + \mathcal{P}_T(T)$     ○ Transfer fidelity



Electric and magnetic control for the final spin projection of the entangled pair

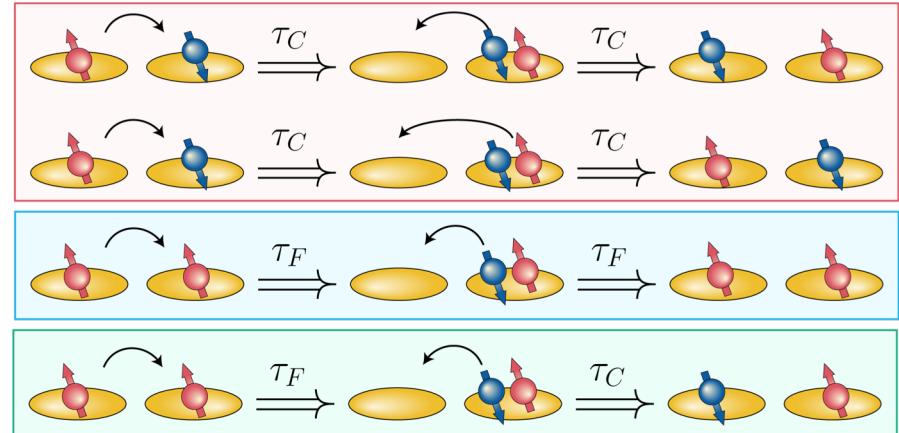
# Triple QD: HH half filling

- We can transfer quantum information by **moving the spin**, and not the charge

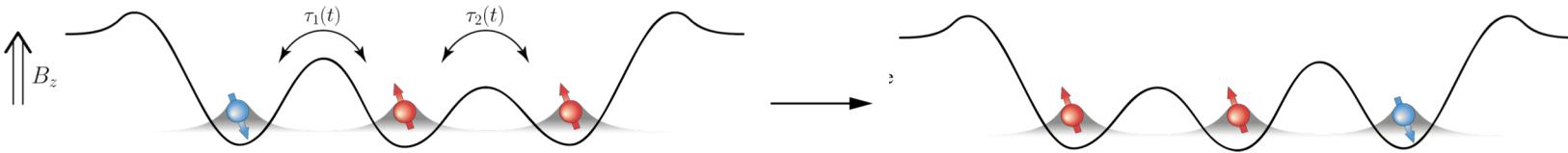


$$\begin{aligned}
 H_{\text{eff}} = & E_Z \sum_i^N \sigma_z^i + \sum_i^{N-1} [J_i^{NN} (\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1} + \sigma_z^i \sigma_z^{i+1} - 1/4) \\
 & + J_i^{FF} (-\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1} - \sigma_z^i \sigma_z^{i+1} - 1/4) \\
 & + 2J_i^{NF} (\sigma_x^i \sigma_z^{i+1} - \sigma_z^i \sigma_x^{i+1})]
 \end{aligned}$$

$$J_i^{ab} \equiv \tau_i^a \tau_i^b / U$$



# Triple QD: HH half filling



For  $E_Z \gg J^{ab}$  the subspaces with a total fixed spin projection are far apart in energy

$$E_Z/J_0 = 100 \quad S_z = +1/2$$

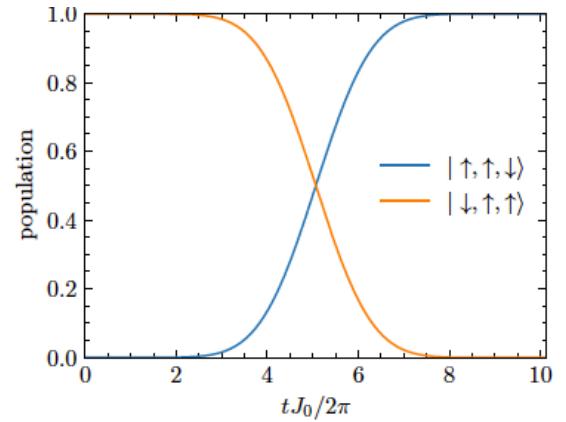
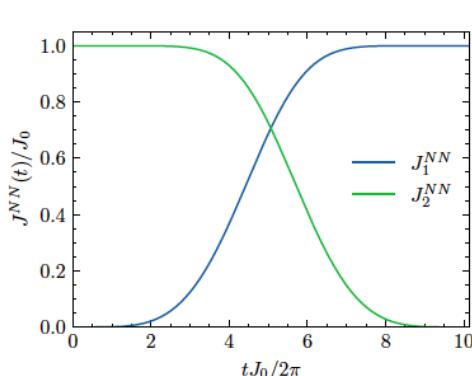
**STA**

$$H_{\text{eff}} = \begin{pmatrix} -J_1^{NN} - J_2^{FF} & J_1^{NN} & 0 \\ J_1^{NN} & -J_1^{NN} - J_2^{NN} & J_2^{NN} \\ 0 & J_2^{NN} & -J_2^{NN} - J_1^{FF} \end{pmatrix}$$

$$J^{ab} \equiv \tau_a \tau_b / U \quad a, b = \{N, F\}$$

$$J_0 \equiv \max(J_1(t), J_2(t))$$

$$J_i^{NN} = J_i^{FF} = J_i$$



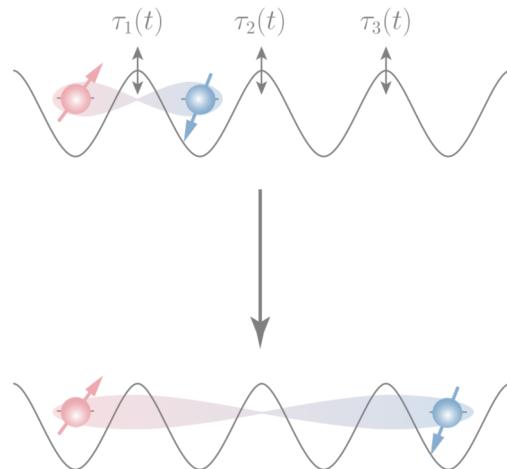
$$|\text{DS}\rangle = \sin \theta |\downarrow, \uparrow, \uparrow\rangle - \cos \theta |\uparrow, \uparrow, \downarrow\rangle$$

$$\tan \theta \equiv J_2/J_1$$

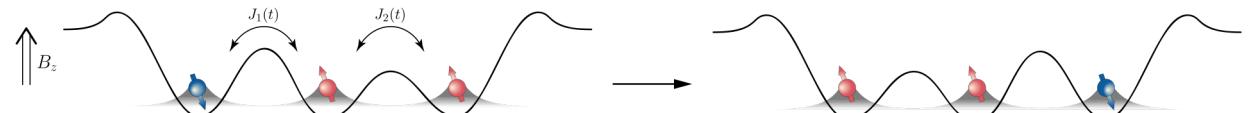
D. Fernández et al., in progress

# Summary

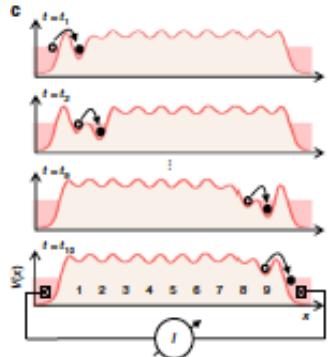
- We have applied **STA** techniques to **hole spin qubits**, with **strong SOC**
- **SOC** provides a **new control parameter** for the long-range **quantum information transfer**



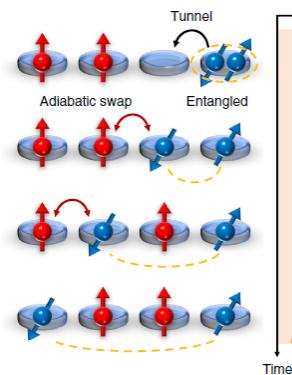
- We can perform a **one-qubit gate in parallel** to the state transfer
- Strong SOC also allows for **quantum state distribution** in large arrays
- Long-range **spin swapping**



# Quantum State Transfer in QD arrays



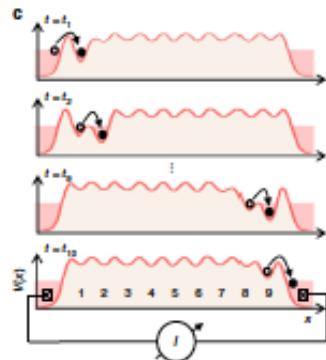
Nakajima et. al.,  
Nat. Comm. 2018



Adiabatic quantum state transfer in a semiconductor quantum-dot spin chain, Y. P. Kandel et al., Nature Comm. 2021

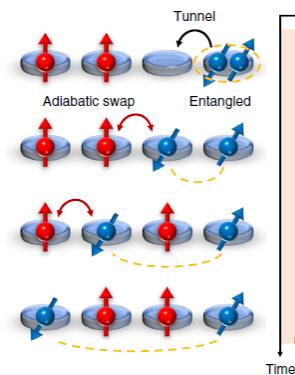
Coherent transport of spin by adiabatic passage in quantum dot arrays, MJ Gullans, J. Petta, PRB 2020

# Quantum State Transfer in QD arrays



A.R. Mills e. al, Nature Comm, 2019

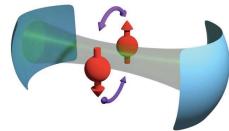
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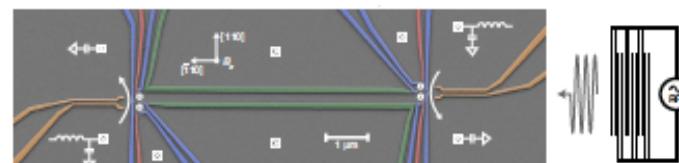
## A Coherent Spin-Photon Interface in Silicon



X. Mi et al., Nature, 2018

## QDs and SAW: Distant spin entanglement via fast and coherent electron shuttling,

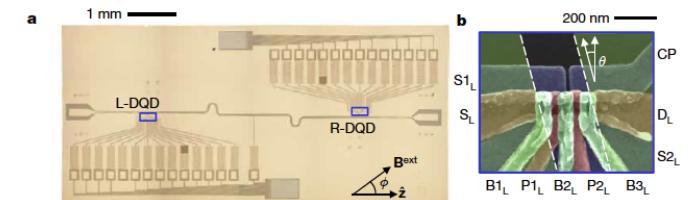
Jadot et al., , Nature Nanotech, 2021



## Strong coupling between a photon and a hole spin in silicon, CX Yu et al, 2022

### Towards cavity mediated spin-spin coupling

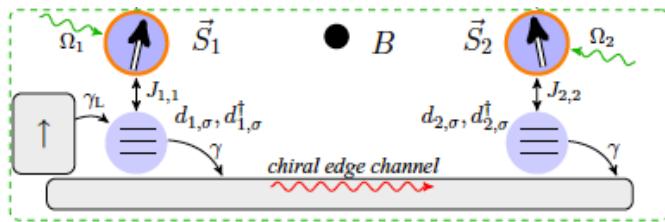
F. Borjans et al., Nature, 2020



# Quantum state transfer by topological edge states

Mesoscopic One-Way Channels for Quantum State Transfer via the Quantum Hall Effect

Stace et al., PRL, 2004



**Long-distance entanglement of spin qubits via quantum Hall edge states,** G. Yang et al., PRB 2016

**Long-range entanglement generation between electronic spins,** M. Benito et al., PRB 2016

**Entangling Nuclear Spins in Distant Quantum Dots via an Electron Bus,** M. Bello et al., Phys. Rev. Applied, 2022.

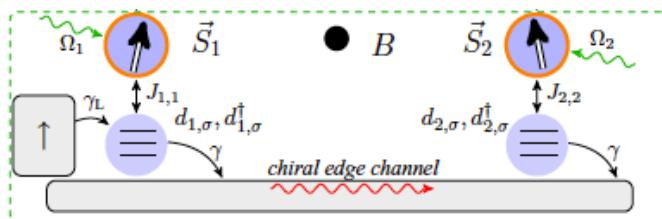
## Simulation of a 1D topological insulator in a driven quantum dot array

B. Pérez-González et al., PRL 2019

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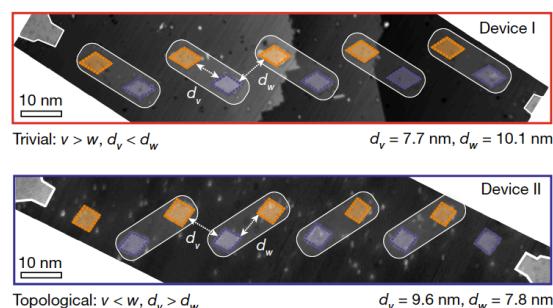
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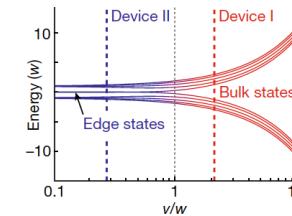
### A dimer chain



Engineering topological states in atom-based semiconductor quantum dots

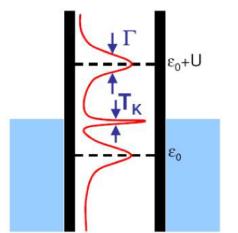
M. Kiczynski et al.  
Nature, June 2022

Floquet theory:  
see M. Grifoni and P. Hanggi,  
Phys. Reports, **304** (1998) 229–354



# QDs as Quantum Simulators

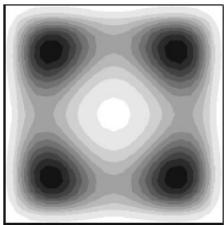
A Tunable Kondo Effect in Quantum Dot  
S. Cronnenwelt et al., Science 98



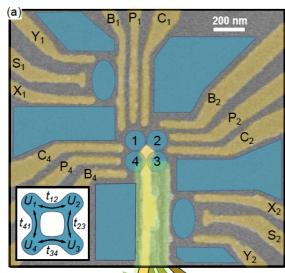
Kondo effect and SO coupling in  
Graphene QDs,  
A. Kurzman et al., Nat. Comm., 2021

Dynamical control of  
correlated states in a  
square quantum dot

C. Creffield and GP,  
PRB 2002

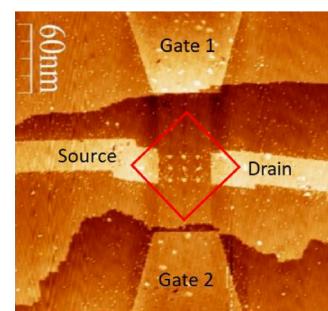


Nagaoka ferromagnetism  
observed in a QD plaquette,  
JP Dehollais et al., Nature  
2020



Quantum Simulation of Antiferromagnetic Heisenberg Chain with  
Gate defined QDs, PRX, C.J. van Diepen et al., 2021

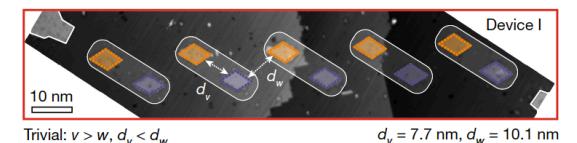
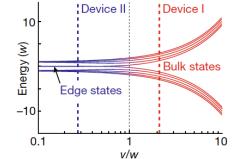
Quantum Simulation of the Fermi-Hubbard model using QD arrays,  
T. Henggels et al., Nature, 2017



Quantum Simulation of an Extended  
Fermi-Hubbard Model Using a 2D  
Lattice of Dopant-based Quantum  
Dots ,  
X. Huang et al., arXiv: 2110.08982.

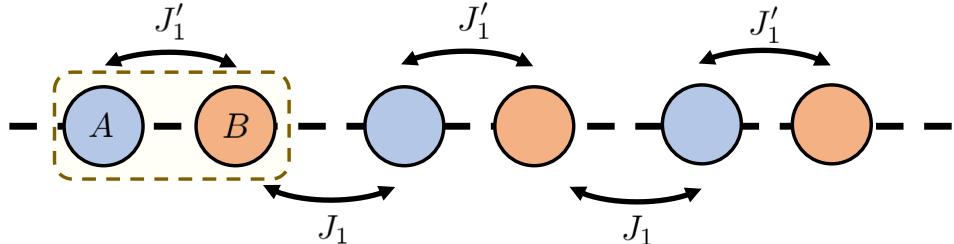
Engineering topological states in atom-based  
semiconductor quantum dots

M. Kiczynski et al.  
Nature, June 2022



# Topology

## Dimer chain (SSH model)

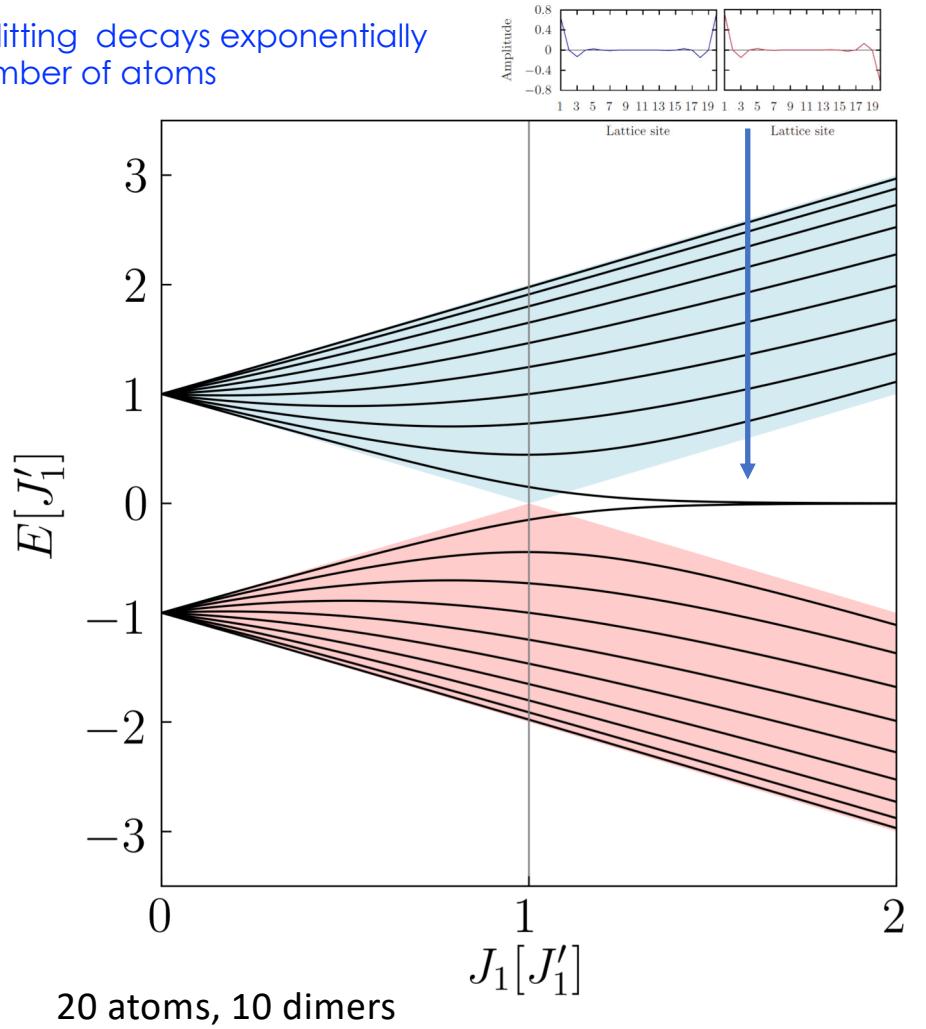


$$H = -J'_1 \sum_{i=1,\sigma}^M c_{2i\sigma}^\dagger c_{2i-1\sigma} - J_1 \sum_{i=1,\sigma}^{M-1} c_{2i+1\sigma}^\dagger c_{2i\sigma} + \text{h.c.}$$

$$\left(\frac{J'_1}{J_1}\right)_c = 1 - \frac{1}{M+1} \quad N_{\text{bulk}} = 2M \quad \frac{J'_1}{J_1} > \left(\frac{J'_1}{J_1}\right)_c$$

$$N_{\text{bulk}} = 2(M-1) \quad \frac{J'_1}{J_1} < \left(\frac{J'_1}{J_1}\right)_c$$

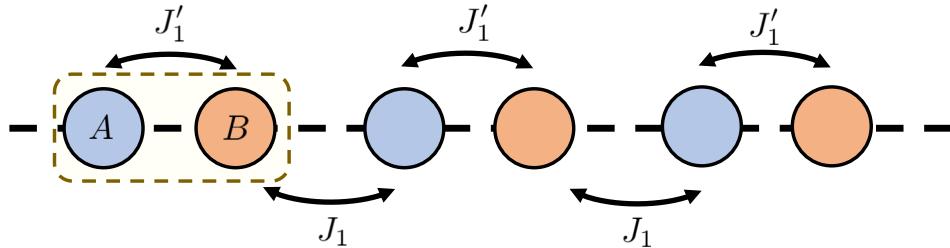
The level splitting decays exponentially with the number of atoms



"A short Course of Topological Insulators", J.K. Asboth et al., Springer

20 atoms, 10 dimers

### Dimer chain (SSH model)



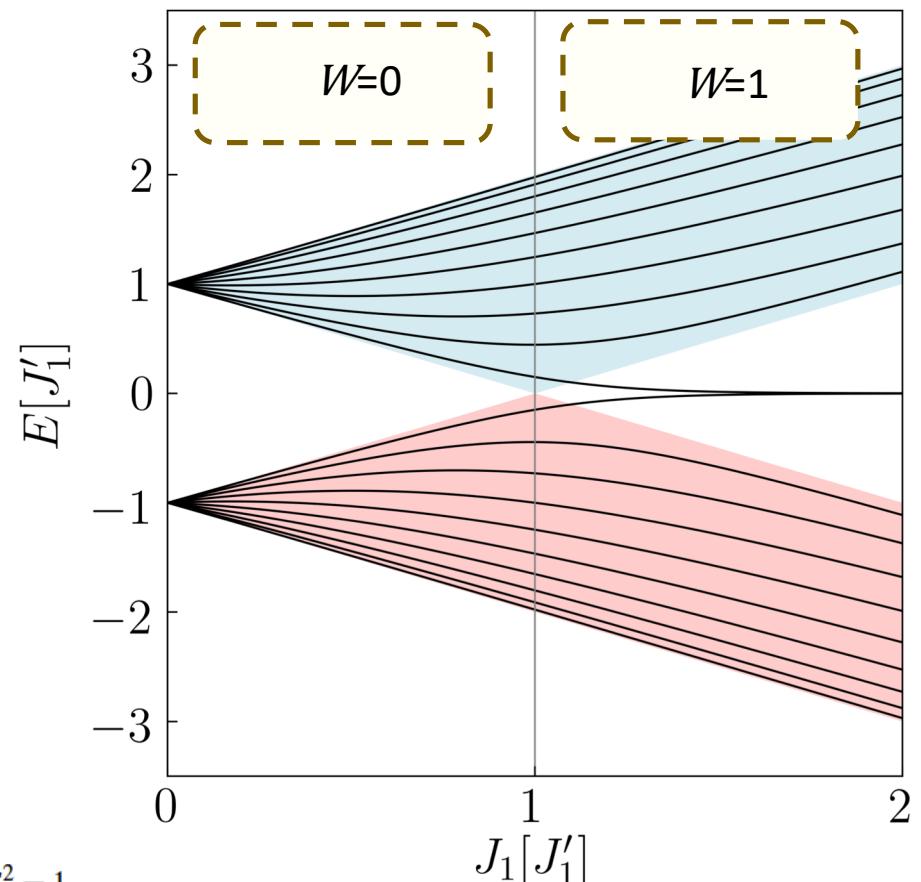
$$\mathcal{W} = \frac{1}{2\pi} \int \langle u_{\alpha,k} | i\partial_k | u_{\alpha,k} \rangle dk$$

Berry's phase picked up by a particle moving across the Brillouin zone.

Measured in cold atoms by I. Bloch and coworkers:  
Nature Physics, M. Atala et al. 2013

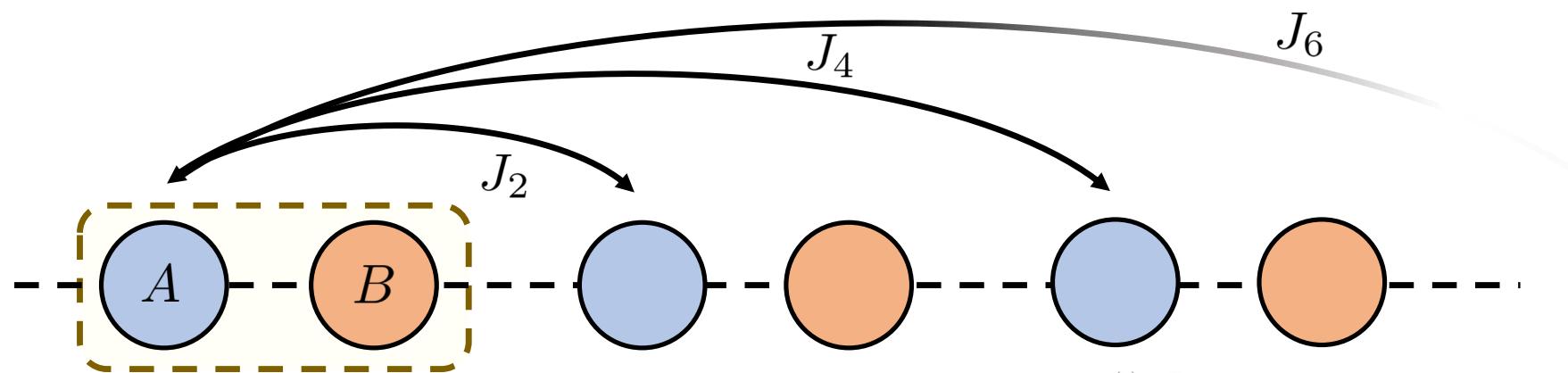
$$\hat{\Gamma} \hat{H} \hat{\Gamma}^\dagger = -\hat{H} \quad \hat{\Gamma}^\dagger \hat{\Gamma} = \hat{\Gamma}^2 = 1$$

BDI class in the Altland-Zirnbauer classification



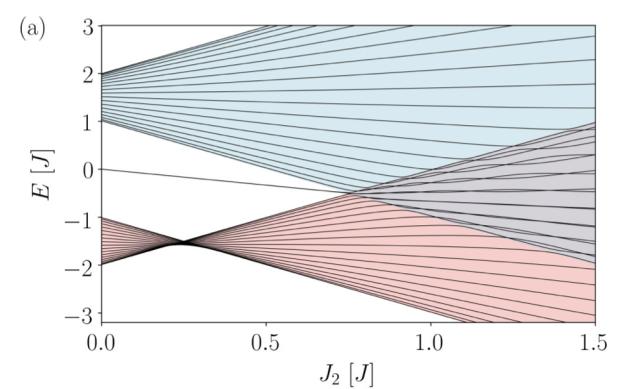
# Extended SSH model

even-neighbor hoppings



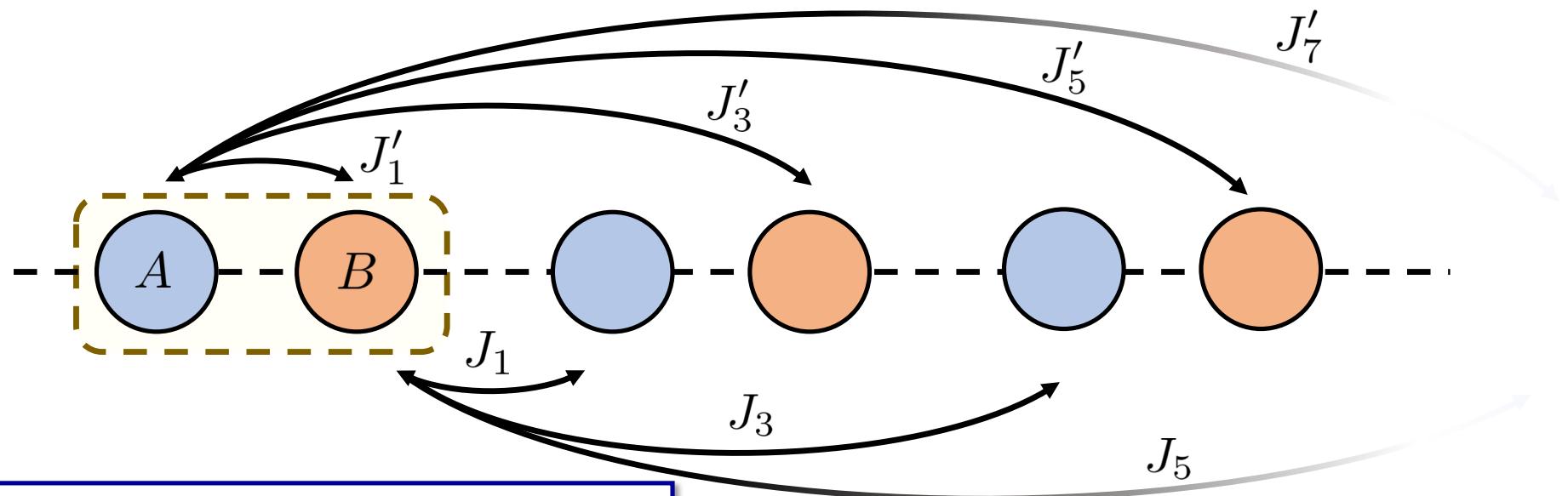
broken chiral symmetry

B. Pérez-González et al., Phys. Rev. B, **99**, 035146 (2019).



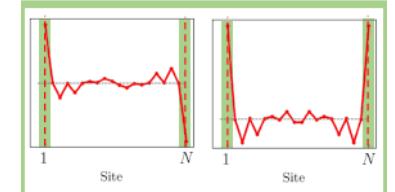
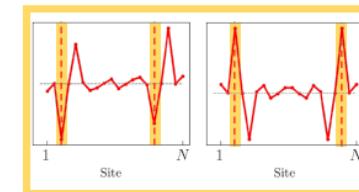
# Extended SSH model

## odd-neighbor hoppings



✓ chiral symmetry  $\mathcal{W} = 0, 1, 2, 3, \dots$

first and third  
neighbors



## What is a Floquet System?

- Exhibits discrete time translation symmetry:

$$H(t + T) = H(t)$$

- For such a system, the Floquet theorem must hold

Floquet Theorem:

$$H(t + T) = H(t) \implies \Psi_\lambda(\mathbf{r}, t) = e^{-i\omega_\lambda t} u_\lambda(\mathbf{r}, t),$$

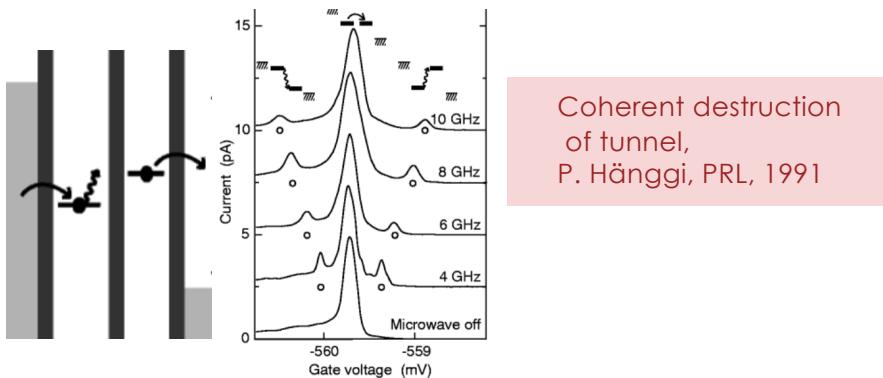
$$u_\lambda(\mathbf{r}, t + T) = u_\lambda(\mathbf{r}, t)$$

Energy  $\rightarrow$  Quasienergy  $\quad \hbar\omega_\lambda \in \left[ -\frac{\pi\hbar}{T}, \frac{\pi\hbar}{T} \right]$

(Note: totally analogous to Bloch's theorem!)

# Driving with periodic AC fields $\rightarrow H_{ac}(t)$ : Floquet Engineering

## □ Photoassisted Tunneling (PAT) in quantum dots

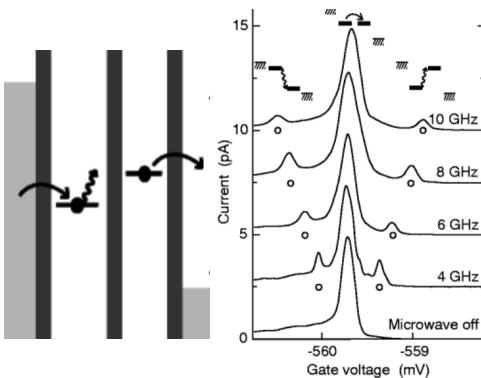


Coherent destruction  
of tunnel,  
P. Hänggi, PRL, 1991

T. H. Oosterkamp et al., Nature 395, 873-876, 1998

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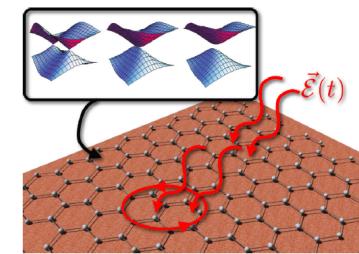


Coherent destruction  
of tunnel,  
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Bonds renormalization

T. H. Oosterkamp et al., Nature 395, 873-876, 1998

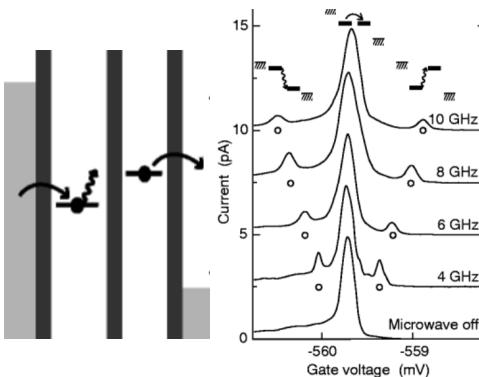
- Tuning electronic and topological properties in driven systems



P. Delplace,A.  
Gómez León, GP  
PRB 88, 245, 2013

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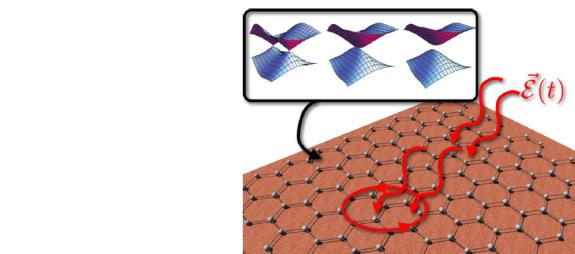


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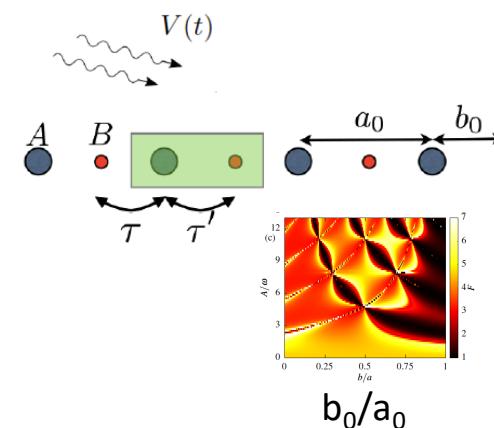
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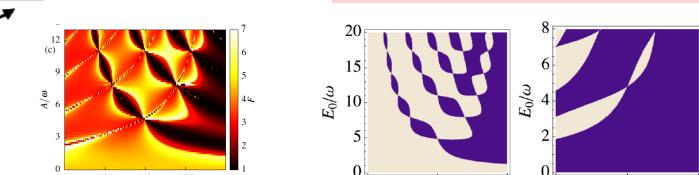
P. Delplace,A.  
Gómez León, GP  
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A. Gómez León and GP,  
B. PRL,110, 200403, 2013

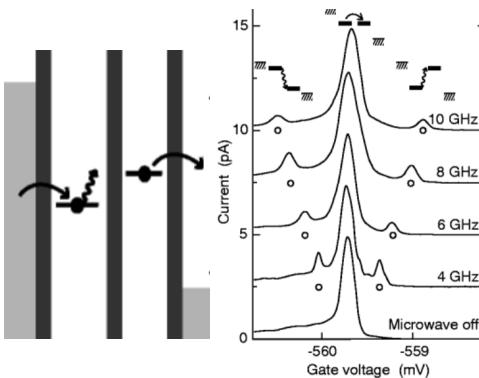
M. Niklas et al.,  
Nanotech., 2017

M. Bello, C.E. Creffield,  
GP, Scientific Rep, 6,  
22562, 2016



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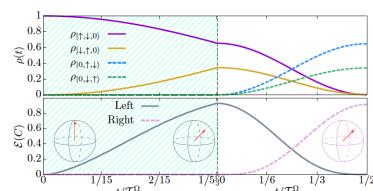
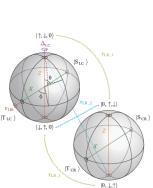


Coherent destruction  
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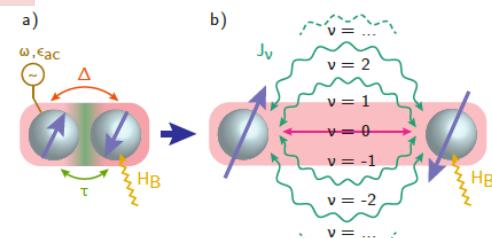
## Bonds renormalization

T. H. Oosterkamp et al., Nature 395, 873-876, 1998

$$\cos\left(\frac{\theta_L}{2}\right)|\uparrow,\downarrow,0\rangle + \sin\left(\frac{\theta_L}{2}\right)e^{i\phi_L}|\downarrow,\uparrow,0\rangle$$

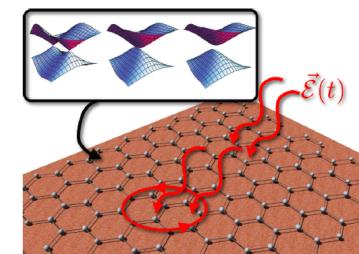


J. Picó-Cortés et al.,  
PRB 2019

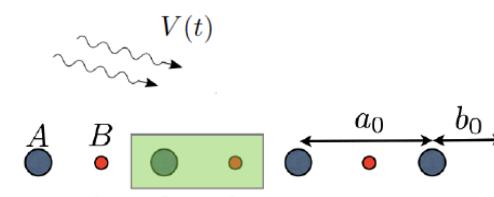


J. Picó-Cortés, G.P., Quantum 5, 607, 2021

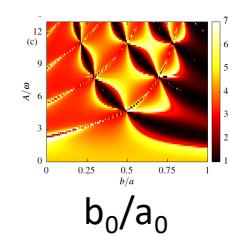
## □ Tuning electronic and topological properties in driven systems



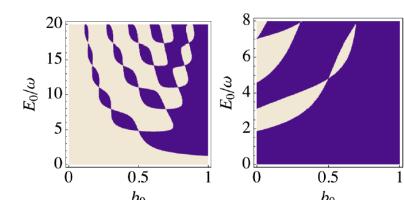
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# Model

$$H(t) = \sum_{|i-j| < R} J_{ij} c_i^\dagger c_j + \sum_i A_i f(t) c_i^\dagger c_i \quad f(t) = \begin{cases} 1 & \text{if } 0 \leq t < T/2 \\ -1 & \text{if } T/2 \leq t < T \end{cases}$$

$$\downarrow \omega \gg J_{ij}$$

$$H_{\text{eff}} = \sum_{|i-j| < R} \tilde{J}_{ij} c_i^\dagger c_j$$

renormalized hopping amplitudes in terms of the field parameters

$$\tilde{J}_{ij} = J_{ij} \frac{i\omega}{\pi(A_i - A_j)} \left[ e^{-i\pi \frac{A_i - A_j}{\omega}} - 1 \right]$$

- periodically spaced zeroes, located at  
 $A_i - A_j = 2\omega q \quad (q = 0, 1, 2\dots)$
- functions of the difference between on-site potentials

# Simulation of the extended SSH model

**our driving protocol:**

- creates a dimerized structure
- suppresses all even hoppings

$$\tilde{J}_{ij} = J_{ij} \frac{i\omega}{\pi(A_i - A_j)} \left[ e^{-i\pi \frac{A_i - A_j}{\omega}} - 1 \right]$$

$$A_j - A_i = \alpha + \beta$$

$$|i - j| = 2n$$

$$n = 1, 2, \dots$$

$$\alpha + \beta = 2\omega q$$

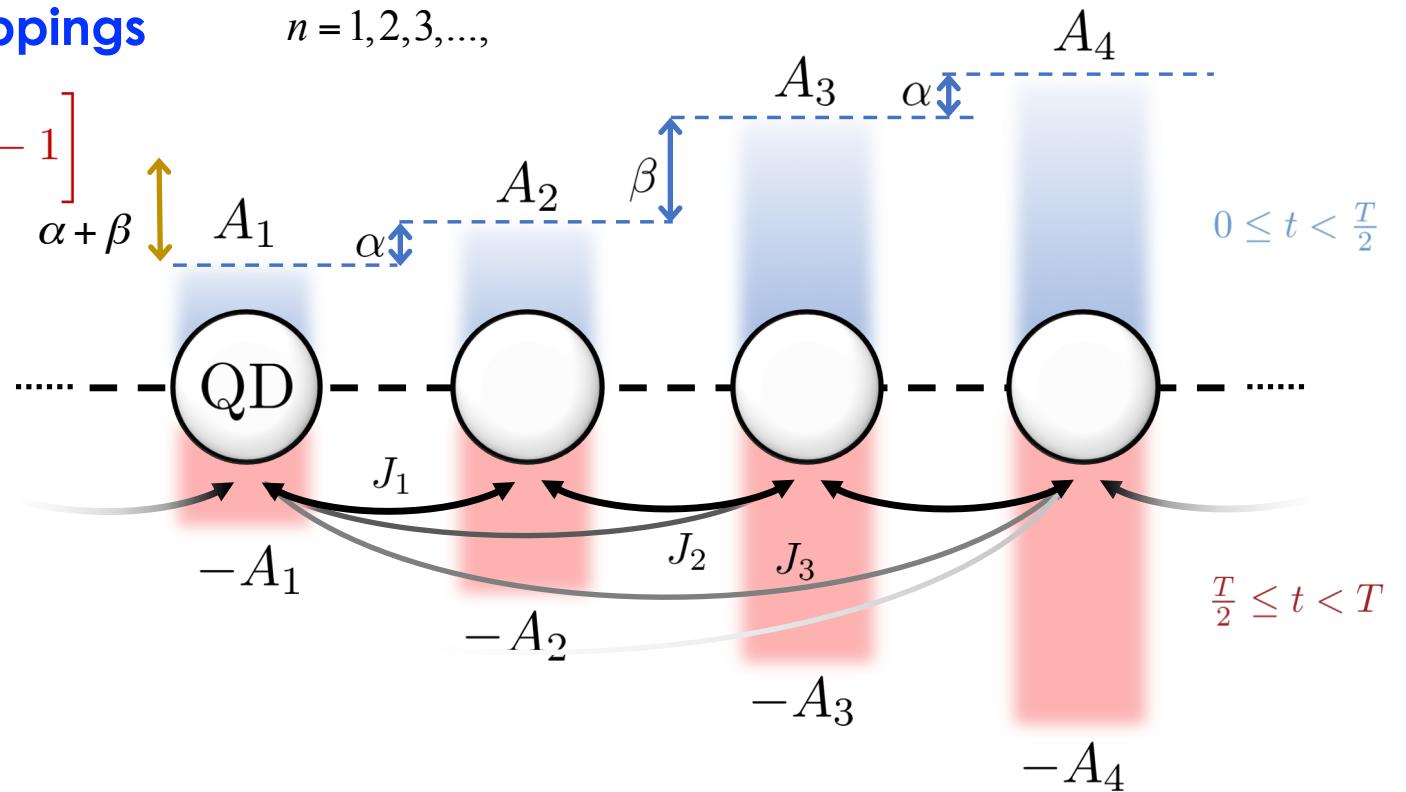
$$J_{i,i+2n} = 0$$

$$A_{2n} = n(\alpha + \beta)$$

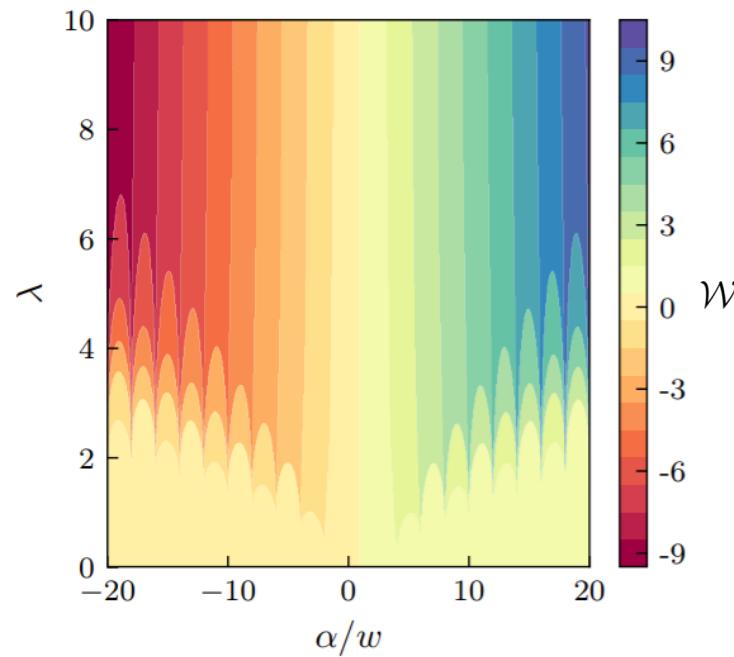
$$A_{2n-1} = n(\alpha + \beta) - \alpha$$

$$n = 1, 2, 3, \dots,$$

$\alpha \rightarrow$  effective field amplitude



# Simulation of the extended SSH model

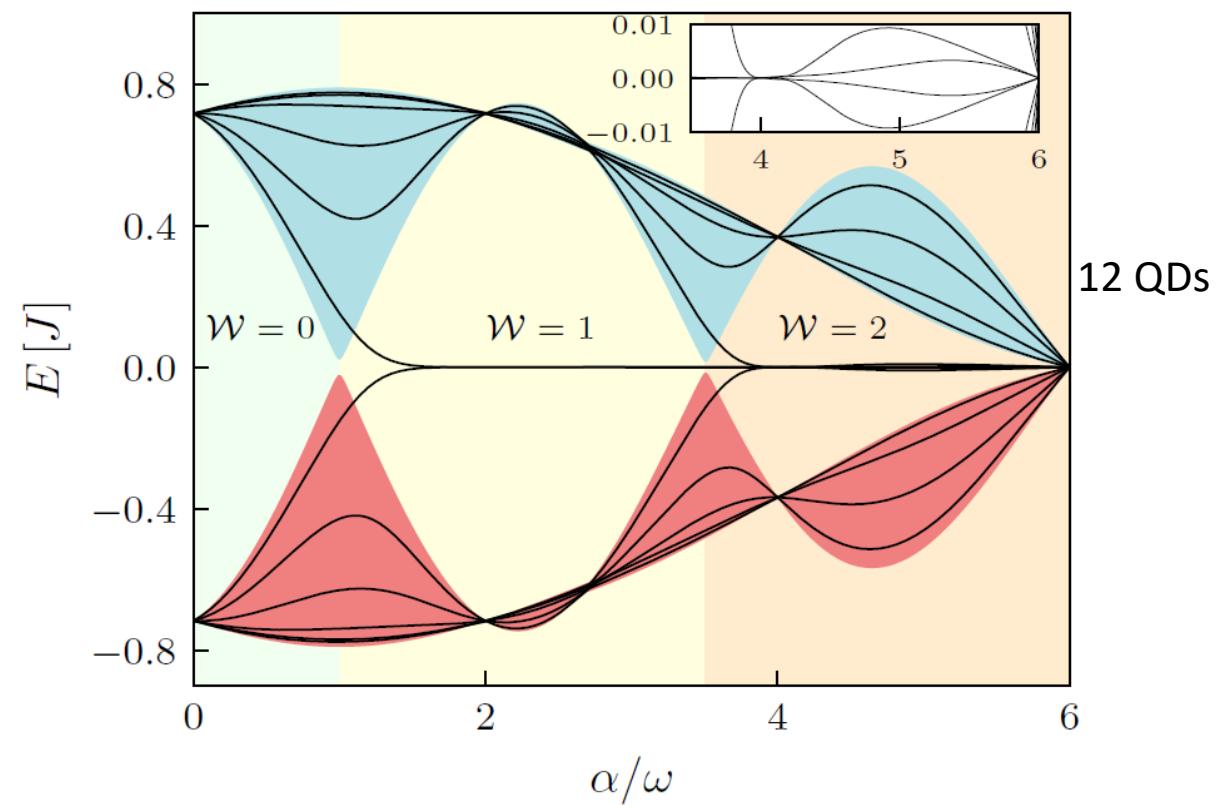


☐ exponential decay with distance

$$J_{ij} = J e^{-d_{ij}/\lambda}$$

☐ R defined as a function of  $\lambda$

$$\frac{J_{\text{longest}}}{J_{\text{shortest}}} = 10^{-8}$$

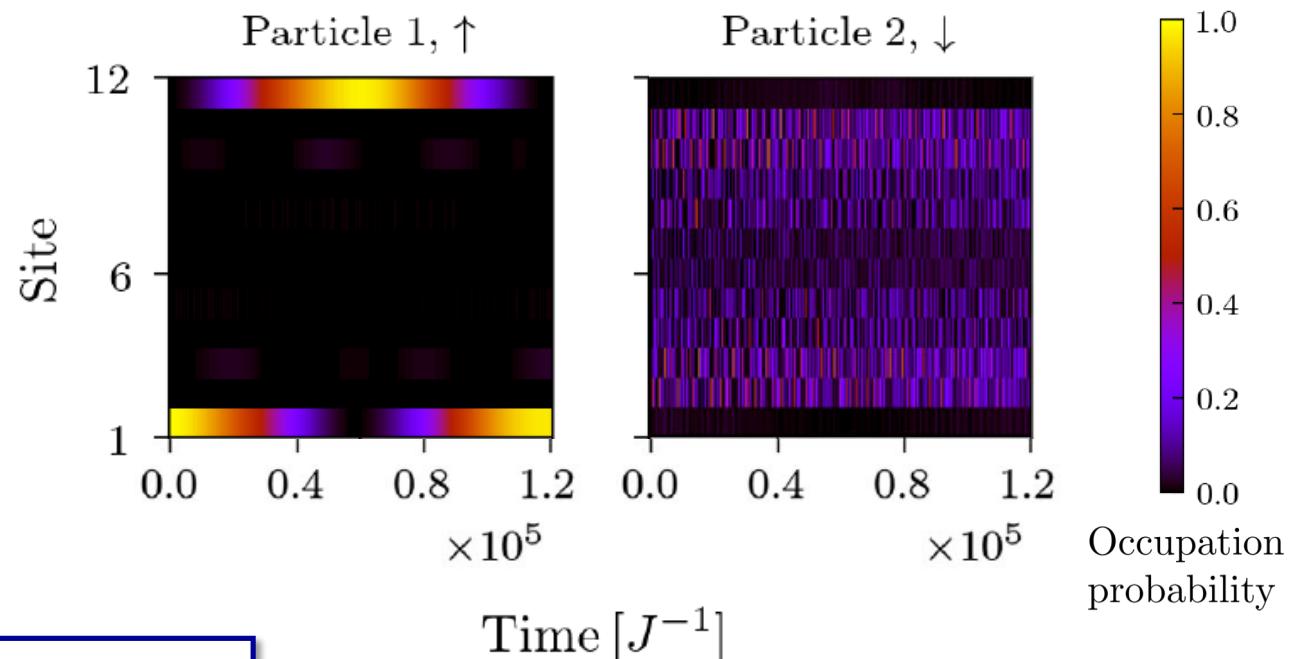
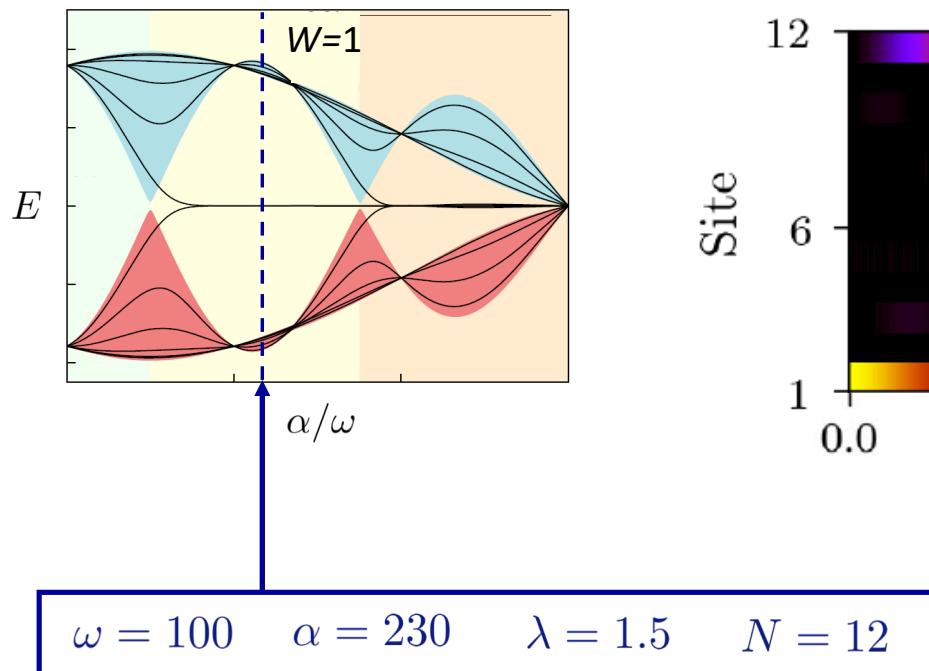


$$\lambda = 1.5$$

# Simulation of the extended SSH model

dynamics of two interacting particles with opposite spin loaded into the system as  $|\uparrow_1\downarrow_3\rangle$

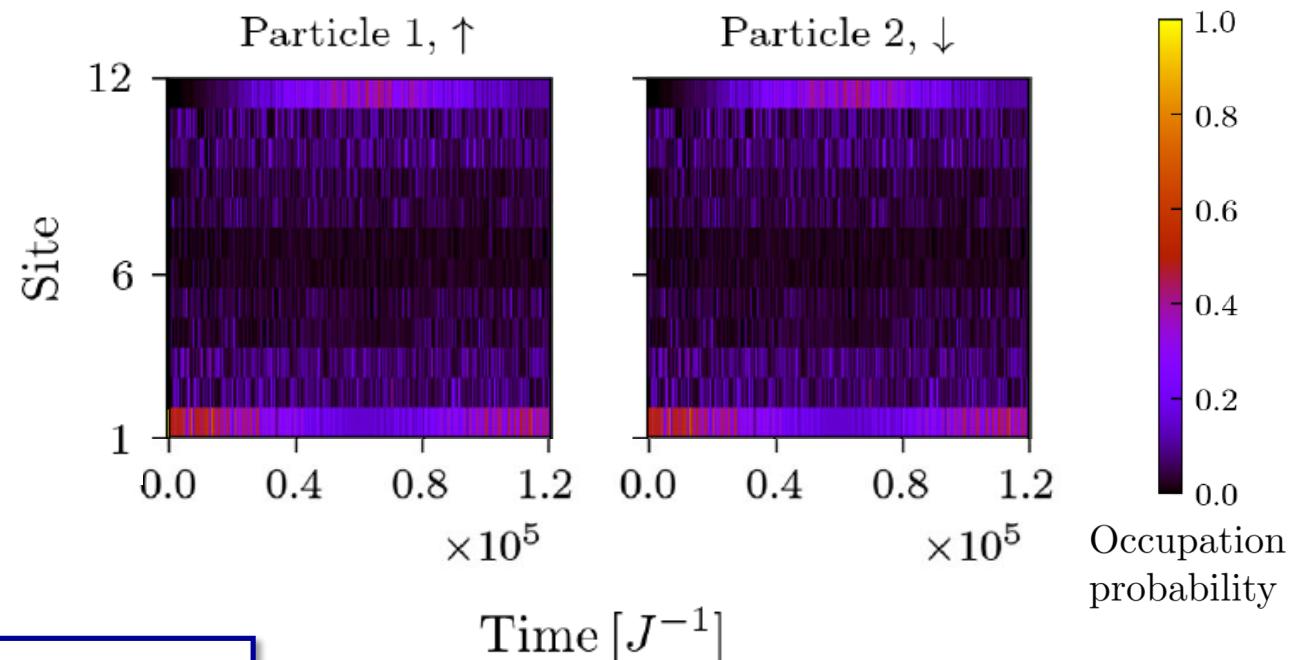
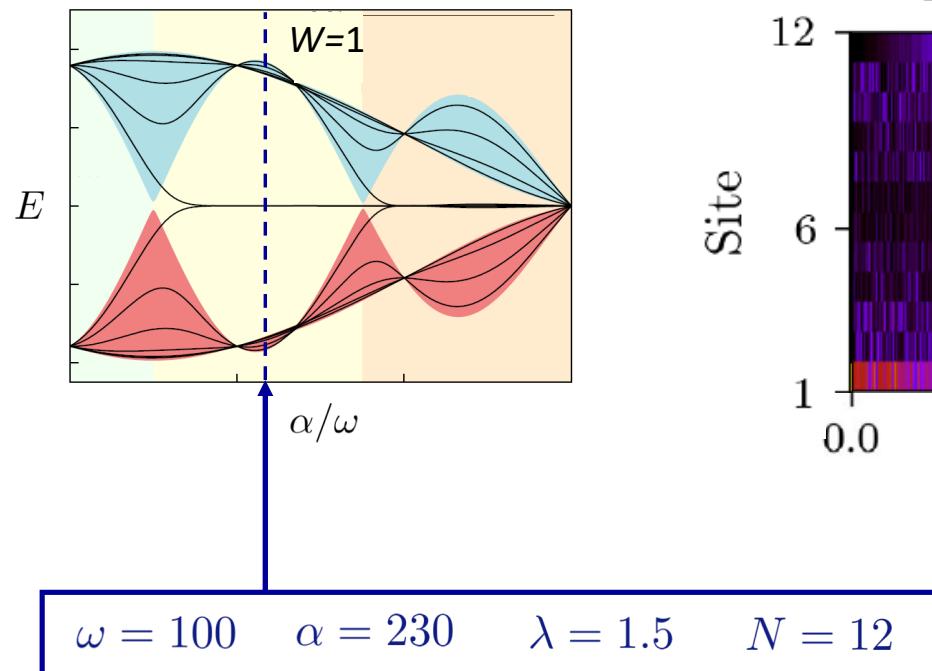
$U = 0$



# Simulation of the extended SSH model

dynamics of two interacting particles with opposite spin loaded into the system as  $|\uparrow_1\downarrow_3\rangle$

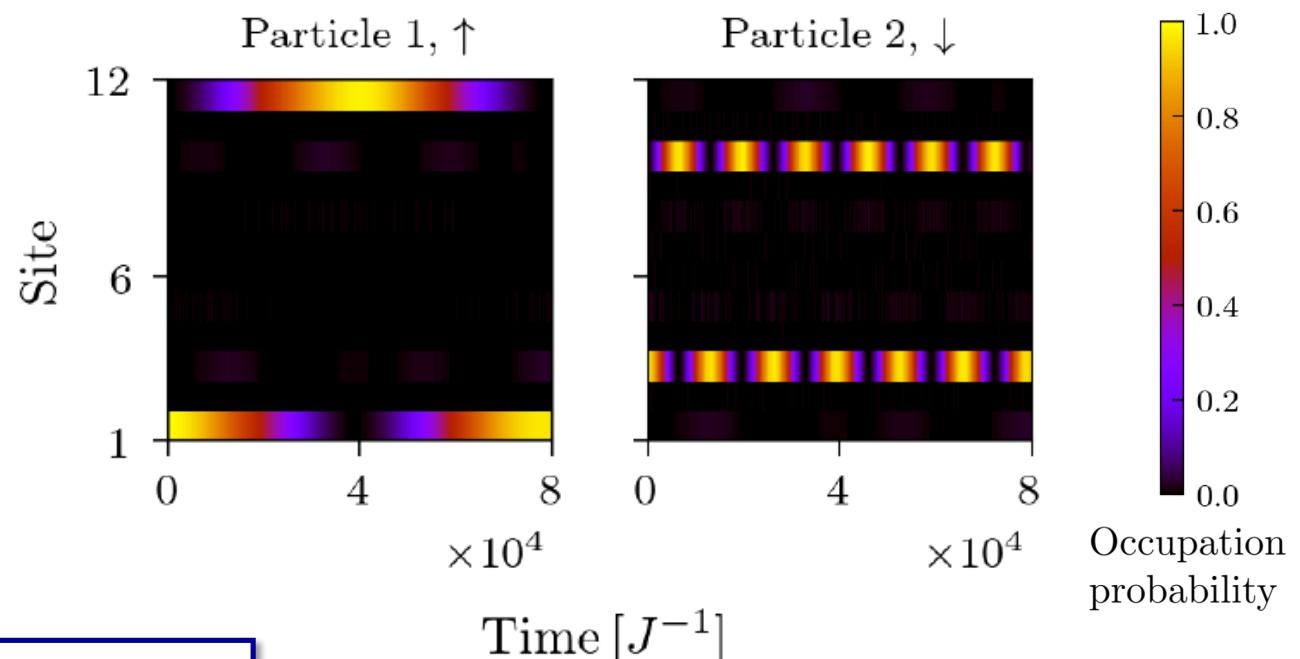
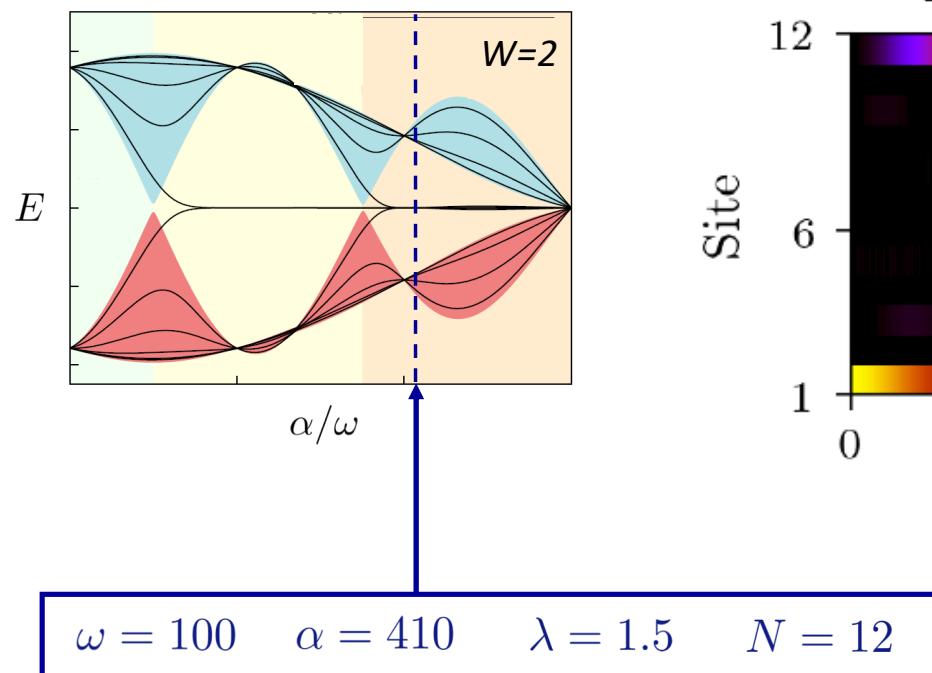
□  $U = 5J$



# Simulation of the extended SSH model

dynamics of two interacting particles with opposite spin loaded into the system as  $|\uparrow_1\downarrow_3\rangle$

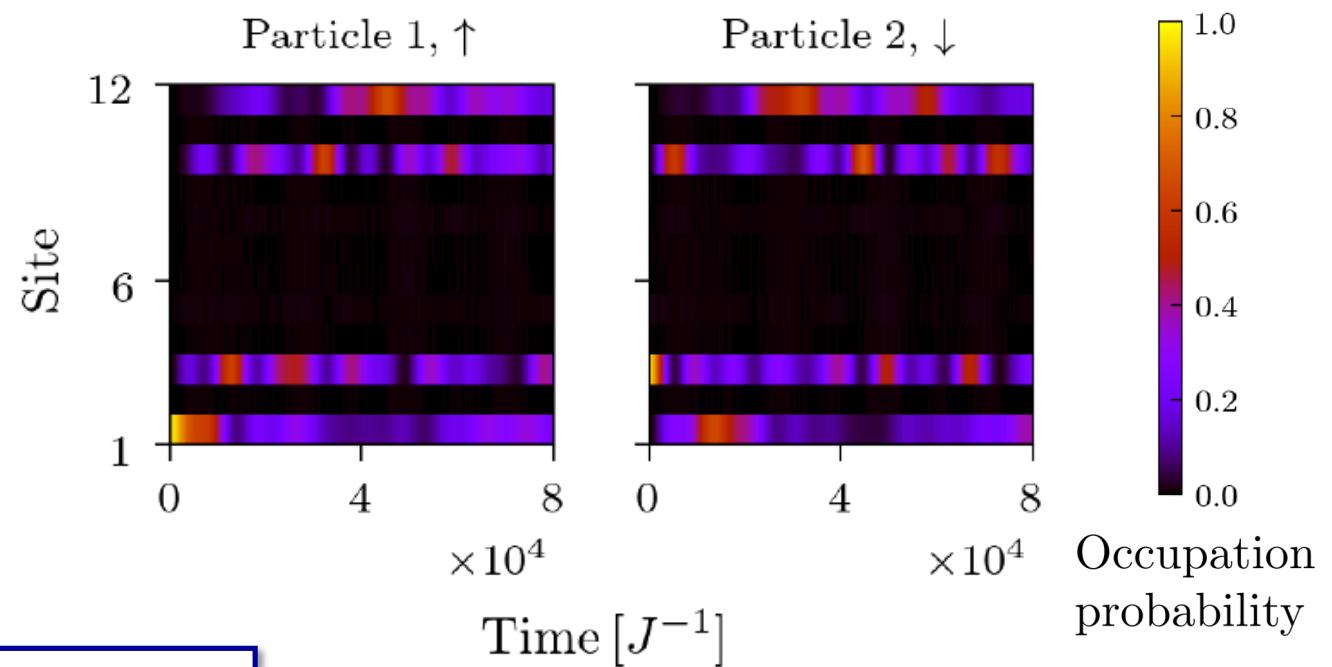
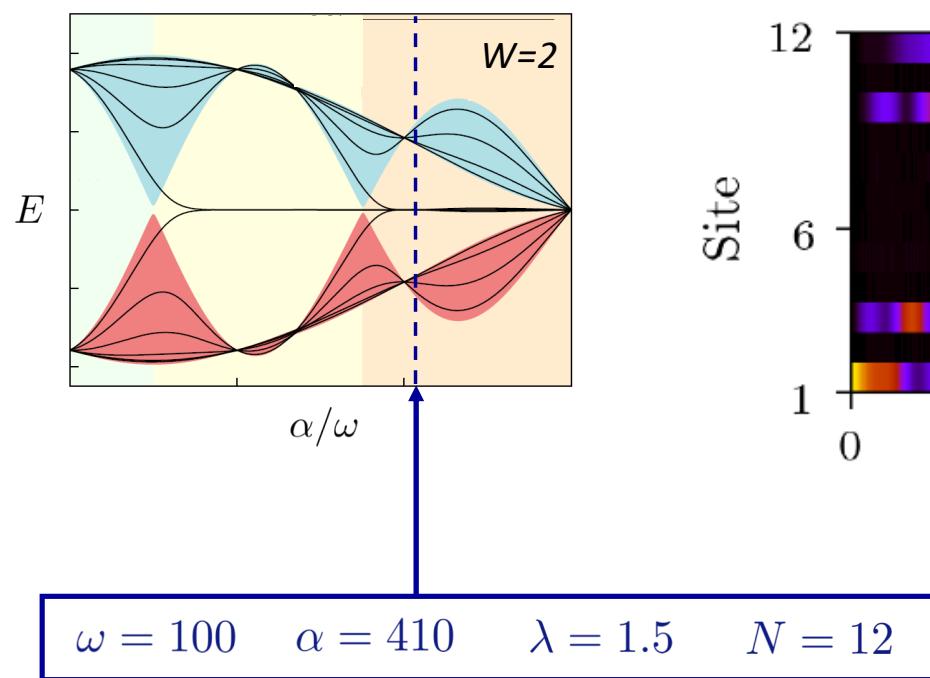
$U = 0$



# Simulation of the extended SSH model

dynamics of two interacting particles with opposite spin loaded into the system as  $|\uparrow_1\downarrow_3\rangle$

$U = 5J$



B. Pérez-González et al., PRL, 123, 126401 (2019)

# Summary

Quantum dot networks: solid state platform for quantum state transfer and quantum simulation:  
Hole spin qubits

ac driven protocol to simulate the extended SSH in a QD array with new topological phases and edge states

# Summary

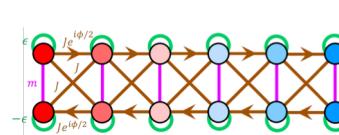
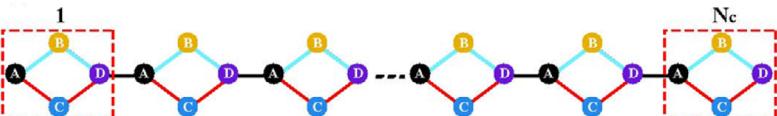
Quantum dot networks: solid state platform for quantum state transfer and quantum simulation:  
Hole spin qubits

ac driven protocol to simulate the extended SSH in a QD array with new topological phases and edge states

## Outlook

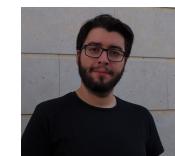
Topological domain walls as quantum amplifiers (J. Zurita et al., arXiv:2208.00797)

Investigate other quasi-1D hamiltonians for transferring Information mediated by edge states



J. Zurita, et al., Advances Quantum Tech., 3, 1900105 (2020).

J. Zurita et al., Quantum 5, 591 (2021)



# Acknowledge to my collaborators



**David Fernández**

ICMM-CSIC



**Beatriz Pérez**

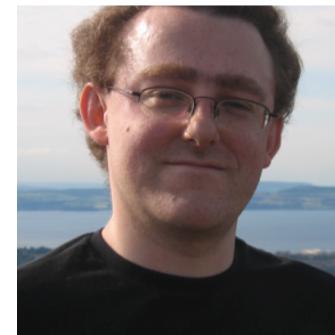
ICMM (CSIC)



**Yue Ban**  
Bilbao University



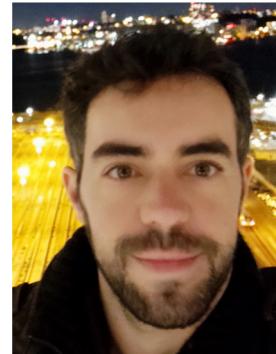
**Juan Zurita,** ICMM-CSIC



**Charles Creffield** UCM

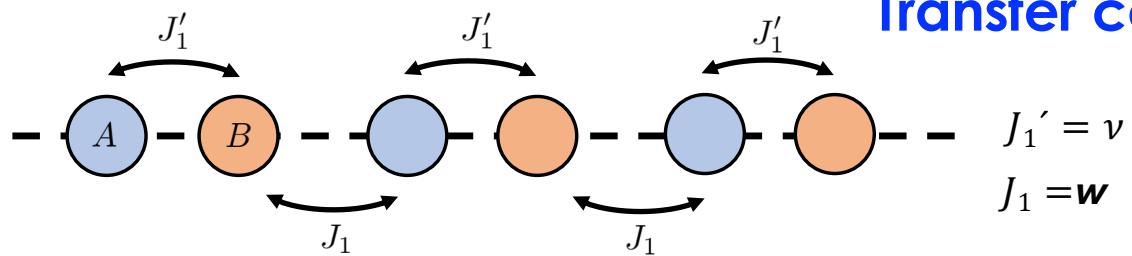


**Miguel Bello**  
MPI, Garching

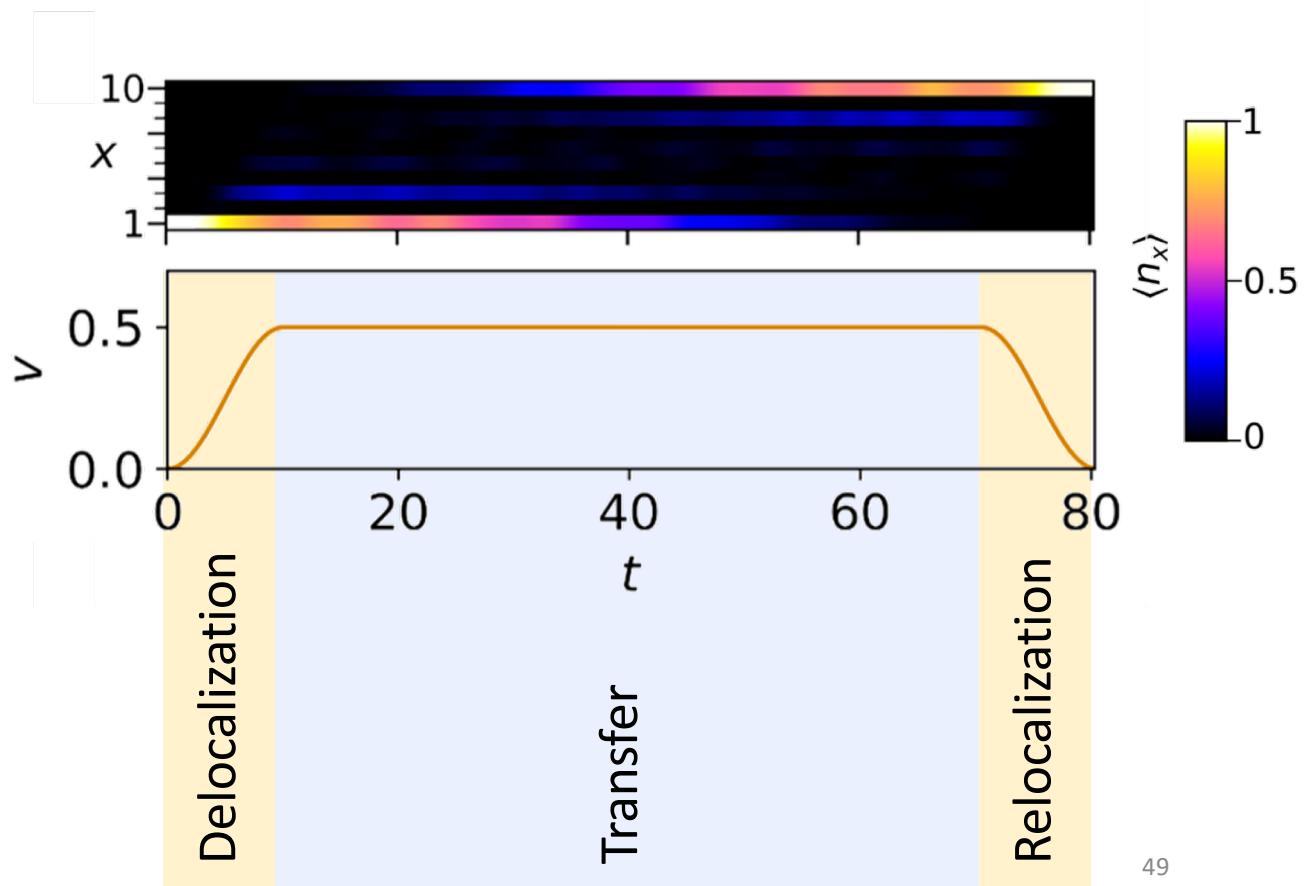
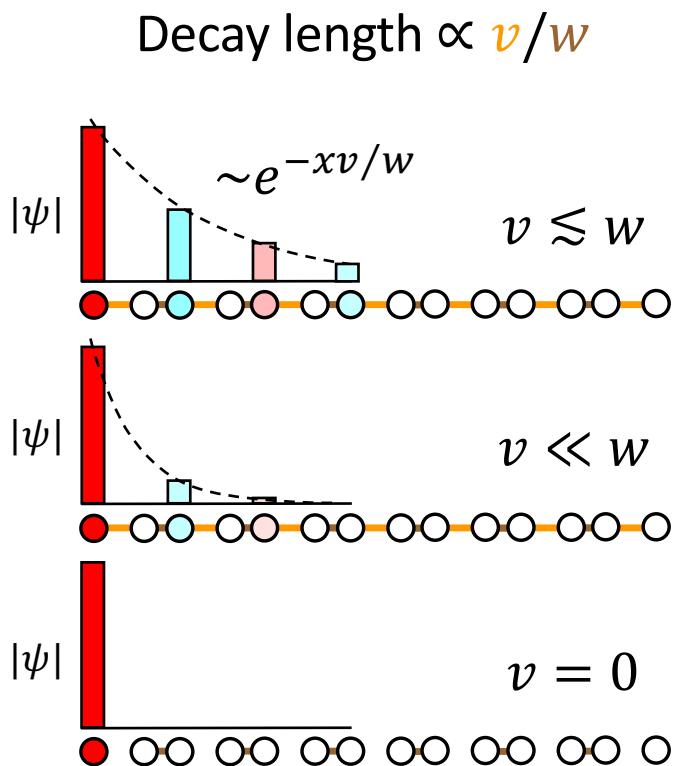


**Álvaro Gómez Léon**  
IFF (CSIC)

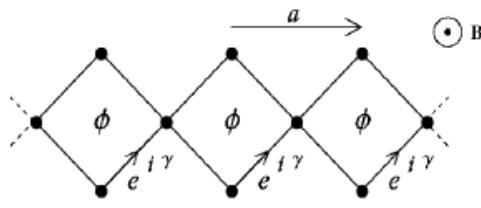
**... thank you  
for your attention!**



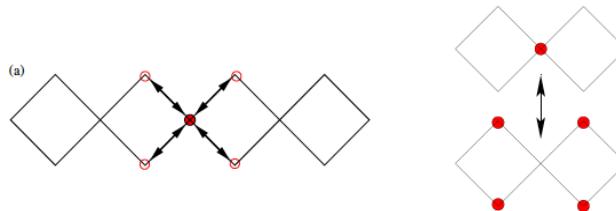
## Transfer control in a dimer molecule (SSH)



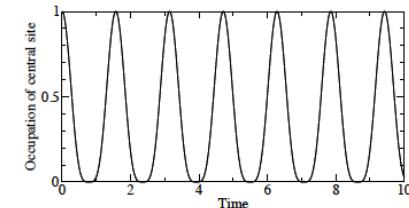
# Aharanov-Bohm caging



<sup>†</sup> Vidal et al, PRL 81, 5888 (1998)



When  $\phi = \pi$ , interference cancels the tunneling

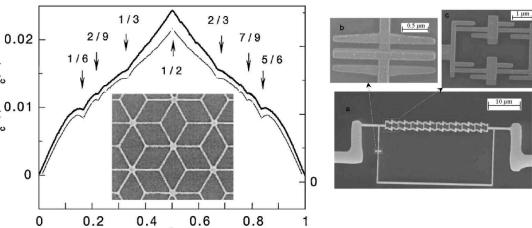
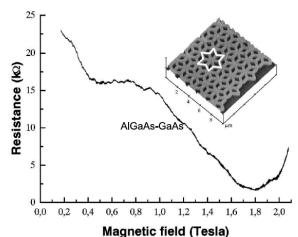


CEC & GP, PRL 105, 086804 (2010)

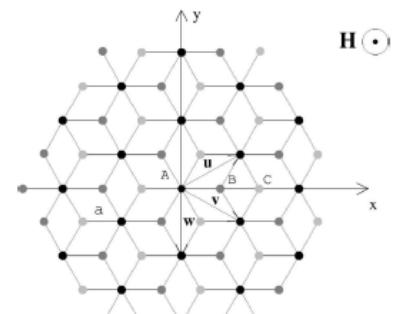
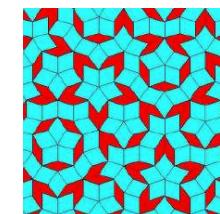
AB-caging does not occur on every lattice –

- square, triangular, honeycomb – no caging
- Penrose, octagonal,  $T_3$  – caging possible

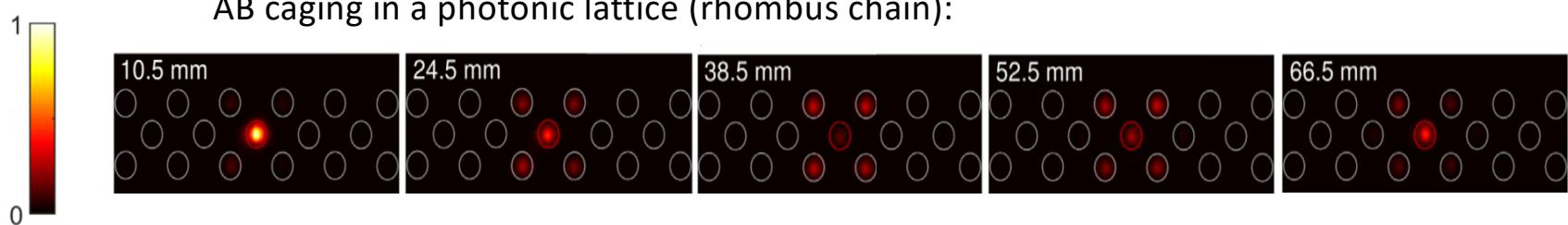
AB caging has been observed in a variety of systems



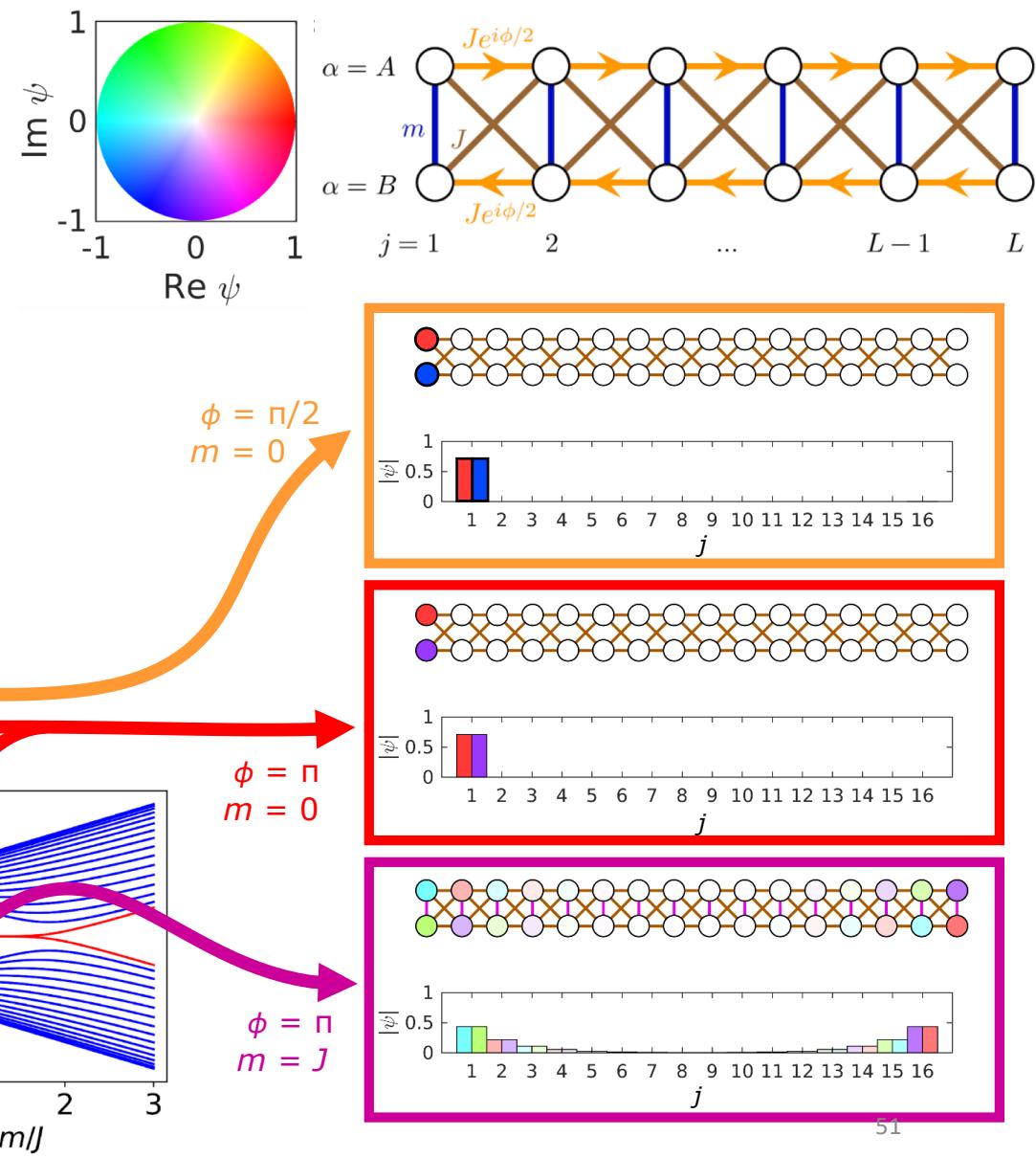
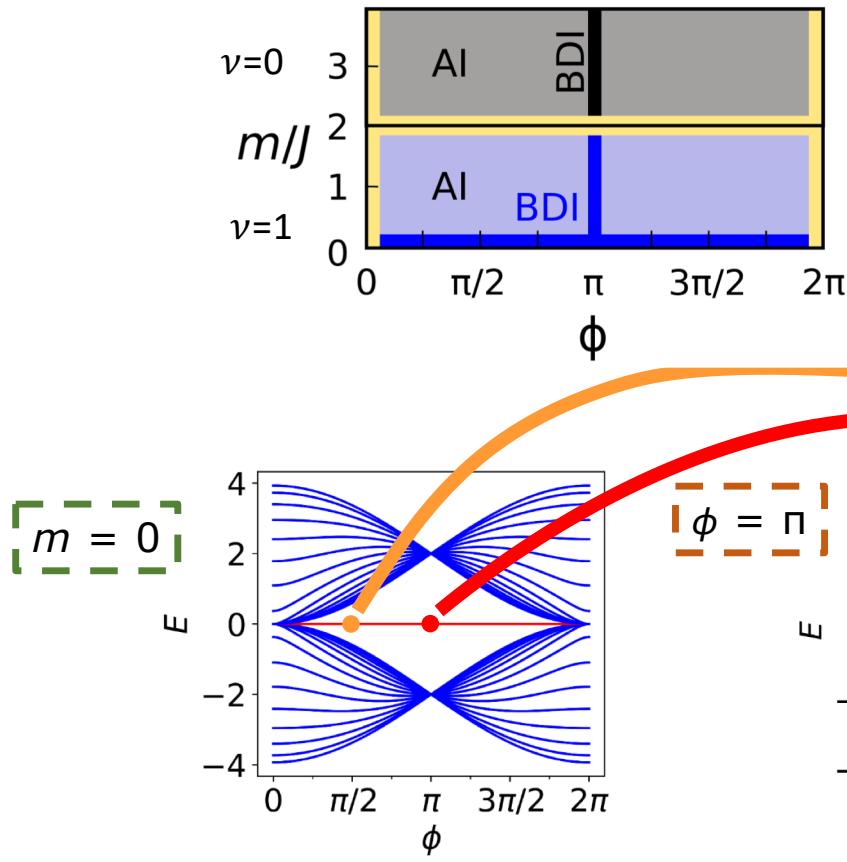
S. Mukherjee et al. PRL (2018)



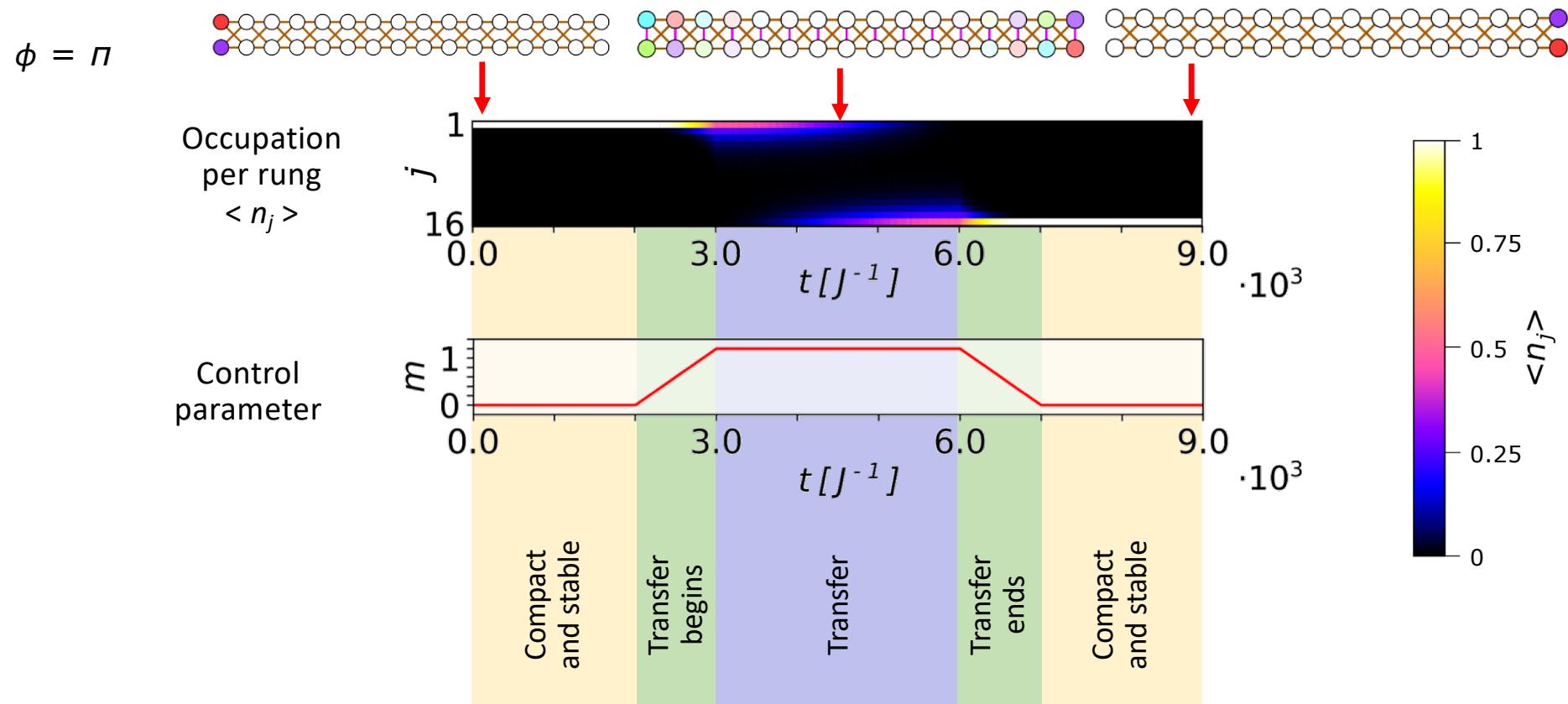
AB caging in a photonic lattice (rhombus chain):



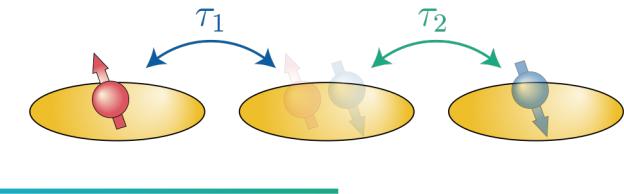
# Topological phases



# Topological quantum state transfer

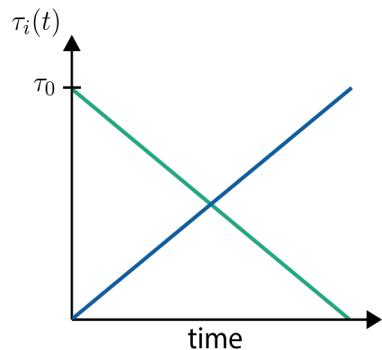


## 5. Transfer protocols



- The **driving parameters** are the tunneling rates
- There exists multiple **adiabatic transfer protocols**, some of the most known ones are

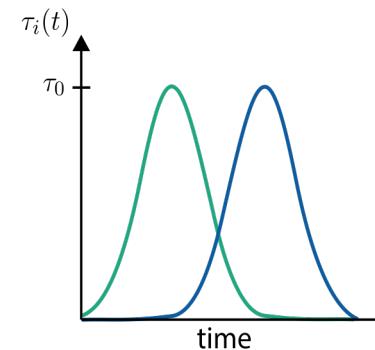
Linear ramp



- Easy to implement in experimental device
- Analytical results based on LZ passage
- **Robust** against error in the tunneling

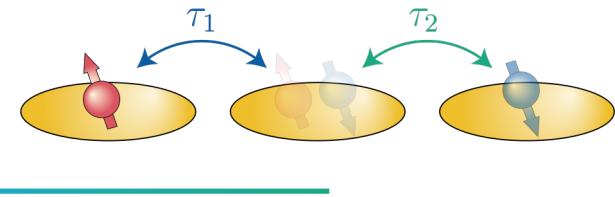
CTAP  
Coherent Transfer by Adiabatic Passage

A. D. Greentree, et al., PRB **70**, 235317 (2004)



- **Smooth** pulses
- **Robust** against error in the detuning
- High expressivity

## 8. Noise model



- Noise in the **detuning** as  $\varepsilon_i^n(t) = \varepsilon_i + \delta_\epsilon \nu_i(t)$
- Noise in the **tunneling rates** as  $\tau_i^n(t) = \tau_i(t) + \delta_\tau \tilde{\nu}_i(t)$

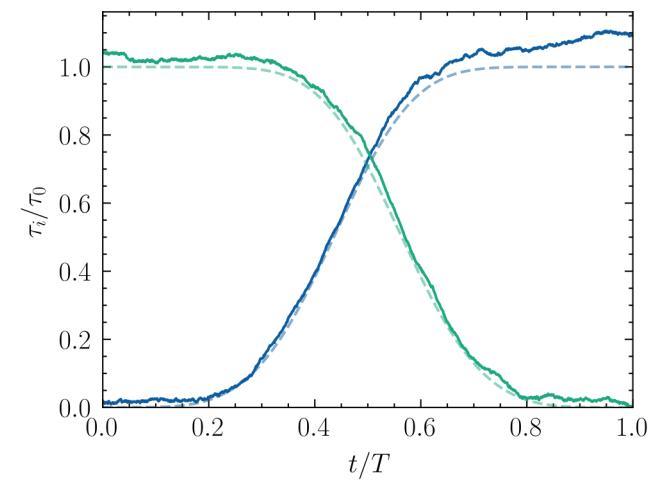
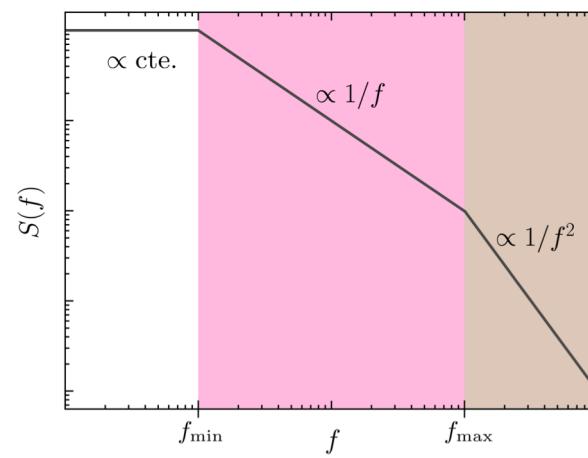
- Uncorrelated errors
 
$$\langle \nu_i(t) \nu_j(t) \rangle = \delta_{i,j}$$

$$\langle \nu_i(t) \tilde{\nu}_j(t) \rangle = 0$$

- White noise:  $f < f_{\min}$
- Pink noise:  $f_{\min} \leq f \leq f_{\max}$
- Brown noise:  $f_{\max} < f$

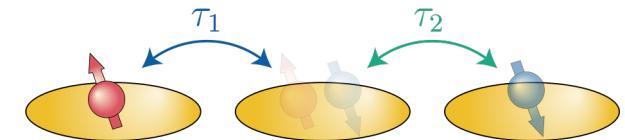
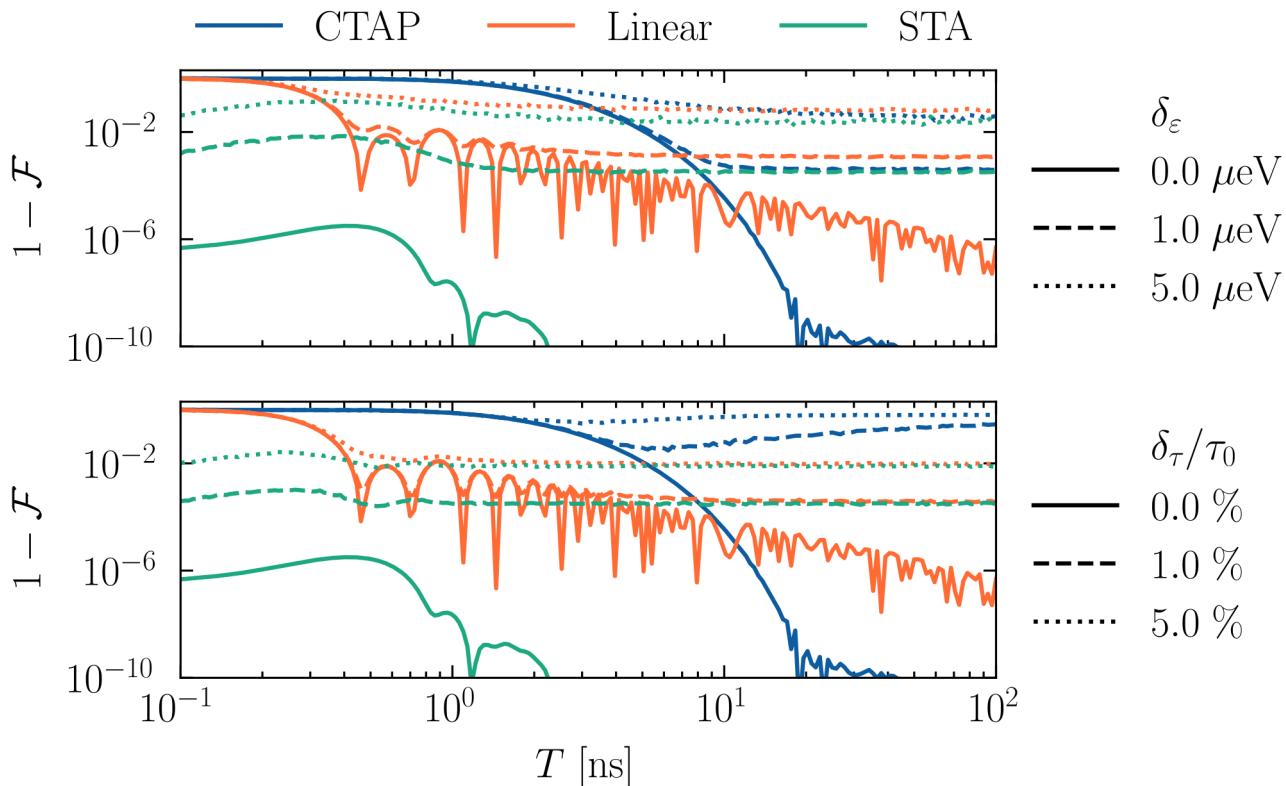
E. Paladino, et al., Rev. Mod. Phys. **86**, 361 (2014)

M. J. Gullans, and J. R. Petta, PRB **100**, 085419 (2019)



# Noisy transfer

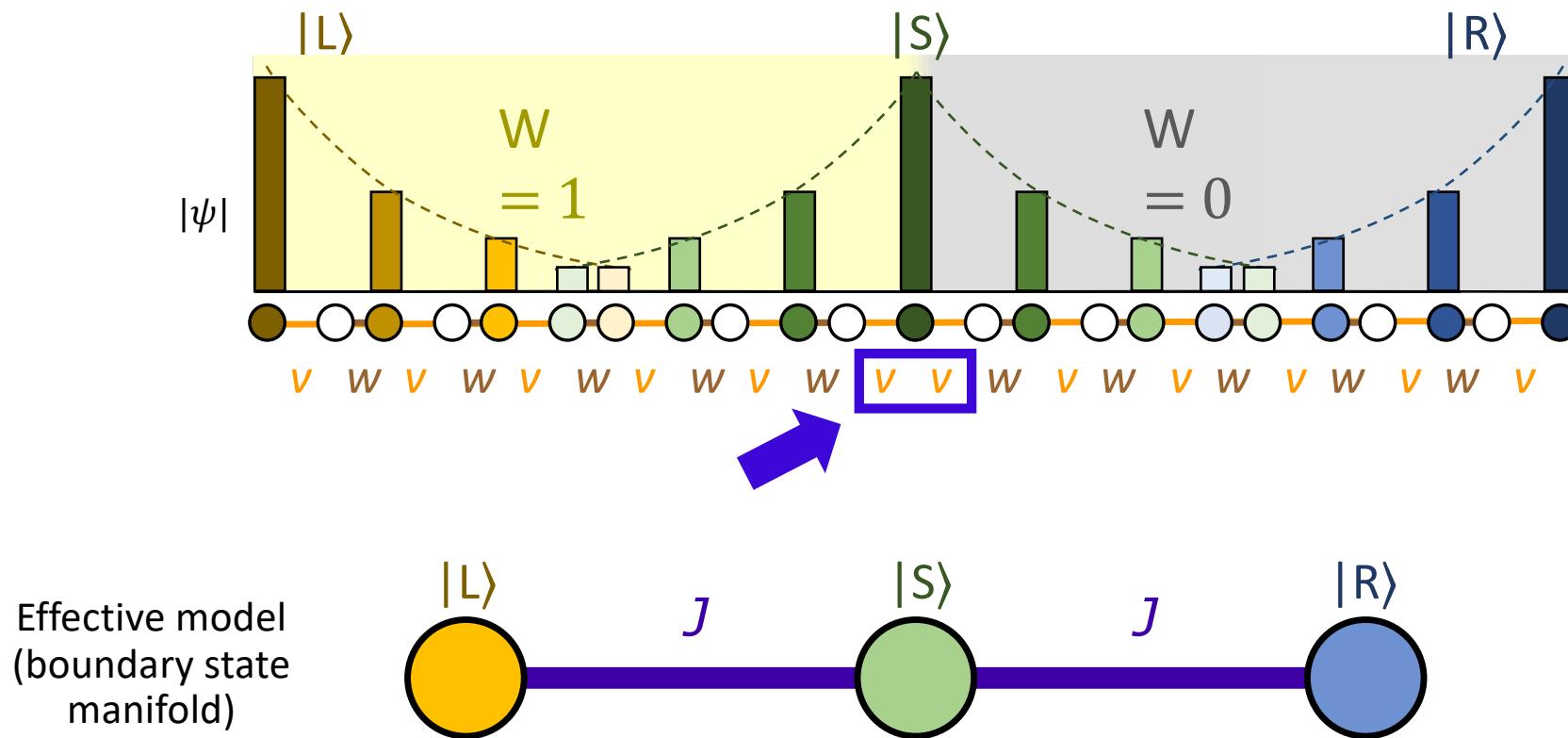
- Transfer fidelity defined as:  $\mathcal{F} \equiv |\langle 0, 0, \downarrow | \Psi(T) \rangle|^2$



Parameters	
$x_{\text{SOC}}$	= 1
$\sigma$	= $T/6$
$f_{\min}$	= 0.16 mHz
$f_{\max}$	= 0.1 MHz
$\tau_0$	= 10 $\mu\text{eV}$

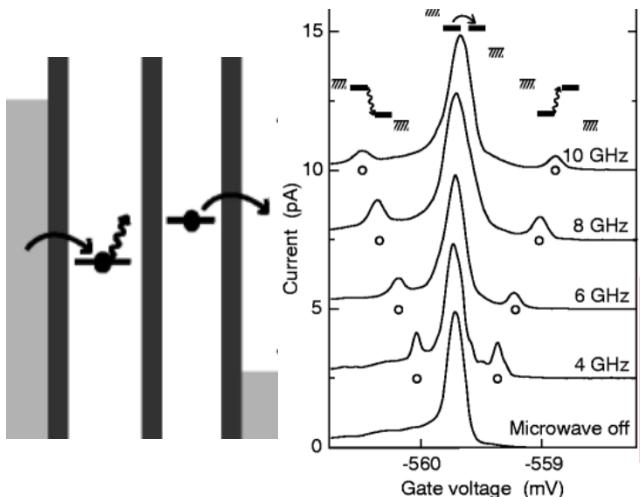
- CTAP** highly sensitive to error in the tunneling rates
- Linear** pulse obtain good results if the transfer time is large enough
- STA** is the best among all the protocols for low transfer times

### III. Topological domain walls



# Driving with periodic AC fields

- Photoassisted Tunneling (PAT) in quantum dots

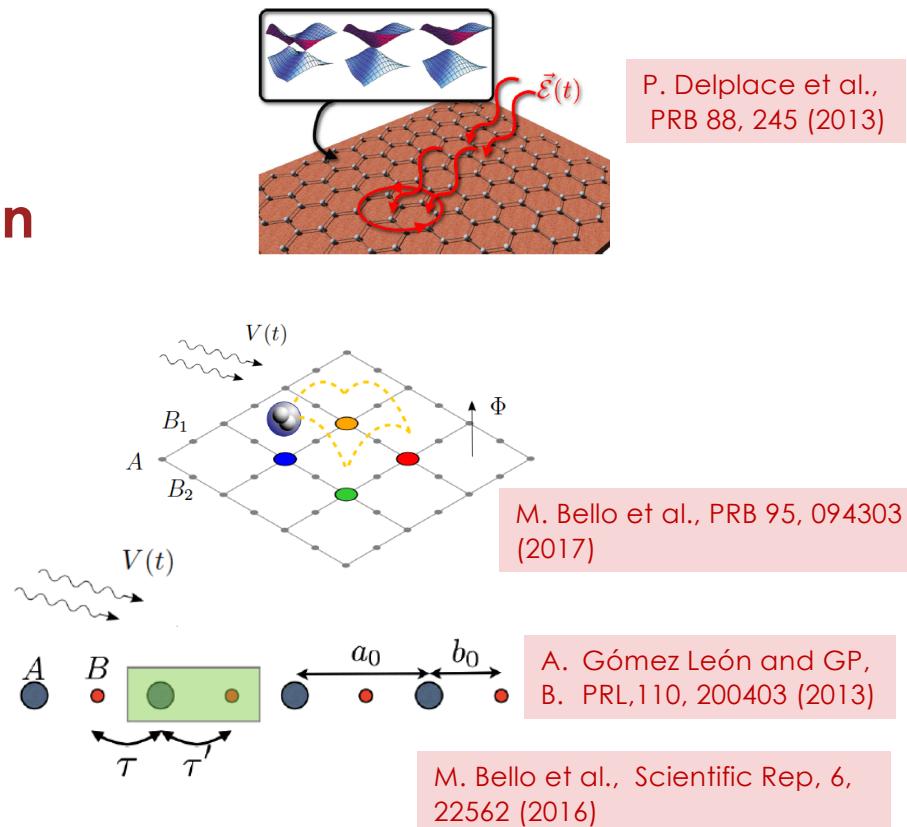


## Bonds renormalization

Coherent destruction  
of tunnel,  
P. Hänggi, PRL, 1991

T. H. Oosterkamp et al.,  
Nature 395, 873-876, 1998

- Tuning electronic and topological properties in driven systems

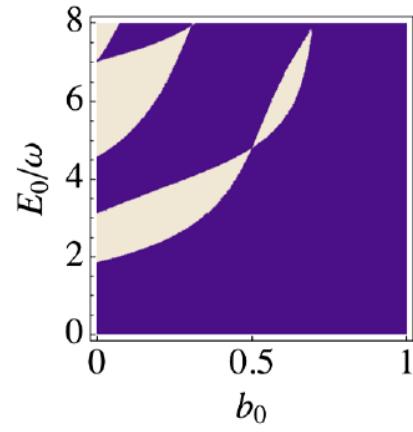
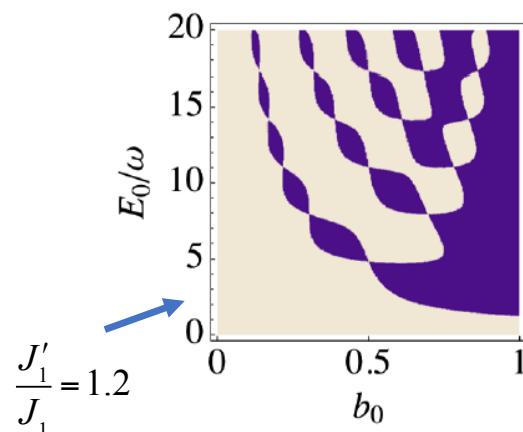


# Driving with periodic AC fields

## AC Driven Dimer Chain

A. Gómez León and G.P., PRL, 2013

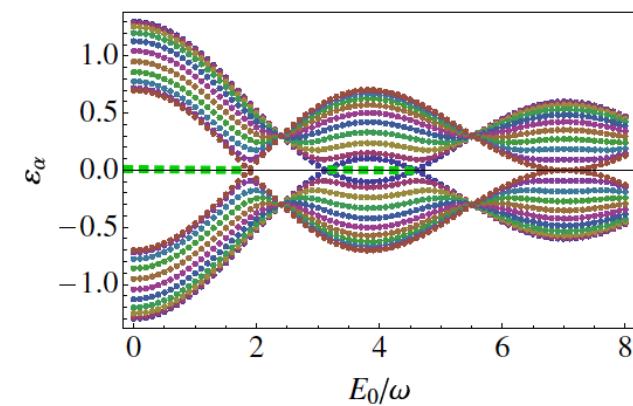
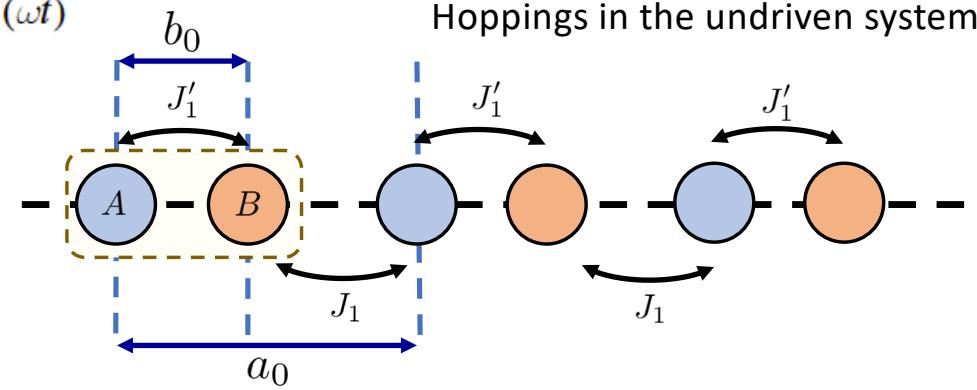
**High-frequency regime**  $H(t) = H_{\text{SSH}} + A \sum_{n=1}^N x_n c_n^\dagger c_n \cos(\omega t)$



Undriven case :

$W=0, J'_1/J_1 > 1$  **Light brown**

$W=1, J'_1/J_1 < 1$  **Blue**



# Driving with periodic AC fields

## AC driven transport to characterize topology

Dimer chain coupled to electron source and drain

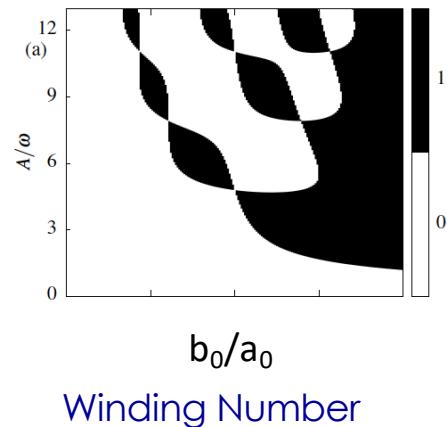
High Frequency

$$\tau'_{\text{eff}} = J_0(Ab/\omega)(\tau_0 - \delta\tau),$$

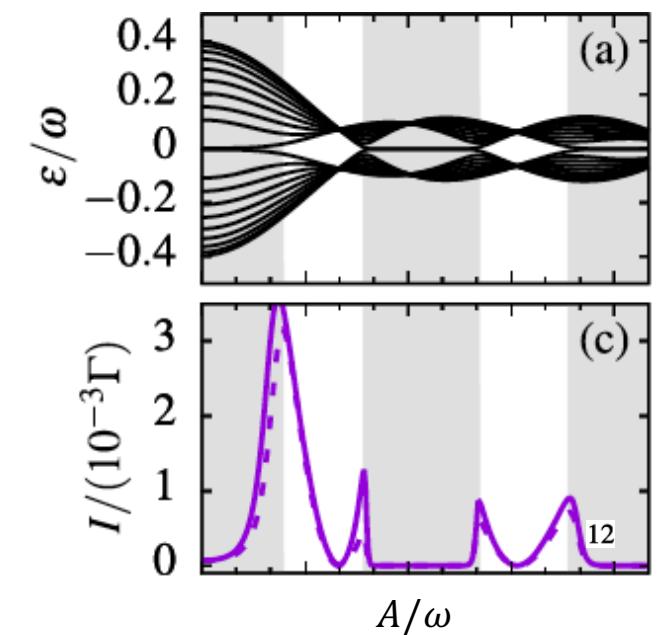
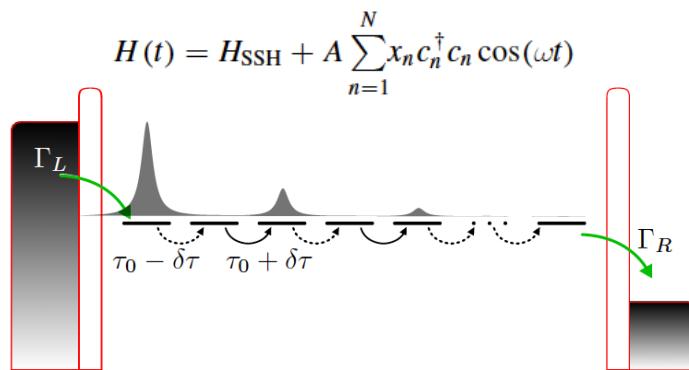
$$\tau_{\text{eff}} = J_0(A(a-b)/\omega)(\tau_0 + \delta\tau),$$

$\tau'_{\text{eff}} > \tau_{\text{eff}}$ : Trivial

$\tau'_{\text{eff}} < \tau_{\text{eff}}$  Non-trivial



A. Gómez  
León  
and G.P.,  
PRL, 2013



M. Niklas et al., Nanotechnology, 2017

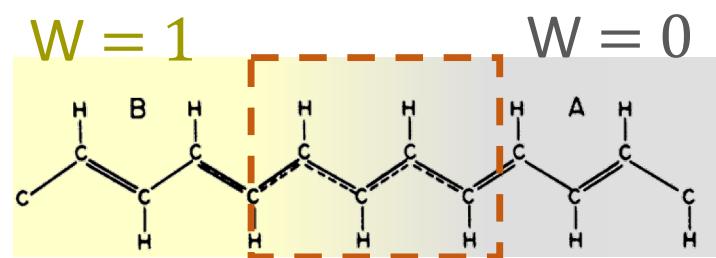
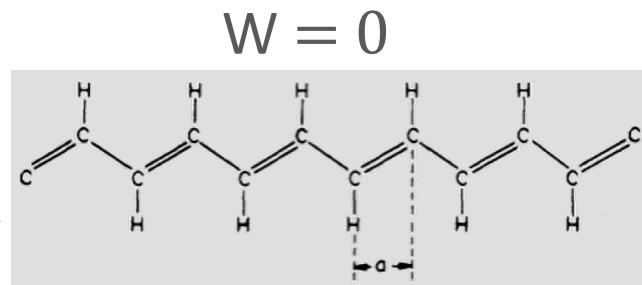
### Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger

*Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104*

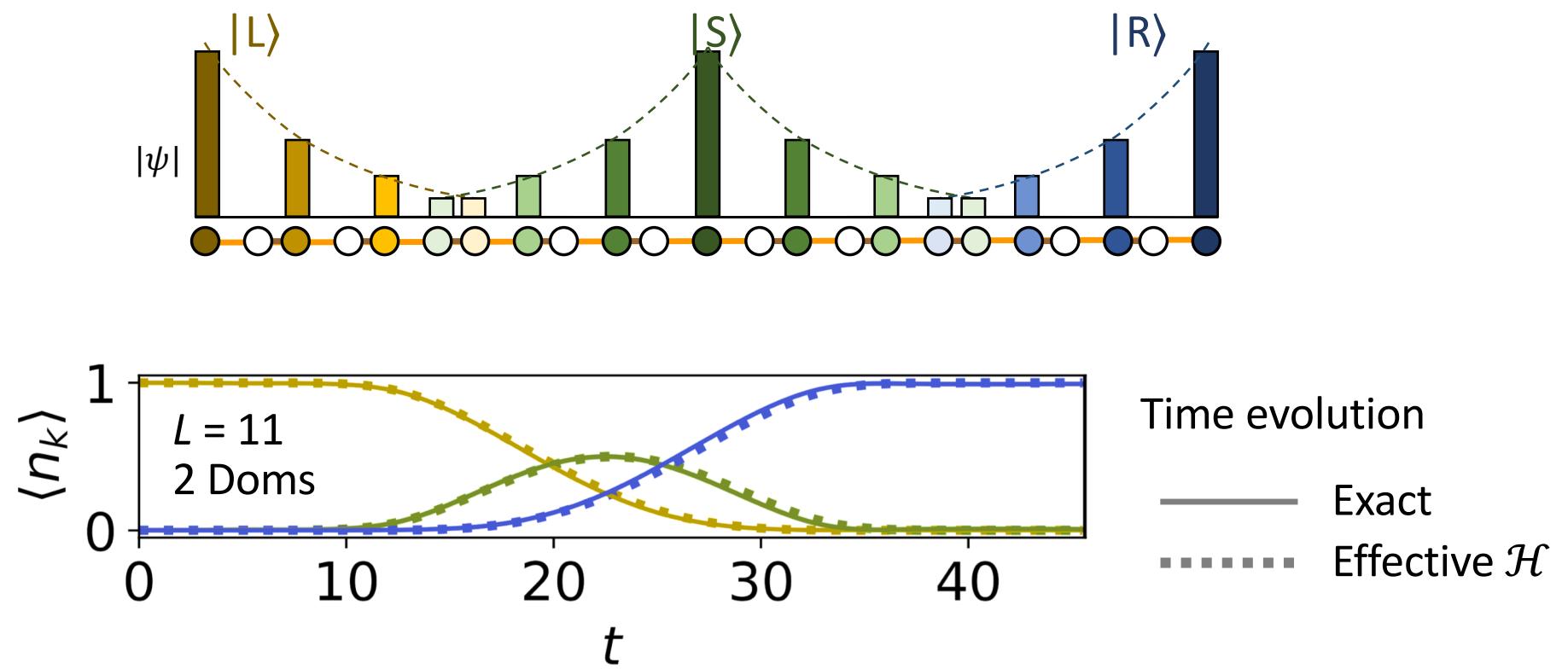
(Received 15 March 1979)

We present a theoretical study of soliton formation in long-chain polyenes, including the energy of formation, length, mass, and activation energy for motion. The results provide an explanation of the mobile neutral defect observed in undoped  $(CH)_x$ . Since the soliton formation energy is less than that needed to create band excitation, solitons play a fundamental role in the charge-transfer doping mechanism.

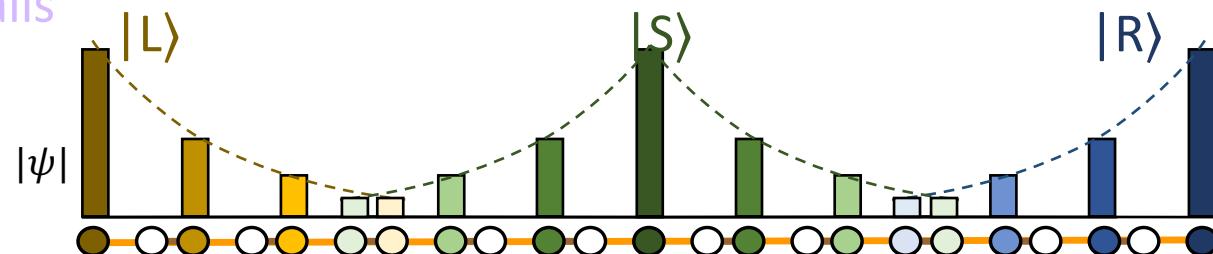


Topological  
domain wall

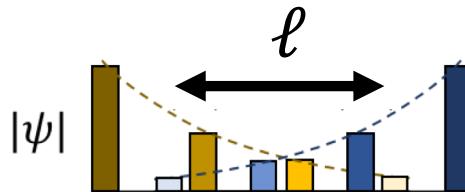
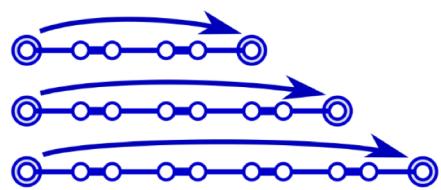
### III. Topological domain walls



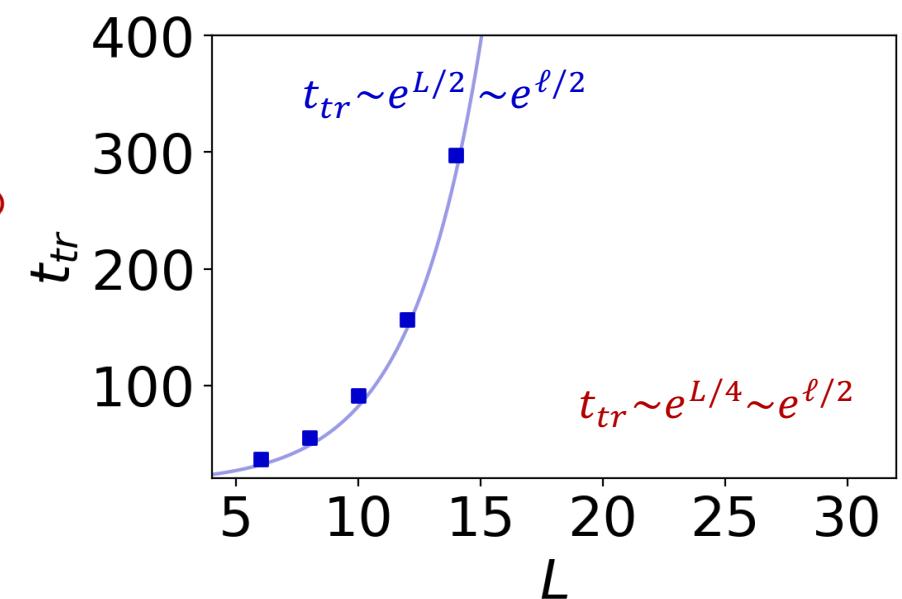
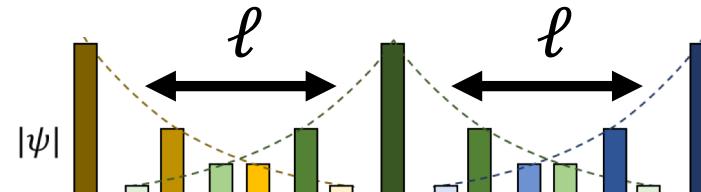
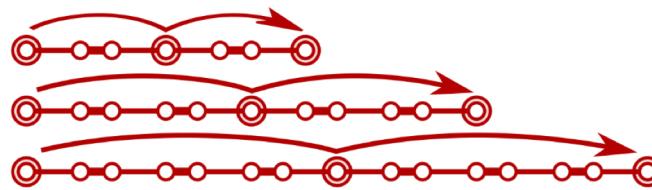
### III. Topological domain walls



One domain

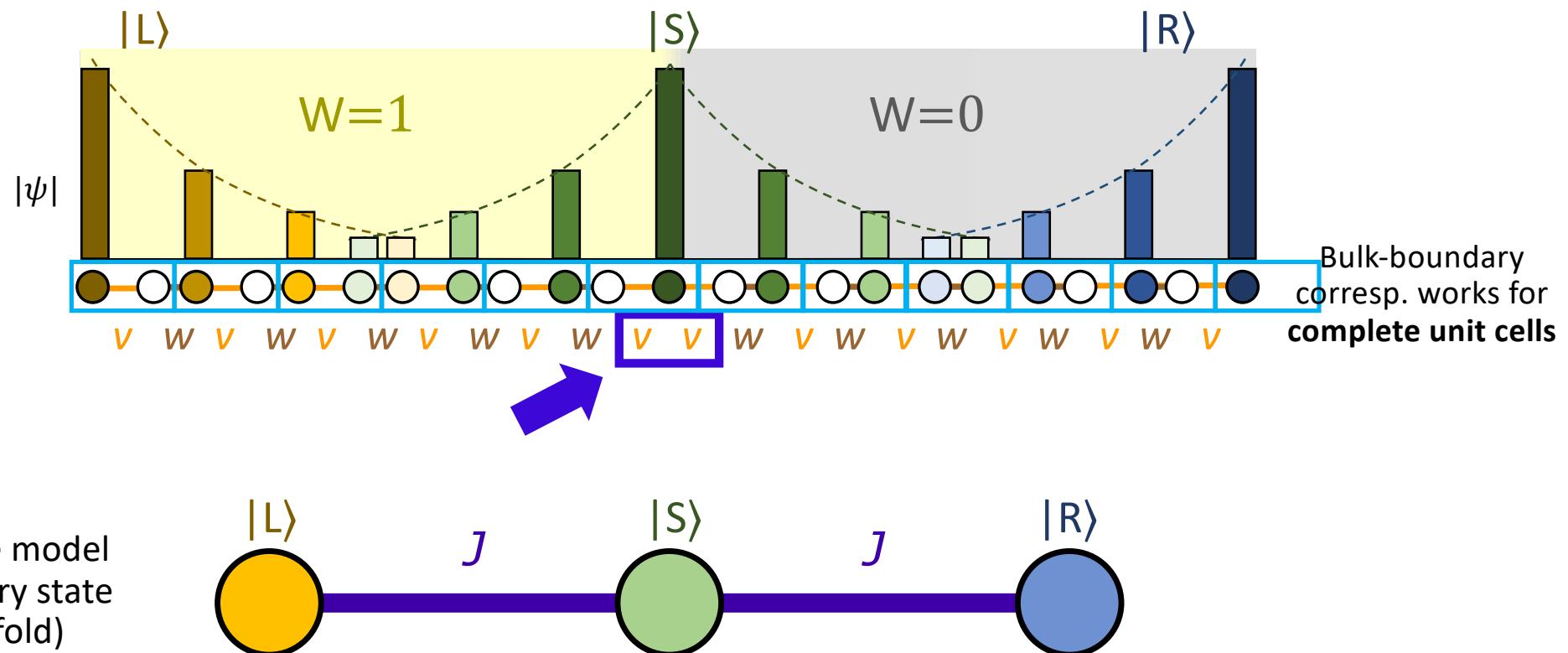


Two domains



Transfer time depends exponentially  
on domain length  $\ell$  only

### III. Topological domain walls



Effective model  
(boundary state  
manifold)

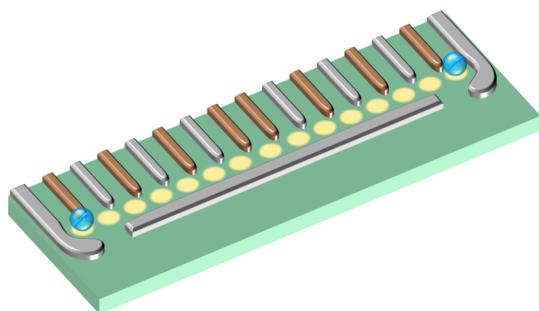
# Summary

Quantum dot networks: solid state platform for quantum state transfer and quantum simulation

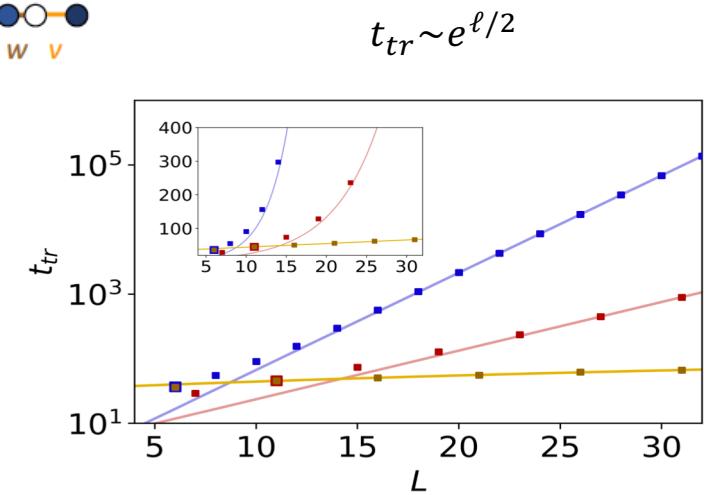
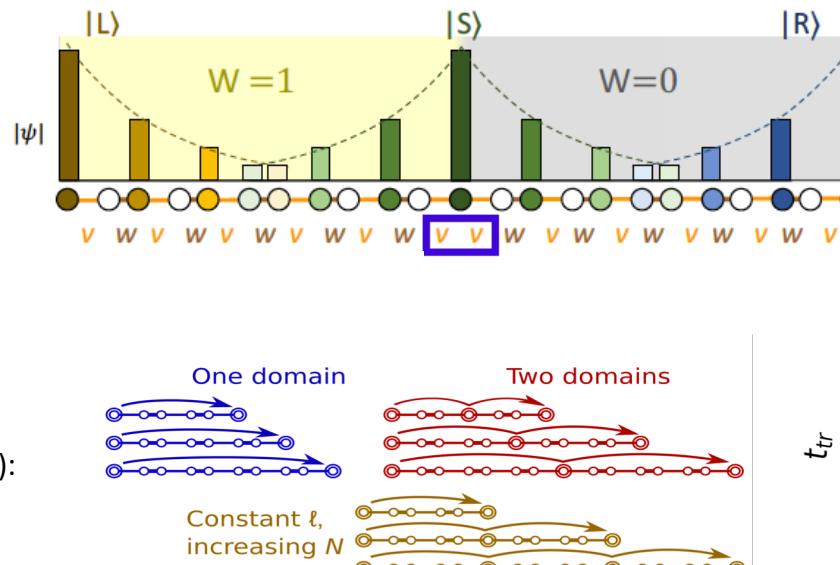
ac driven protocol to simulate the extended SSH in a QD array with new topological phases and edge states

## Outlook

Topological domain walls as quantum amplifiers (J. Zurita et al., arXiv:2208.00797)



Transfer time ( $L = 26$ ):  
1 dom: 1131.2 ns  
5 doms: 4.1 ns



## Summary

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Topological domain walls as quantum amplifiers (J. Zurita et al., arXiv:2208.00797)

## Outlook

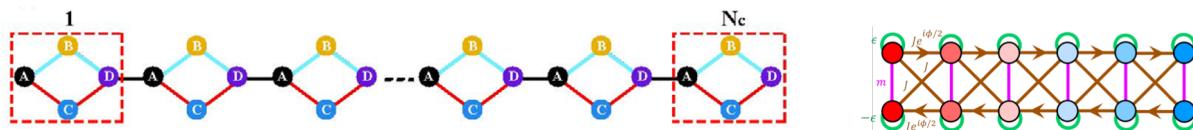
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Investigate other quasi-1D hamiltonians for transferring Information mediated by edge states



J. Zurita, et al., Advances Quantum Tech., 3, 1900105 (2020).

J. Zurita et al., Quantum 5, 591 (2021)

# Electron Dipole Spin Resonance (EDSR)

## Spin Orbit Interaction (SOI)

$$SOI + E(t) = E_0 \cos(\omega t) \rightarrow B_0 \cos(\omega t) \quad \text{Effective magnetic field}$$

- **Rashba SOI:** Structural inversion asymmetry

$$H_{so} = \alpha(p_x\sigma_y - py\sigma_x) + \beta(-px\sigma_x + py\sigma_y)$$

- **Dresselhaus SOI:** Lack of bulk inversion symmetry

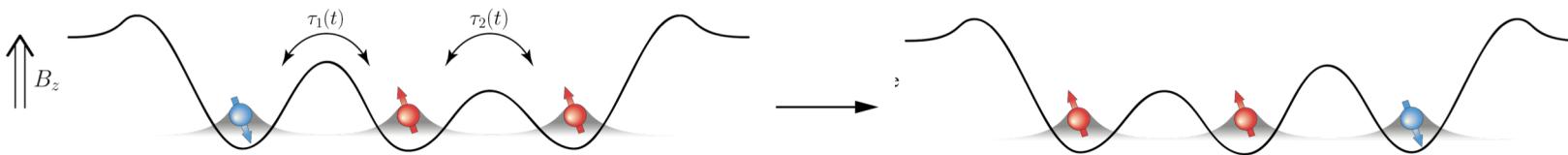
(GaAs, InAs, In Sb..)

$$\mathbf{B}_{\text{eff}}(\mathbf{x}, \mathbf{y}) = \mathbf{n} \otimes \mathbf{B}_{ext}$$

$$\mathbf{x}(t) \rightarrow \mathbf{B}_{\text{eff}}(t)$$

$$n_x = (-\alpha y - \beta x) \frac{2m^*}{\hbar}; n_y = (\alpha x + \beta y) \frac{2m^*}{\hbar}; n_z = 0$$

# Triple QD: HH half filling



$$J^{ab} \equiv \tau_a \tau_b / U \quad a, b = \{N, F\}$$

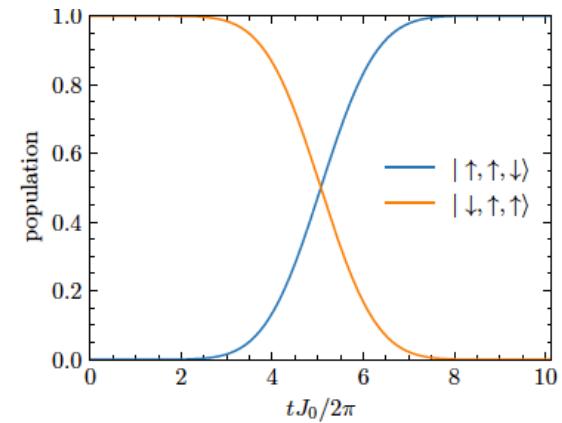
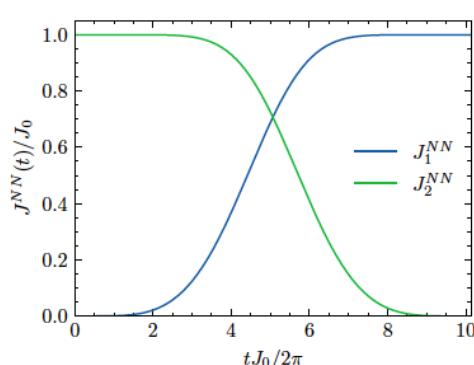
**STA**

$$J_0 \equiv \max(J_1(t), J_2(t))$$

For  $J_i^{NN} = J_i^{FF} = J_i$

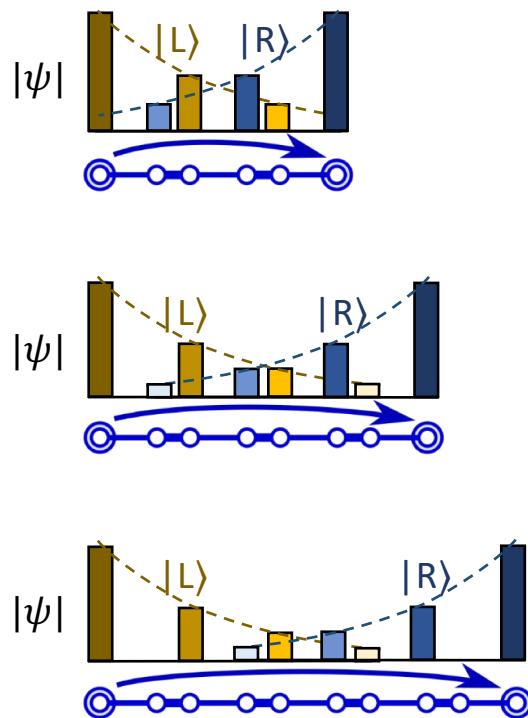
$$|DS\rangle = \sin \theta |\downarrow, \uparrow, \uparrow\rangle - \cos \theta |\uparrow, \uparrow, \downarrow\rangle$$

$$\tan \theta \equiv J_2/J_1$$



D. Fernández et al., in progress

## Slow dynamics



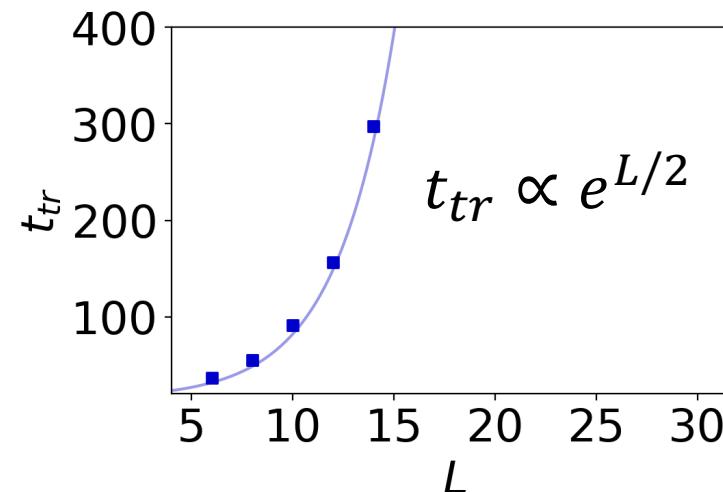
$L = 6$

$L = 8$

$L = 10$

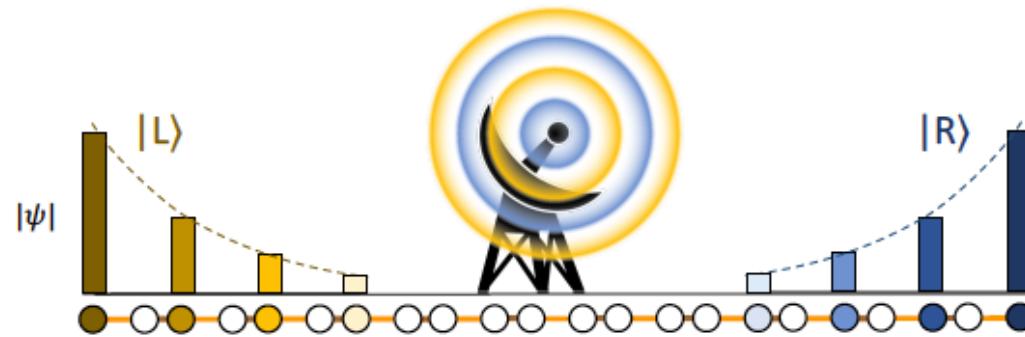
Transfer times

$$t_{tr} \propto (\langle L | \mathcal{H} | R \rangle)^{-1}$$

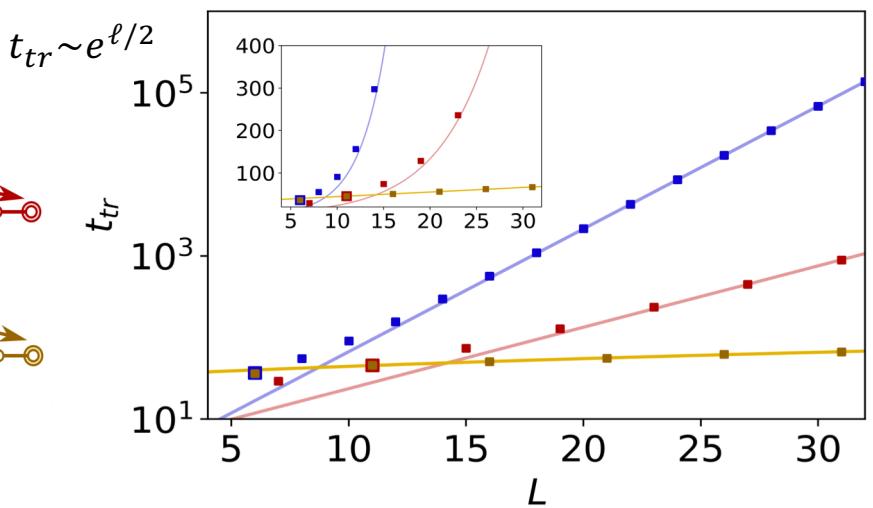
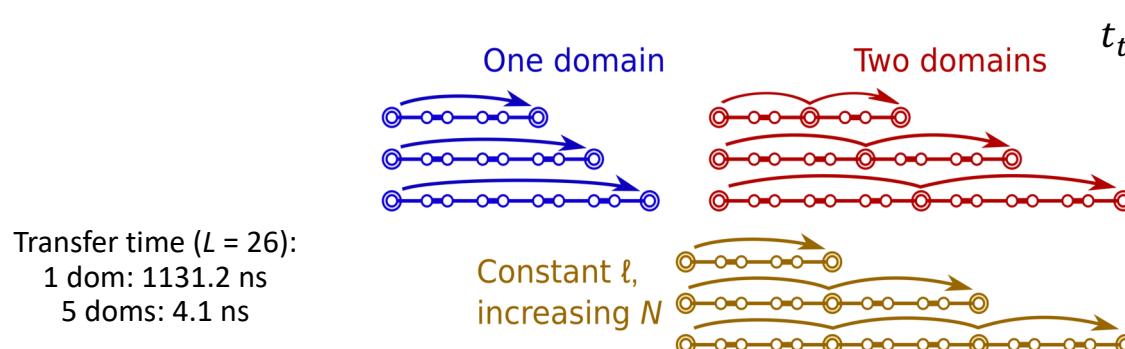
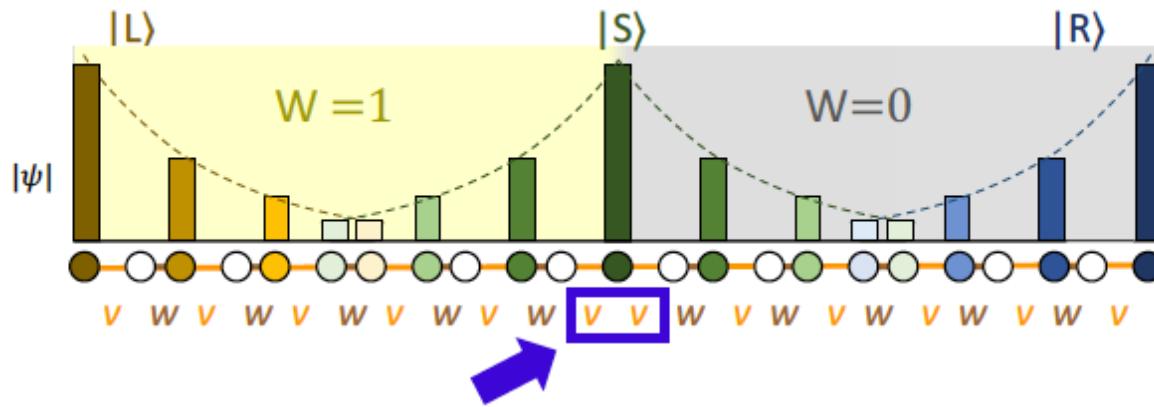


## Topological domain walls as quantum amplifiers (J. Zurita et al., arXiv:2208.00797)

Can we add amplifiers?



## Topological domain walls as quantum amplifiers (J. Zurita et al., arXiv:2208.00797)



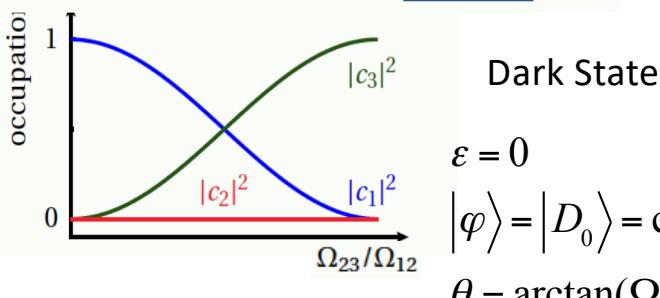
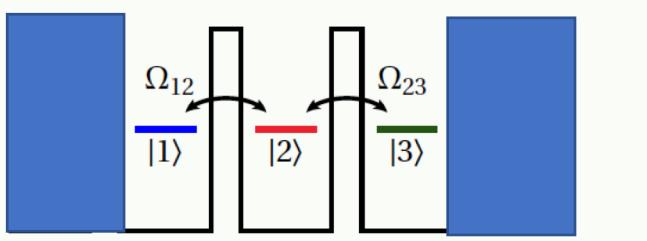
# Quantum State Transfer in QDs

CTAP

A. Greentree et al., PRB, 70, 235317 (2004)

$$\Omega_{12}(t) = \Omega^{\max} \exp \left[ - \left( t - \frac{t_{\max} + \sigma}{2} \right)^2 / (2\sigma^2) \right]$$

$$\Omega_{23}(t) = \Omega^{\max} \exp \left[ - \left( t - \frac{t_{\max} - \sigma}{2} \right)^2 / (2\sigma^2) \right]$$



J. Huneke et al., PRL  
110,036802 (2013)

$$|\mathcal{E}_0 - \mathcal{E}_{\pm}| \gg |\langle \dot{\mathcal{D}}_0 | \mathcal{D}_{\pm} \rangle|.$$

Shortcuts to Adiabaticity (STA): versatile ways to speed up adiabatic passages.

(D. Guéry-Odelin et al., Rev. Modern Phys., 91, 045001, 2019)

Inverse Engineering: impose the desired evolution of the occupation and infer from it the time evolution of the parameters.

$$\tilde{H}(t) = \tilde{\Omega}_{12}(t)c_1^+c_2 + \tilde{\Omega}_{23}(t)c_2^+c_3 + h.c$$

$$|\Psi(t)\rangle = \cos\chi \cos\eta|1\rangle - i \sin\eta|2\rangle - \sin\chi \cos\eta|3\rangle$$

Boundary conditions  $\chi(0) = 0, \chi(t_f) = \pi/2, \eta(0) = 0, \eta(t_f) = 0$

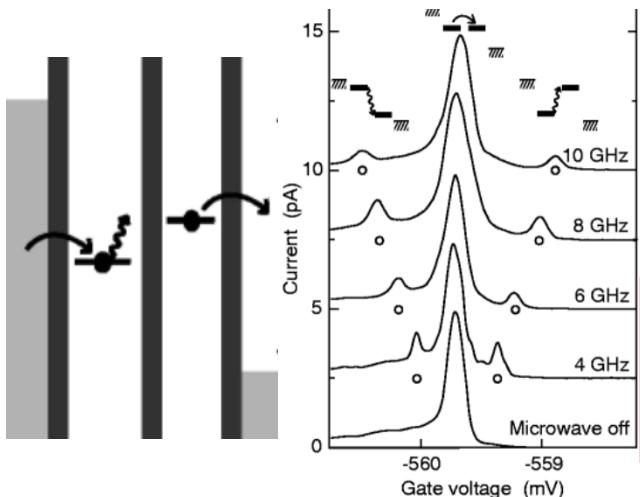
+ Ansatz for  $\chi, \eta$

$$i\hbar\partial_t\Psi(t) = \tilde{H}(t)\Psi(t) \longrightarrow \tilde{\Omega}_{12}(t), \tilde{\Omega}_{23}(t)$$

Y. Ban, et al., Nanotechnology, 29, 505201 (2018)

# Driving with periodic AC fields

- Photoassisted Tunneling (PAT) in quantum dots

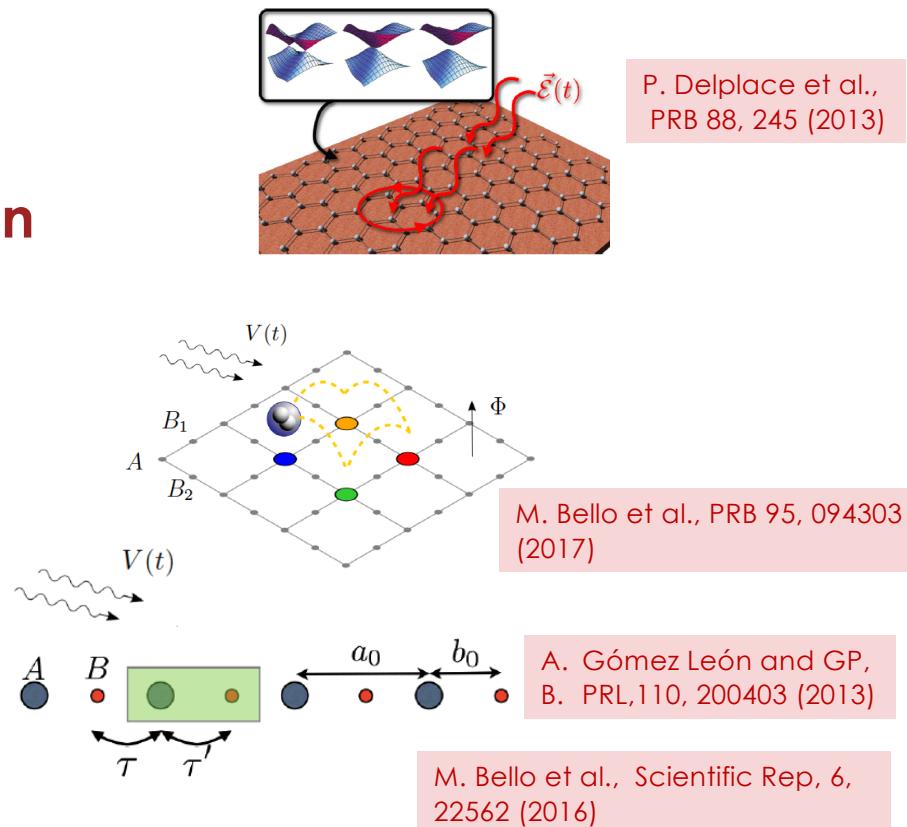


T. H. Oosterkamp et al.,  
Nature 395, 873-876, 1998

## Bonds renormalization

Coherent destruction  
of tunnel,  
P. Hänggi, PRL, 1991

- Tuning electronic and topological properties in driven systems



P. Delplace et al.,  
PRB 88, 245 (2013)

M. Bello et al., PRB 95, 094303  
(2017)

A. Gómez León and GP,  
B. PRL, 110, 200403 (2013)

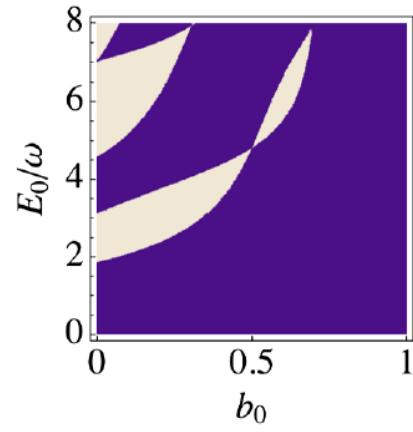
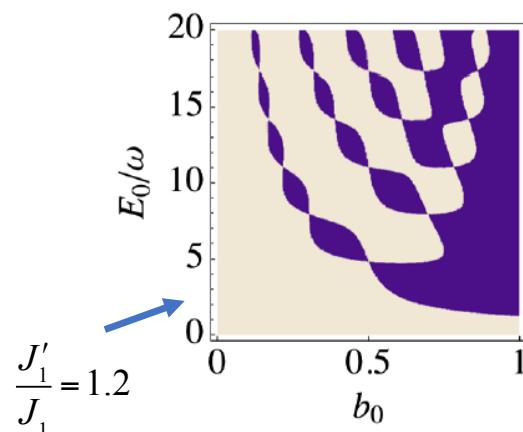
M. Bello et al., Scientific Rep, 6,  
22562 (2016)

# Driving with periodic AC fields

## AC Driven Dimer Chain

A. Gómez León and G.P., PRL, 2013

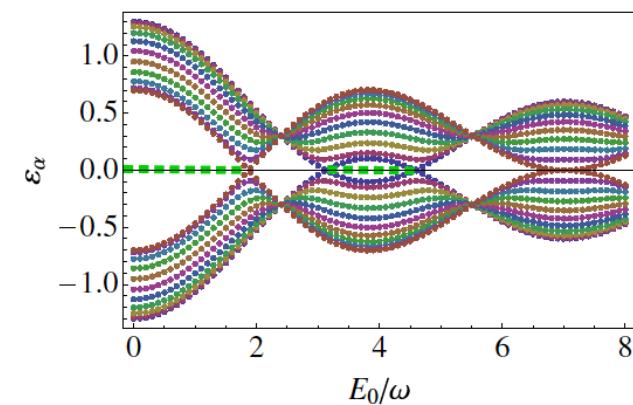
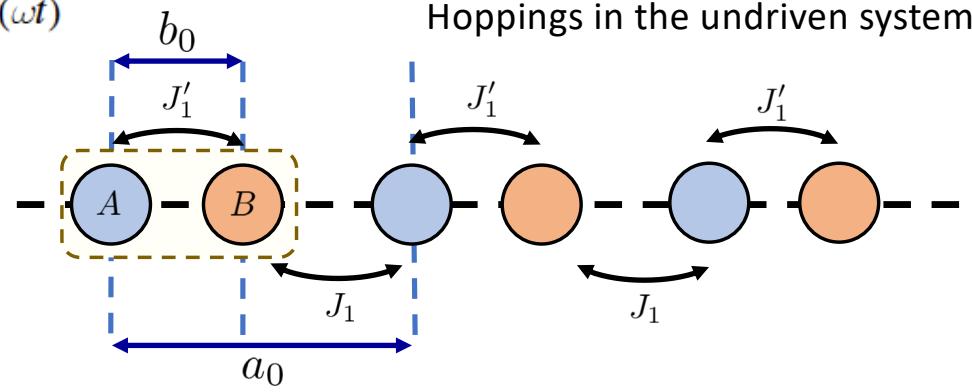
**High-frequency regime**  $H(t) = H_{\text{SSH}} + A \sum_{n=1}^N x_n c_n^\dagger c_n \cos(\omega t)$



Undriven case :

$W=0, J'_1/J_1 > 1$  Light brown

$W=1, J'_1/J_1 < 1$  Blue



# Driving protocol

## what we have...

- ❑ monomer array of QDs with long-range hopping



- ❑ Exponentially decaying hoppings with distance

## what we can achieve...

- ❑ spatially modulated hoppings that create bond ordering
- ❑ certain key symmetries that provide for topological protection
- ❑ control and tunability of long-range hoppings