

I. Phys. Mater. **6** 034004 (2023)

Photo-assisted spin transport in double quantum dots with spin–orbit interaction

David Fernández Fernández, Jordi Picó Cortés,

Sergio Vela Liñán, Gloria Platero

Instituto de Ciencia de Materiales de Madrid



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Semiconductor spin qubits

High fidelity one- and two-qubit gates

Long dephasing and decoherence times

Promise of **scalability** for error correction codes



01.

A. R. Mills, et al., Nat. Commun. 10, 1063 (2019)



PA. Mortemousque, et al., Nat. Nanotechnol. **16**, 296-301 (2021)



D. Jirovec, et al., PRL. 128, 126803 (2022)

Spin-orbit coupling

+ Fast **spin control**

02.

- + Simplify the experimental device (remove oscillating magnetic fields)
- + New and interesting phenomena
- Usually seen as a source of error
- Couples spin to electric noise (charge noise)

Dresselhaus SOC

$$H = \beta(\sigma_x k_x - \sigma_y k_y)$$



- Bulk inversion asymmetry
- Fixed at device growing

Rashba SOC

$$H = \alpha(\boldsymbol{\sigma} \times \boldsymbol{k}) \cdot \hat{z}$$



- Structure inversion asymmetry
- Tunable with electric fields

02. Spin-orbit coupling + **interdot motion**

Spin conserving

 $/_{|\mathrm{R}\rangle} \qquad \tau_0 \equiv \langle L\sigma | \, \hat{H}_{\mathrm{DQD}} \, | R\sigma \rangle$ $|L\rangle$ $\chi \equiv \frac{1}{\tau_0 / \tau_{\rm sf} + 1} \longrightarrow \chi = 0, \ \tau_{\rm sf} = 0$ Spin flip (Rashba SOC) $\tau \equiv \tau_0 + \tau_{\rm sf}$ $\tau_{\rm sf} \equiv \left\langle L\sigma \right| \hat{H}_{\rm SOC} \left| R\sigma' \right\rangle$ $= \langle \sigma | \boldsymbol{\alpha} \cdot \hat{\boldsymbol{\sigma}} | \sigma' \rangle \langle L | \hat{k}_x | R \rangle$ Preserves time-reversal symmetry Parallel to SOC vector

02. Spin-orbit coupling + intradot motion

Single dot Hamiltonian

 $\hat{H}(x,t) = \hat{H}_0 + e\hat{x}E(t) + \boldsymbol{\alpha} \cdot \hat{\boldsymbol{\sigma}}\hat{k}_x + E_z\hat{\sigma}_z/2$

Quantized Hamiltonian for a **harmonic oscillator** \hat{M}

 $\hat{H}_{\text{osc}} = \omega_0(\hat{a}^{\dagger}\hat{a} + 1) + el_0E(t)(\hat{a}^{\dagger} + \hat{a}) + \frac{i}{l_0}\boldsymbol{\alpha}\cdot\hat{\boldsymbol{\sigma}}(\hat{a}^{\dagger} - \hat{a}) + E_z\hat{\sigma}_z/2$

Effective Hamiltonian (Schrieffer-Wolff transformation) $\hat{H}_{eff} = \hat{H}_0 + E_z \left[1 - \frac{|\boldsymbol{\alpha}|^2}{2l_0(\omega_0^2 - E_z^2)} \right] \hat{\sigma}_z + \frac{E_z eE(t)}{\omega_0^2 - E_z^2} \boldsymbol{\alpha}^{\perp} \cdot \hat{\boldsymbol{\sigma}}$ Time-reversal symmetry must be broken + Perpendicular to SOC vector



03. Model for double quantum dot

$$\begin{array}{c}
 & \overbrace{f} B_z \\
 & F_z \\
 & F_z$$

$$\hat{H}(t) = \begin{pmatrix} \epsilon_L(t) + E_z/2 & \beta(t)/2 & -\tau_0 & -\tau_{\rm sf} \\ \beta(t)/2 & \epsilon_L(t) - E_z/2 & \tau_{\rm sf} & -\tau_0 \\ -\tau_0 & \tau_{\rm sf} & \epsilon_R + E_z/2 & \beta(t)/2 \\ -\tau_{\rm sf} & -\tau_0 & \beta(t)/2 & \epsilon_R - E_z/2 \end{pmatrix}$$

04. Open system

- Unpolarized contacts
- Infinite bias approximation
- Current is **unidirectional**



Master equation in Lindblad form

$$i\dot{\hat{\rho}}(t) = \left[\hat{H}(t), \hat{\rho}(t)\right] + \Gamma \sum_{\sigma} \left(\hat{\mathcal{D}}\left[\hat{d}_{L,\sigma}^{\dagger}\right](\hat{\rho}(t)) + \hat{\mathcal{D}}\left[\hat{d}_{R,\sigma}\right](\hat{\rho}(t))\right) \\ \hat{\mathcal{D}}\left[\hat{o}\right](\hat{\rho}(t)) \equiv \hat{o}\hat{\rho}(t)\hat{o}^{\dagger} - \frac{1}{2}\left\{\hat{o}\hat{o}^{\dagger}, \hat{\rho}(t)\right\}$$

04. Open system

The Liouvillian operator define the system dynamic

 $\hat{\mathcal{L}}(t)\hat{\rho}(t)=i\dot{\hat{\rho}}(t)$

The **steady state** is given by the eigenstate with zero eigenvalue





Since the system is periodic in time, we compute the **average populations** $\rho_{\eta,\sigma}^{\infty} \equiv \frac{1}{T} \int_{0}^{T} dt \langle \eta, \sigma | \hat{\rho}_{\rm ss}(t) | \eta, \sigma \rangle$

The **current intensity** is defined in terms of the population of the right dot $I^{\infty} = e\Gamma \left(\rho_{R,\uparrow}^{\infty} + \rho_{R,\downarrow}^{\infty}\right)$

05. Photo-assisted tunneling (PAT)

Two-level system with driving in the detuning



06. Landau-Zener-Stückelberg interf.

- Coherent destruction of tunneling (CDT): $J_n(\epsilon_{\rm ac}/\omega) = 0$
- Main resonances for spin-conserving processes: $\delta = n\omega$
- Satellite peaks for spin-flip processes: $\delta \sim n\omega \pm E_z$





 $\omega = 10\tau = 100\Gamma$ $\chi = 0.2$ $E_z = 0.3\omega \quad \beta_{SO} = 0$

07. Current through the DQD

2

2





- Four different processes
- **Spin polarized current** for spin-flip processes
- Highly tunable polarization with detuning

08. Effect of EDSR

When the **magnetic field is in resonance**, EDSR produces spin rotations

Two satellite peaks appear due to the **renormalization of the magnetic field**

However, these two peaks are **asymmetric**, indicating a **spin-imbalance** process



 $\begin{array}{l} \underline{Parameters}\\ \omega=10\tau=100\Gamma\\ \epsilon_{\rm ac}=1.2\omega \ \chi=0.2\\ E_z=\omega \end{array}$



- Spin-flip tunneling
- Spin-imbalance tunneling

$$\begin{split} & \tilde{\hat{H}} = \frac{\tilde{E}_z}{2} \hat{\sigma}_z - \delta \hat{\eta}_z - \tilde{\tau}_{\rm sf} \hat{\eta}_x \hat{\sigma}_x \\ & - \left[\tilde{\tau}_{\uparrow} (\mathbb{1}_{\sigma} + \hat{\sigma}_z) + \tilde{\tau}_{\downarrow} (\mathbb{1}_{\sigma} - \hat{\sigma}_z) \right] \frac{\hat{\eta}_x}{2} \end{split}$$



 $\tau_0^{\rm RWA} \equiv \tau_0 J_0(\epsilon_{\rm ac}/\omega)$

Effective model 09

$$\hat{\tilde{H}} = \begin{pmatrix} (\tilde{E}_z - \delta)/2 & 0 & -\tilde{\tau}_{\uparrow} & -\tilde{\tau}_{\mathrm{sf}} \\ 0 & (-\tilde{E}_z - \delta)/2 & -\tilde{\tau}_{\mathrm{sf}} & -\tilde{\tau}_{\downarrow} \\ -\tilde{\tau}_{\uparrow} & -\tilde{\tau}_{\mathrm{sf}} & (\tilde{E}_z + \delta)/2 & 0 \\ -\tilde{\tau}_{\mathrm{sf}} & -\tilde{\tau}_{\downarrow} & 0 & (-\tilde{E}_z + \delta)/2 \end{pmatrix}$$



The resonances appear when the **spin levels** are aligned $\tilde{E}_z = \beta_{\rm SO}/2 + \mathcal{O}\left((\beta_{\rm SO}/\omega)^5\right)$ $\delta_{\rm res} \sim \pm \beta_{\rm SO}/2$

Each spin channel has a **different tunneling rate**, producing the asymmetry in the current intensity

 $> I^{\infty}$

 $\tilde{\tau}_{\uparrow} \simeq 0.06\omega > \tilde{\tau}_{\downarrow} \simeq 0.04\omega \qquad \tilde{\tau}_{\rm sf} \simeq 0.002\omega$





10. Dark states

A dark state is an **eigenstate uncoupled** to the right contact

 $|\mathrm{DS}\rangle = \cos(\vartheta/2) |\mathrm{L}\uparrow\rangle + \sin(\vartheta/2)e^{i\phi} |\mathrm{L}\downarrow\rangle$

The presence of dark states (in some cases) can be attributed to quantum coherence interferences

Dark states have been extensively studied in **driven system,** but the presence of high **SOC** is still to be explored



Arnau Sala, et al., PRB. **104**, 085421 (2021)



D. V. Khomitsky, et al., PRB. **106**, 195414 (2022)



Yuan Zhou, et al., arXiv 2209.14528v1 (2022)

Coherence interferences

$$\hat{H}_{\text{RWA}}^{(n)} = \begin{pmatrix} 0 & 0 & -\tau_0 J_0(\epsilon_{\text{ac}}/\omega) & \text{Direct transfer} \\ \tau_{\text{sf}} \to \tau_{\text{sf}} J_{-n}(\epsilon_{\text{ac}}/\omega) & \text{Emits } n \text{ photons} \\ \tau_{\text{sf}} \to \tau_{\text{sf}} J_n(\epsilon_{\text{ac}}/\omega) & \text{Absorbs } n \text{ photons} \end{pmatrix}$$

$$\hat{H}_{\text{RWA}}^{(n)} = \begin{pmatrix} 0 & 0 & -\tau_0 J_0(\epsilon_{\text{ac}}/\omega) & -\tau_{\text{sf}} J_n(\epsilon_{\text{ac}}/\omega) \\ 0 & 0 & \tau_{\text{sf}} J_{-n}(\epsilon_{\text{ac}}/\omega) & -\tau_0 J_0(\epsilon_{\text{ac}}/\omega) \\ -\tau_0 J_0(\epsilon_{\text{ac}}/\omega) & \tau_{\text{sf}} J_{-n}(\epsilon_{\text{ac}}/\omega) & 0 & 0 \\ -\tau_{\text{sf}} J_n(\epsilon_{\text{ac}}/\omega) & -\tau_0 J_0(\epsilon_{\text{ac}}/\omega) & 0 & 0 \end{pmatrix}$$

 $n=2k
ightarrow \hat{H}_{
m RWA}^{(n)}\propto au_{
m sf}\hat{\eta}_y\hat{\sigma}_y
ightarrow$ Perpendicular to spin conserving

 $n=2k+1
ightarrow \hat{H}_{
m BWA}^{(n)}\propto au_{
m sf}\hat{\eta}_x\hat{\sigma}_x
ightarrow$ Parallel to spin conserving



The dark state can be found by **exact diagonalization**

$$\chi_{\rm DS}^{(n)} = \frac{J_0(\epsilon_{\rm ac}/\omega)}{J_0(\epsilon_{\rm ac}/\omega) \pm i^{n+1}J_n(\epsilon_{\rm ac}/\omega)}$$

Recall the definition: $\chi \equiv \frac{1}{\tau_0/\tau_{\rm sf}+1} \in \mathbb{R}$

The dark state is only present for an **odd** number of photons

12. **Even-odd effect** Analytical prediction 1.00.4n = 2n = 10.3 ≈ 0.5 0.20.10.0 0.022 0 () $\epsilon_{\rm ac}/\omega$

- Close to $\chi \sim 1$ the dark state is not produced due to coherent interferences
- The dark state is only present when • time-reversal symmetry is broken
- Analytical prediction in **agreement** with ٠ numerical results
- **Sharp drop** in the total current

Locate the dark state versus driving amplitude to measure the effective SOC in an experimental device

Parameters $\omega = 10\tau = 100\Gamma$ $\delta = 0$ $\beta_{\rm SO} = 0 \quad E_z = n\omega$

 $\epsilon_{\rm ac}/\omega$

13. Dark state out-of-resonance

$\beta_{\rm SO} = 0$

- The presence of a dark state is **sensitive to the magnetic field**
- Out of resonance the only dark state corresponds to $\chi \sim 1$



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$\beta_{\rm SO} \neq 0$

- The **dark state survives** for all magnetic field values
- The location of the dark state has not analytical solution



14. Floquet theory



 $\begin{array}{l} \underline{Parameters}\\ \omega=10\tau=100\Gamma\\ \delta=0 \ \beta_{\mathrm{SO}}=0.2E_z \end{array}$

For a time-periodic Hamiltonian, define the operator

 $\hat{\mathcal{H}}(t) \equiv \hat{H}(t) - i\partial_t$

The Floquet modes and quasienegies are given by

 $\hat{\mathcal{H}}(t)\Phi_{\alpha}(t) = \varepsilon_{\alpha}\Phi_{\alpha}(t)$

The **crossing of quasienergies** indicates that the driving restores some system's symmetry (Von Neumann-Wigner theorem)



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Agreement between numerical results from dynamics and Floquet theory

 $\begin{array}{l} \underline{Parameters}\\ \omega=10\tau=100\Gamma\\ \delta=0 \ \beta_{\mathrm{SO}}=0.2E_z \end{array}$

15. Flopping mode qubit

Encode the qubit in the **spin** of the particle

Use the **spatial extension** of the wave function to **manipulate** the qubit

Due to **the large dipole moment**, the coupling with a **cavity** can be higher than the decoherence rate



M. Benito, et al., PRB. **100**, 125430 (2019)



X. Croot, et al., Phys. Rev. Research 2, 012006(R) (2020)



P. M. Mutter, et al., Phys. Rev. Research **3**, 013194 (2021)

16. Effective driven system

- The Zeeman splitting is in resonance with the driving ($E_z = n\omega$)
- **Time-dependent** Schrieffer-Wolff transformation
- Effective model for large driving amplitude
- Two-axis control by tuning the driving phase, and the detuning

$$\epsilon_{\rm L}(t) = \epsilon_{\rm ac} \cos(\omega t + \varphi)$$
$$\epsilon_{\rm B} = \delta$$

$$\hat{H}_{n}^{(2)}(t) = \frac{\hat{\tau}_{z}}{2} \left\{ -\tilde{\delta}^{(2)} + (E_{z} + \tilde{b}_{z}^{(2)} - n\omega)\hat{\sigma}_{z} + \tilde{b}_{n,\perp}^{(2)} [\cos(n\varphi)\hat{\sigma}_{x} + \sin(n\varphi)\hat{\sigma}_{y}] \right\}$$

$$\tilde{b}_{n,\perp}^{(2)} \equiv \sum_{\nu} J_{\nu} \left(\frac{\epsilon_{ac}}{\omega}\right) J_{\nu+n} \left(\frac{\epsilon_{ac}}{\omega}\right) \left(\frac{\tau_{0}\tau_{sf}}{\delta - \nu\omega} - \frac{\tau_{0}\tau_{sf}}{\delta - E_{z} - \nu\omega}\right)$$

$$-\sum_{\nu} J_{\nu} \left(\frac{\epsilon_{ac}}{\omega}\right) J_{\nu-n} \left(\frac{\epsilon_{ac}}{\omega}\right) \left(\frac{\tau_{0}\tau_{sf}}{\delta - \nu\omega} - \frac{\tau_{0}\tau_{sf}}{\delta + E_{z} - \nu\omega}\right)$$
Virtual second-order PAT
$$\frac{1.00}{0.25} \left(\frac{|L \downarrow\rangle}{0.25} + \frac{|R \downarrow\rangle}{0.00}\right) \left(\frac{|L \downarrow\rangle}{250} + \frac{|R \downarrow\rangle}{500} + Effective model$$

$$\lim_{\nu \to \infty} I_{1} = I_{1} + I_{2} +$$

Parameters

 $\chi = 0.2 \quad E_z = \omega$ $\beta_{\rm SO} = 0$

17. Rabi frequency

The Rabi frequency is given by





18. Conclusions

- Study of a **periodically driven** system with **high SOC**
- Highly polarizable spin current





• System as a **flopping-mode** qubit with **sweep-spots**





