EXCELENCIA MARÍA DE MAEZTU 2023 - 2027

Topological Superconductivity and Majorana Modes in Magnetic Topological Insulators

Daniele Di Miceli and Llorenç Serra

Outline

i. Topology in Condensed Matter Physics

- Quantum Anomalous Hall State
- Topological Insulators

ii. Topological Superconductors in MTIs

- 2D Chiral Superconductors
- 1D Topological Superconductor
- The Emergence of Majorana Modes

iii. Detecting Majorana Excitations

- Antisymmetric Electric Conductance
- Numerical Results

MATERIALS SCIENCE

Higher-order topological insulators

Frank Schindler,[1] Ashley M. Cook,[1] Maia G. Vergniory,[2,3]* Zhijun Wang,[4] Stuart S. P. Parkin,[5] B. Andrei Bernevig,[4,2,6†] Titus Neupert[1†]

Three-dimensional topological (crystalline) insulators are materials with an insulating bulk b
states that are topologically protected by time-reversal (or spatial) symmetries. We extenc
dimensional topological insulators to systems that host no gapless surface states but exhibit t
gapless hinge states. Their topological character is protected by spatiotemporal symmetries of
cases: (i) Chiral higher-order topological insulators protected by the combination of time-re
rotation symmetry. Their hinge states are chiral modes, and the bulk topology is $\mathbb{Z}_2$-classified. (
topological insulators protected by time-reversal and mirror symmetries. Their hinge states com
the bulk topology is $\mathbb{Z}$-classified. We provide the topological invariants for both cases. Furthermc
as well as surface-modified Bi_2TeI, BiSe, and BiTe are helical higher-order topological insulators
experimental setup to detect the hinge states.

ARTICLE

doi:10.1038/nature23268

Topological quantum chemistry

Barry Bradlyn[1]*, L. Elcoro[2]*, Jennifer Cano[1]*, M. G. Vergniory[3,4,5]*, Zhijun Wang[6]*, C. Felser[7], M. I. Aroyo[2] & B. Andrei Bernevig[3,6,8,9]

Since the discovery of topological insulators and semimetals, there has been much research into predicting and experimentally discovering distinct classes of these materials, in which the topology of electronic states leads to robust surface states and electromagnetic responses. This apparent success, however, masks a fundamental shortcoming: topological insulators represent only a few hundred of the 200,000 stoichiometric compounds in material databases. However, it is unclear whether this low number is indicative of the esoteric nature of topological insulators or of a fundamental problem with the current approaches to finding them. Here we propose a complete electronic band theory, which builds on the conventional band theory of electrons, highlighting the link between the topology and local chemical bonding. This theory of topological quantum chemistry provides a description of the universal (across materials), global properties of all possible band structures and (weakly correlated) materials, consisting of a graph-theoretic description of momentum (reciprocal) space and a complementary group-theoretic description in real space. For all 230 crystal symmetry groups, we classify the possible band structures that arise from local atomic orbitals, and show which are topologically non-trivial. Our electronic band theory sheds new light on known topological insulators, and can be used to predict many more.

RESEARCH ARTICLE

TOPOLOGICAL MATTER

All topological bands of all nonmagnetic stoichiometric materials

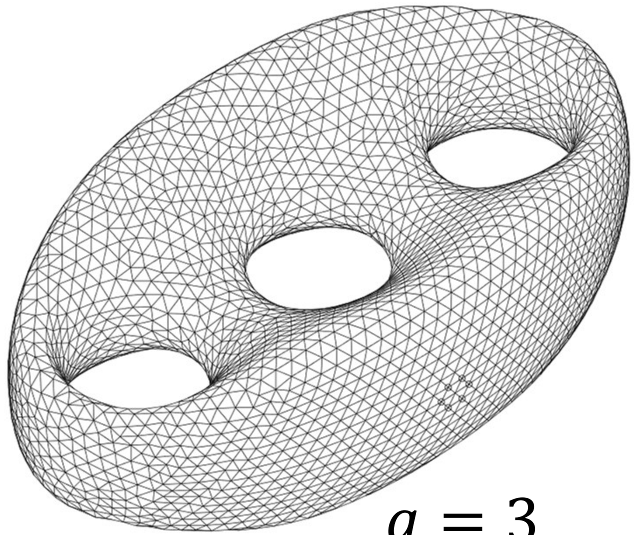
Maia G. Vergniory[1,2,3]*†, Benjamin J. Wieder[4,5,6]*†, Luis Elcoro[7], Stuart S. P. Parkin[8], Claudia Felser[3], B. Andrei Bernevig[6], Nicolas Regnault[6,9]*

Topological quantum chemistry and symmetry-based indicators have facilitated large-scale searches for materials with topological properties at the Fermi energy (E_F). We report the implementation of a publicly accessible catalog of stable and fragile topology in all of the bands both at and away from E_F in the 96,196 processable entries in the Inorganic Crystal Structure Database. Our calculations, which represent the completion of the symmetry-indicated band topology of known nonmagnetic materials, have enabled the discovery of repeat-topological and supertopological materials, including rhombohedral bismuth and Bi_2Mg_3. We find that 52.65% of all materials are topological at E_F, roughly two-thirds of bands across all materials exhibit symmetry-indicated stable topology, and 87.99% of all materials contain at least one stable or fragile topological band.

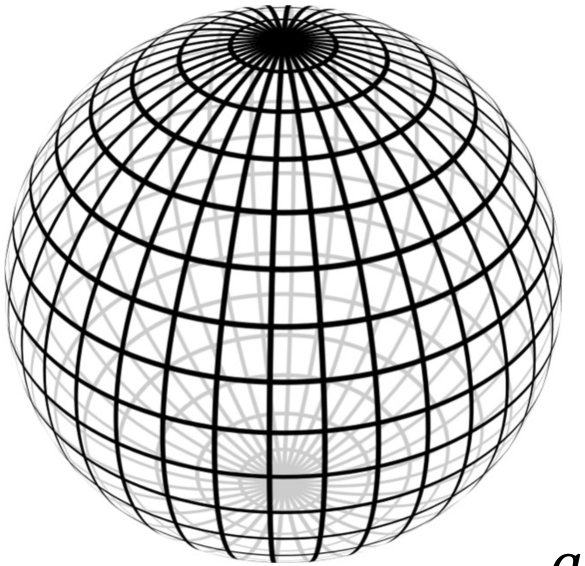
Topology is concerned with the properties of geometric figures that are invariant under continuous deformations (stretching, twisting, ecc.)

- Topologically equivalent shapes can be smoothly deformed into each other

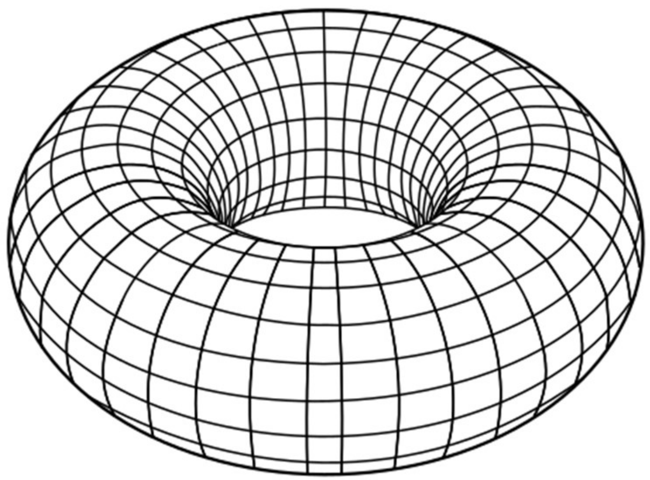
- A discrete topological invariant characterizes the equivalence classes



$g = 3$



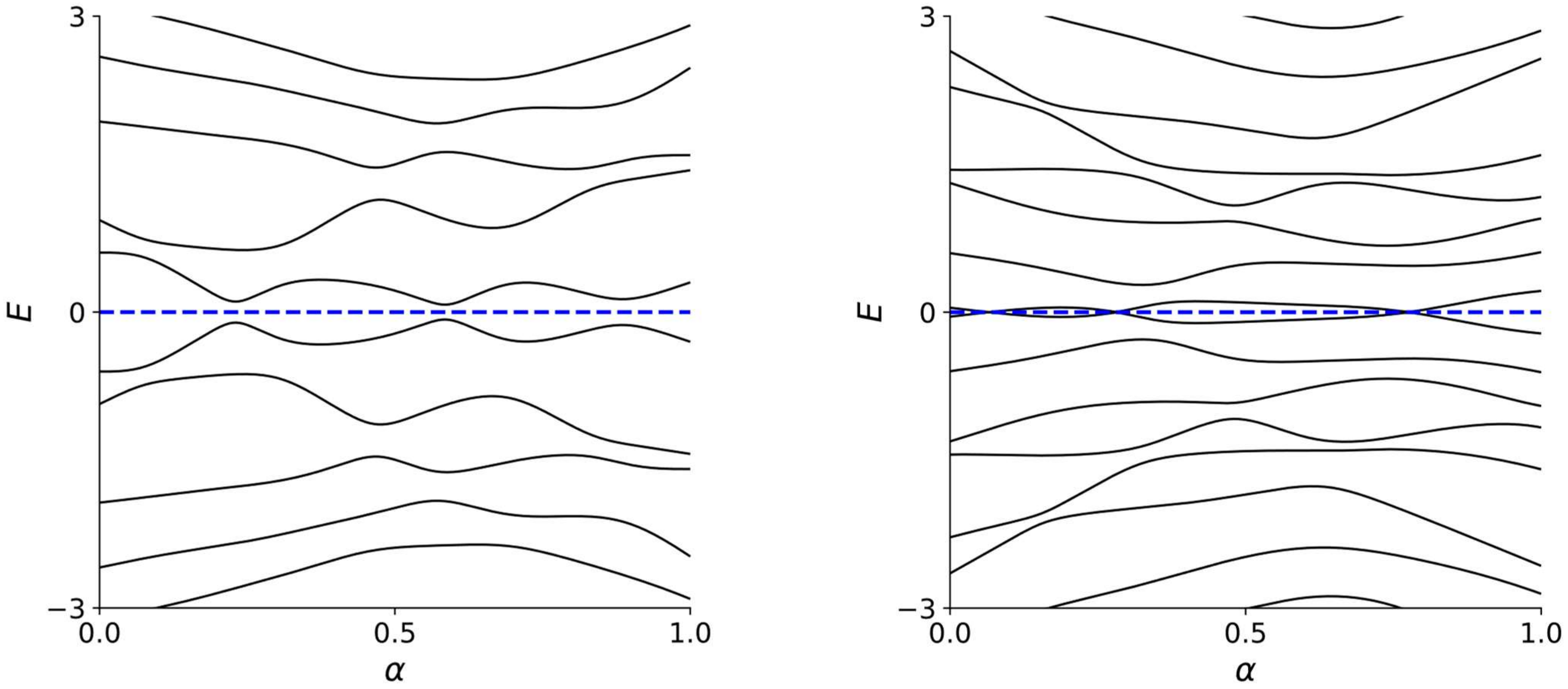
$g = 0$



$g = 1$

Topological classification of insulating Hamiltonians describing gapped band structures

- Topologically equivalent Hamiltonian can be continuously deformed one into each other without closing the energy gap
- A discrete topological invariant characterizes the equivalence classes



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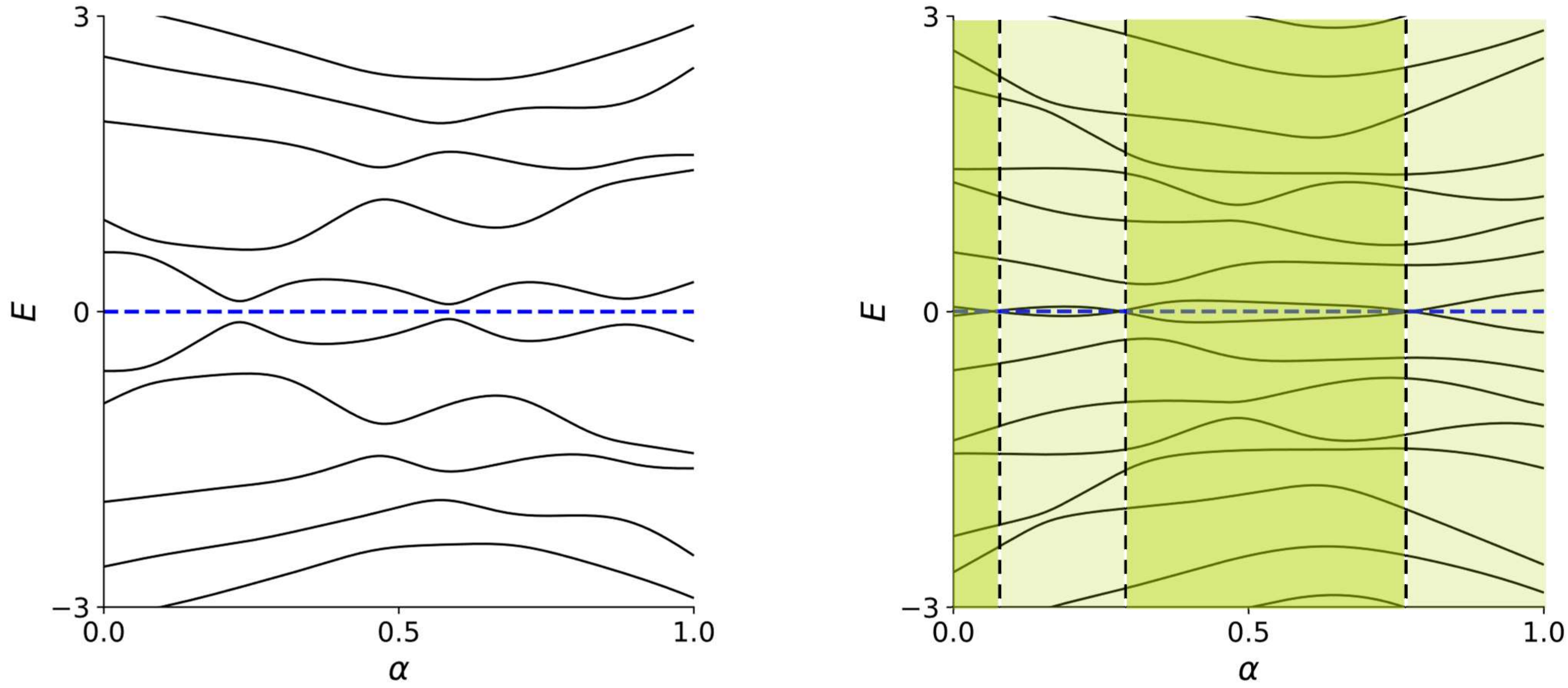


A fundamental consequence of the topological classification is the **bulk-boundary correspondence**

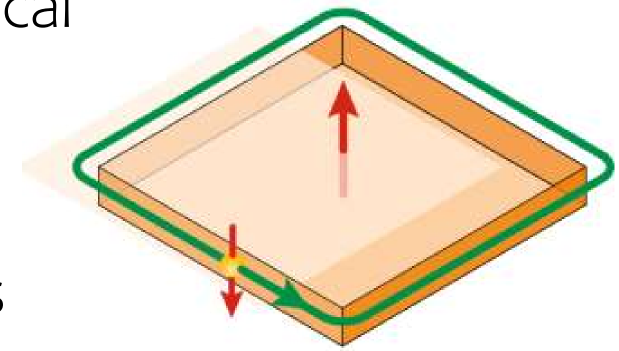
Along the **interfaces** between distinct topological phases the **energy gap** has to **vanish** to allow the topological invariant to change:

- **Zero-energy** **gapless** modes **localized** over the **interfaces** between different topological states of matter

- Topological invariants **count** the number of **zero-energy** surface modes

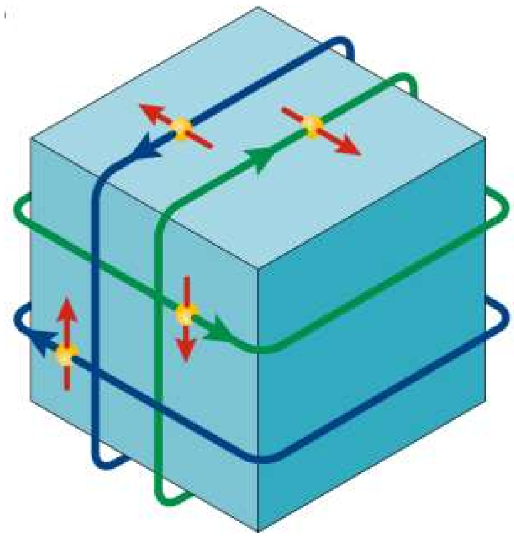
2D topological bulk

1D gapless edge states



3D topological bulk

2D gapless surface states



The quantum anomalous Hall (QAH) state is the simplest 2D topological state

- A single chiral edge mode at the interface with vacuum
- Integer topological invariant called the "Chern number" $\mathcal{C}$

PHYSICAL REVIEW LETTERS

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

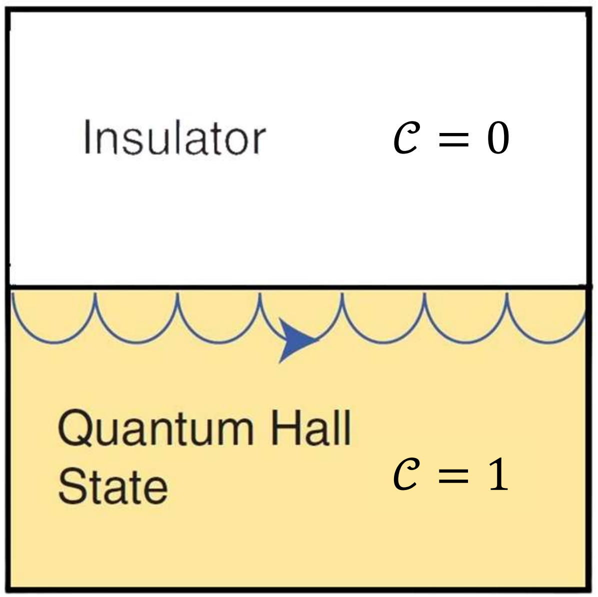
Department of Physics, University of California, San Diego, La Jolla, California 92093

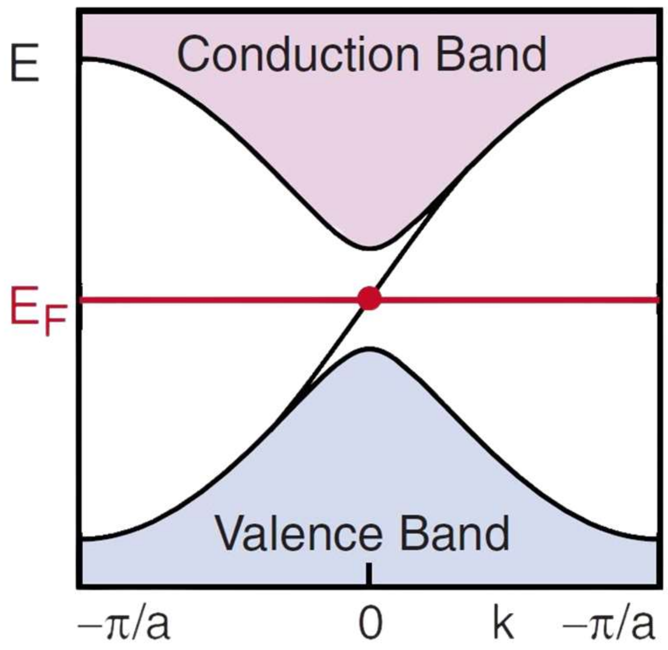
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

The Chern invariant defines a quantized Hall conductance

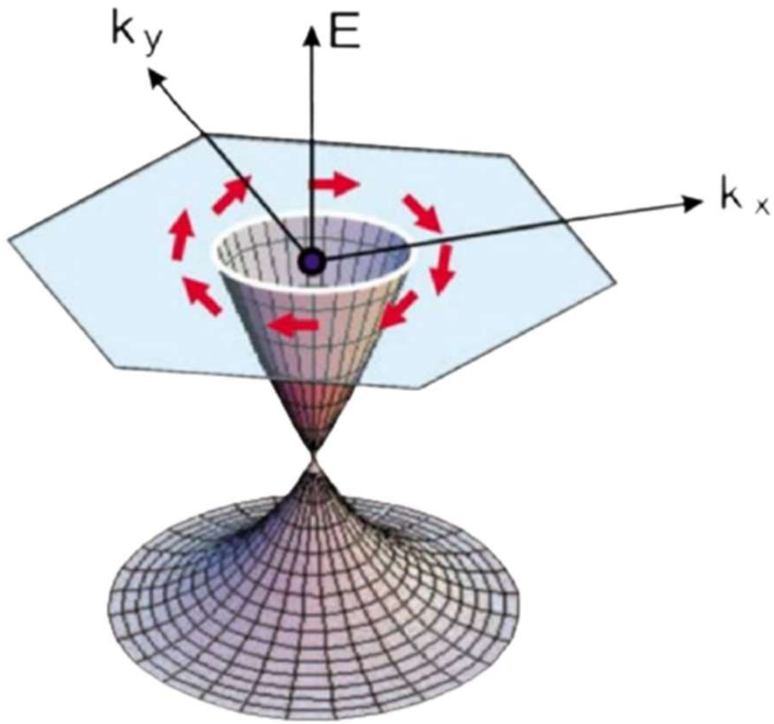
$$\sigma_{xy} = \mathcal{C}\,\frac{e^2}{h}$$





Three-dimensional spinful systems can realize **topological insulating** (TI) phases

- Insulating bulk but **conductive edges**
- Single **Dirac cone** shaped topological surface state
- Topological classification through a set of $\mathbb{Z}_2$ topological invariants



ARTICLES
PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1270
nature physics

Topological insulators in Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3 with a single Dirac cone on the surface

Haijun Zhang[1], Chao-Xing Liu[2], Xiao-Liang Qi[3], Xi Dai[1], Zhong Fang[1] and Shou-Cheng Zhang[3]*

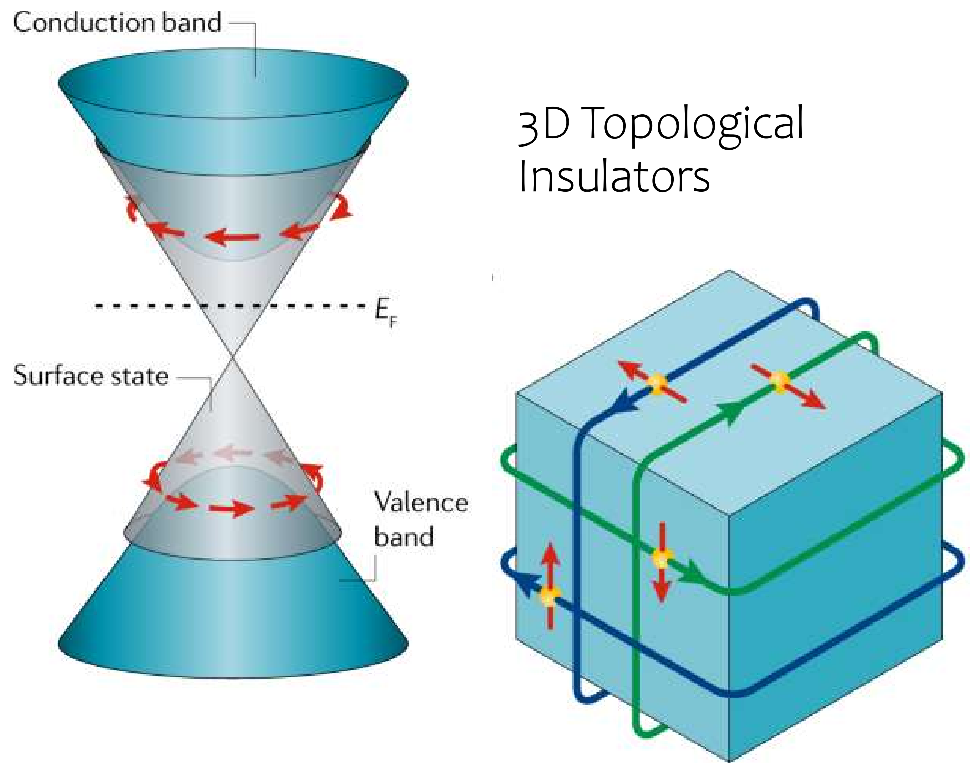
Topological insulators are new states of quantum matter in which surface states residing in the bulk insulating gap of such systems are protected by time-reversal symmetry. The study of such states was originally inspired by the robustness to scattering of conducting edge states in quantum Hall systems. Recently, such analogies have resulted in the discovery of topologically protected states in two-dimensional and three-dimensional band insulators with large spin-orbit coupling. So far, the only known three-dimensional topological insulator is Bi_xSb_{1-x}, which is an alloy with complex surface states. Here, we present the results of first-principles electronic structure calculations of the layered, stoichiometric crystals Sb_2Te_3, Sb_2Se_3, Bi_2Te_3 and Bi_2Se_3. Our calculations predict that Sb_2Te_3, Bi_2Te_3 and Bi_2Se_3 are topological insulators, whereas Sb_2Se_3 is not. These topological insulators have robust and simple surface states consisting of a single Dirac cone at the Γ point. In addition, we predict that Bi_2Se_3 has a topologically non-trivial energy gap of 0.3 eV, which is larger than the energy scale of room temperature. We further present a simple and unified continuum model that captures the salient topological features of this class of materials.

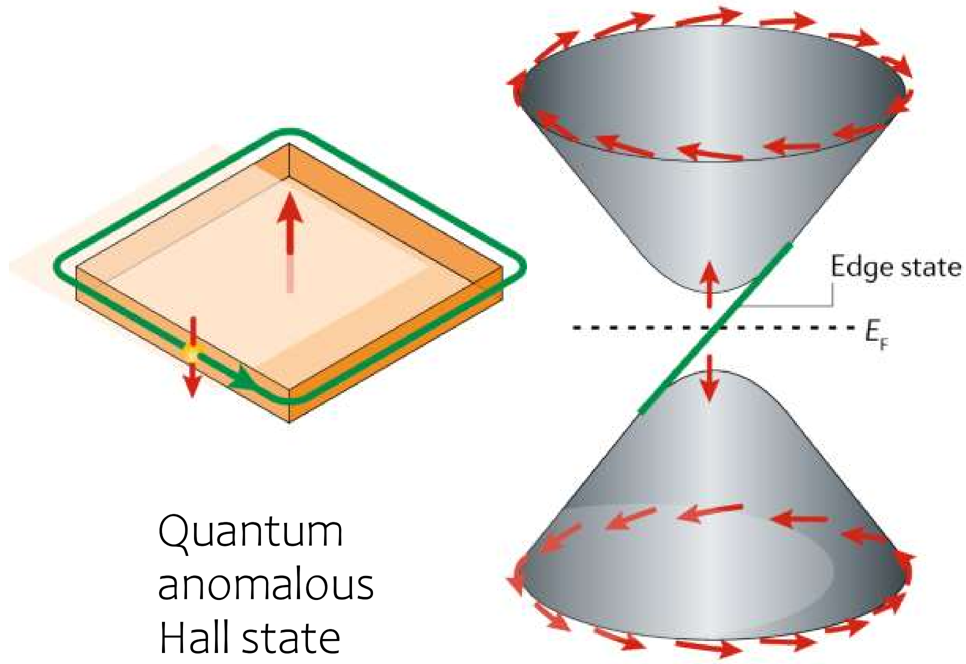
In $\mathrm{Bi_2Se_3}$ the Surface states are protected by a large energy gap around $\approx 0.3\ eV$

Magnetic topological insulators (MTIs) are 3D topological insulators with topological protected **surface states** and **ferromagnetic ordering**

A robust QAH state can be realized when

- TIs are placed in a thin film configuration
- Ferromagnetic **ordering** is induced through magnetic doping





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The topological classification is valid also for superconducting Hamiltonian

- Superconducting (gapped) bulk
- Gapless boundary modes

PHYSICAL REVIEW B **78**, 195125 (2008)

Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]
[1] *Kavli Institute for Theoretical Physics, University of California–Santa Barbara, Santa Barbara, California 93106, USA*
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(Received 11 April 2008; revised manuscript received 13 September 2008; published 26 November 2008)

The topological invariant is determined by symmetry and dimensions

		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
Standard	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
(Wigner-Dyson)	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	−1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
(sublattice)	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	−1	−1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	−1	0	-	$\mathbb{Z}$	-
	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

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Topological Superconductors

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	C	0	−1	0	-	$\mathbb{Z}$	-
	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

2D Chiral Topological Superconductor

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- Gapless boundary modes

PHYSICAL REVIEW B 78, 195125 (2008)

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BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	−1	0	-	$\mathbb{Z}$	-
	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

1D Topological Superconductor

Chiral Topological Superconductor

The **chiral TSC** can be realized through a quantum Hall state in proximity to an ordinary s-wave superconductor

- 2D superconducting (gapped) **bulk**
- 1D counterpropagating **edge modes**

Integer topological invariant $\mathcal{N}$ analogous to the Chern number

PHYSICAL REVIEW B **82**, 184516 (2010)

Chiral topological superconductor from the quantum Hall state

Xiao-Liang Qi,[1,2] Taylor L. Hughes,[1,3] and Shou-Cheng Zhang[1]
[1]*Department of Physics, Stanford University, Stanford, California 94305, USA*
[2]*Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, California 93106, USA*
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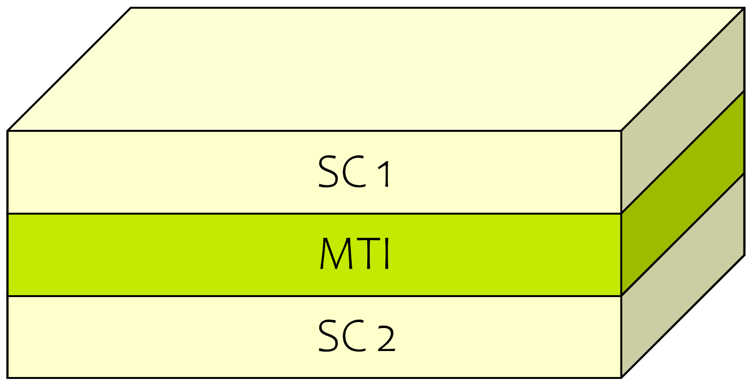
(Received 31 March 2010; revised manuscript received 29 September 2010; published 10 November 2010)

The chiral topological superconductor in two dimensions has a full pairing gap in the bulk and a single chiral Majorana state at the edge. The vortex of the chiral superconducting state carries a Majorana zero mode which is responsible for the non-Abelian statistics of the vortices. Despite intensive searches, this superconducting state has not yet been identified in nature. In this paper, we consider a quantum Hall or a quantum anomalous Hall state near the plateau transition and in proximity to a fully gapped *s*-wave superconductor. We show that this hybrid system may realize the chiral topological superconductor state and propose several experimental methods for its observation.

Described by a **Bogoliubov de Gennes** Hamiltonian

The superconductors induce pairing amplitudes Δ_1 and Δ_2 on top and bottom layers

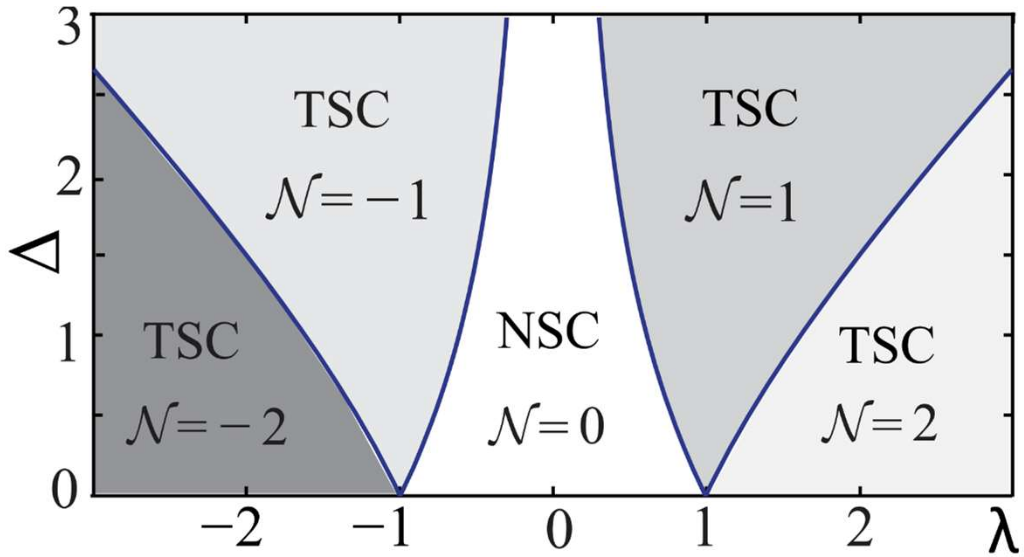
QAH insulator-superconductor heterostructure with MTI thin film



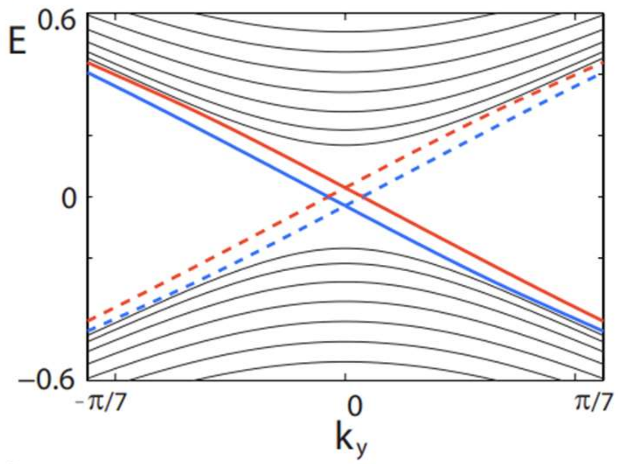
In the BdG language the $\mathcal{C} = 1$ QAH state is **equivalent** to a TSC with Chern invariant $\mathcal{N} = 2\mathcal{C} = 2$

Superconducting pairing $\Delta \neq 0$ leads to a $\mathcal{N} = 1$ chiral TSC

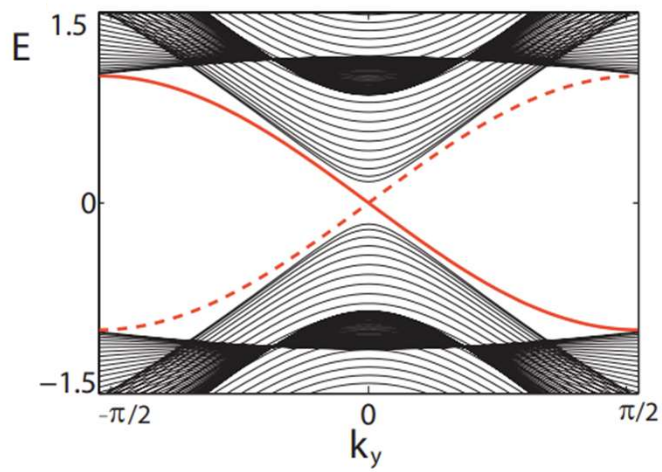
- Equivalent to the QAH state in superconducting systems
- A pair of counterpropagating edge modes



Energy spectrum for a $\mathcal{N} = 2$ proximitized QAH state



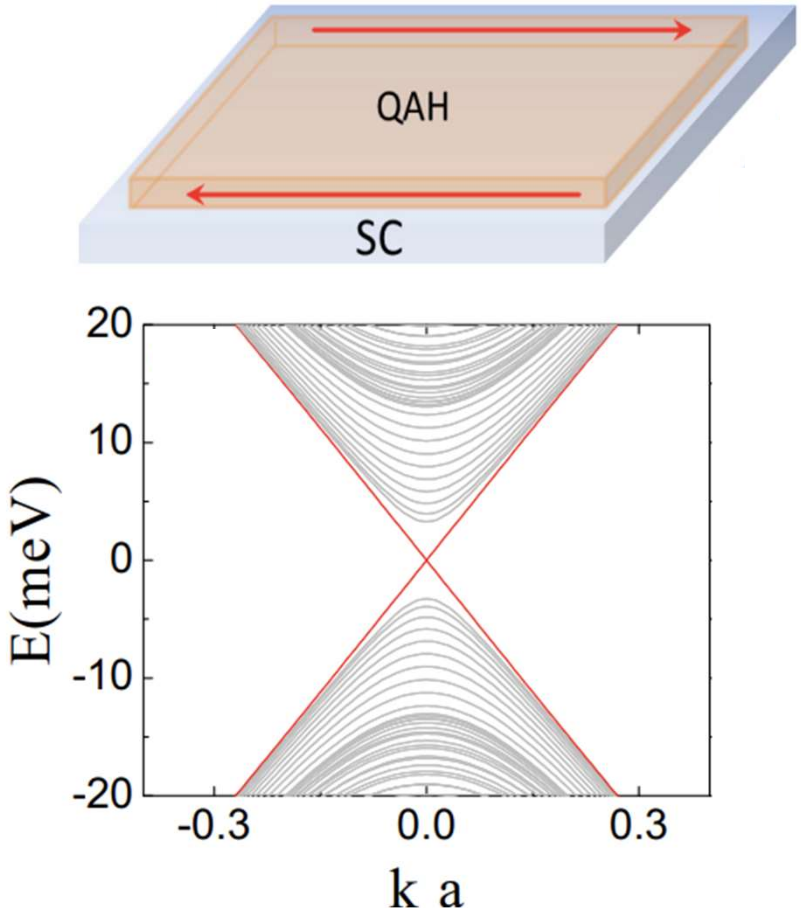
Energy bands for a $\mathcal{N} = 1$ chiral TSC

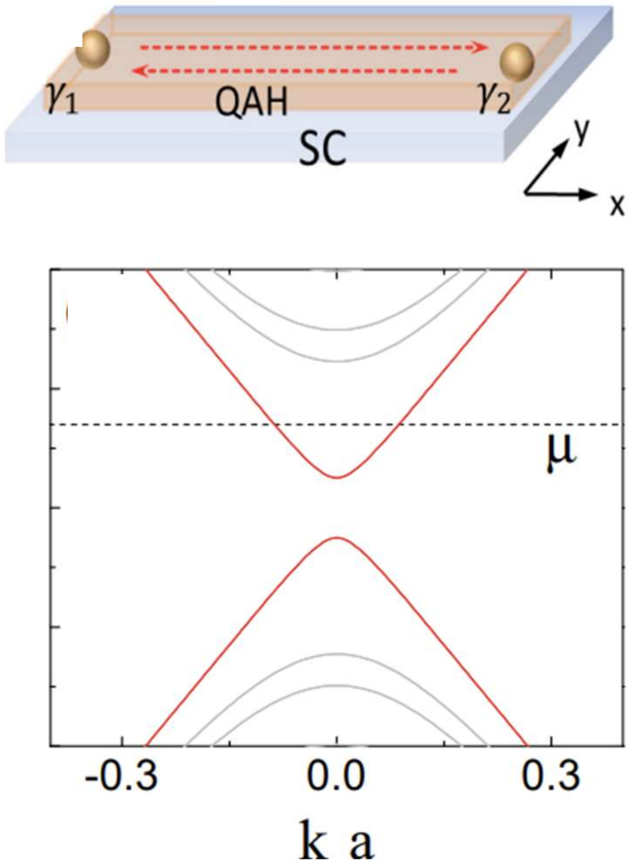


1D Topological Superconductor

A small width of the MTI slab couples opposite edge modes and opens an edge energy gap

- Large thin film:
  2D TSC with **gapless** edge modes
- Narrow thin film:
  1D TSC with **gapped** edge modes





Odd number of intersections: 1D TSC with end-localized zero-energy states

1D topological superconductor

- Characterized by an integer topological invariant N_{BDI}
- Host N_{BDI} zero-energy end-localized modes

Quasi-one-dimensional quantum anomalous Hall systems as new platforms for scalable topological quantum computation

Chui-Zhen Chen,[1] Ying-Ming Xie,[1] Jie Liu,[2] Patrick A. Lee,[3,*] and K. T. Law[1,†]
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[2]Department of Applied Physics, School of Science, Xian Jiaotong University, Xian 710049, China
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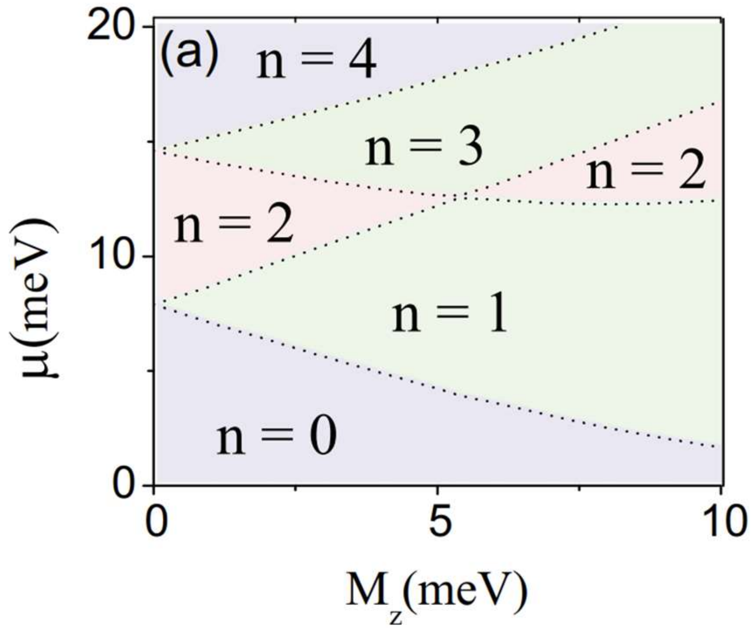
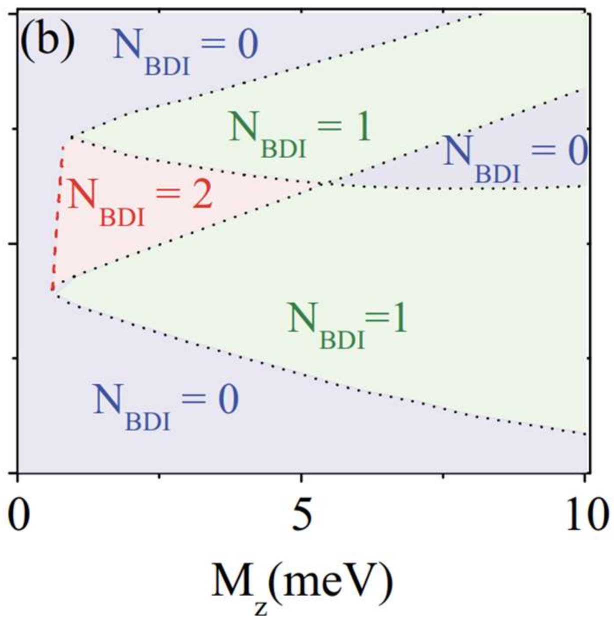
Quantum anomalous Hall insulator/superconductor heterostructures emerged as a competitive platform to realize topological superconductors with chiral Majorana edge states as shown in recent experiments [He *et al.* Science 357, 294 (2017)]. However, chiral Majorana modes, being extended, cannot be used for topological quantum computation. In this work, we show that quasi-one-dimensional quantum anomalous Hall structures exhibit a large topological regime (much larger than the two-dimensional case) which supports localized Majorana zero energy modes. The non-Abelian properties of a cross-shaped quantum anomalous Hall junction is shown explicitly by time-dependent calculations. We believe that the proposed quasi-one-dimensional quantum anomalous Hall structures can be easily fabricated for scalable topological quantum computation.

$\Delta = 0$



The number n of bands at chemical potential μ in the QAH state determines the N_{BDI} topological invariant

$\Delta = 1$ meV

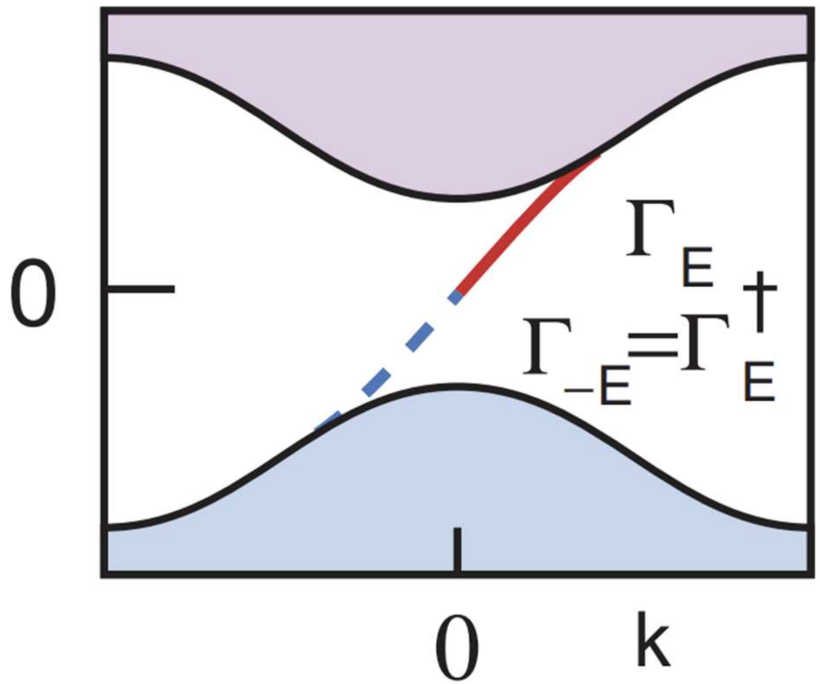


Boundary modes in topological superconductors are Majorana quasiparticles due to the presence of particle-hole symmetry

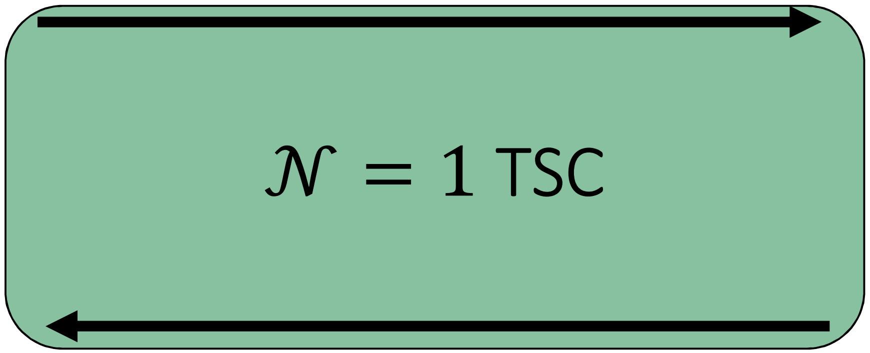
- Unpaired, zero-energy states are described by nonfermionic operators

$$\Gamma = \Gamma^{\dagger}$$

MCEM edge spectrum
for a $\mathcal{N} = 1$ TSC



The $\mathcal{N} = 1$ 2D TSC has a 1D single, unpaired Majorana chiral edge mode (MCEM) counterpropagating on the side of a large thin film

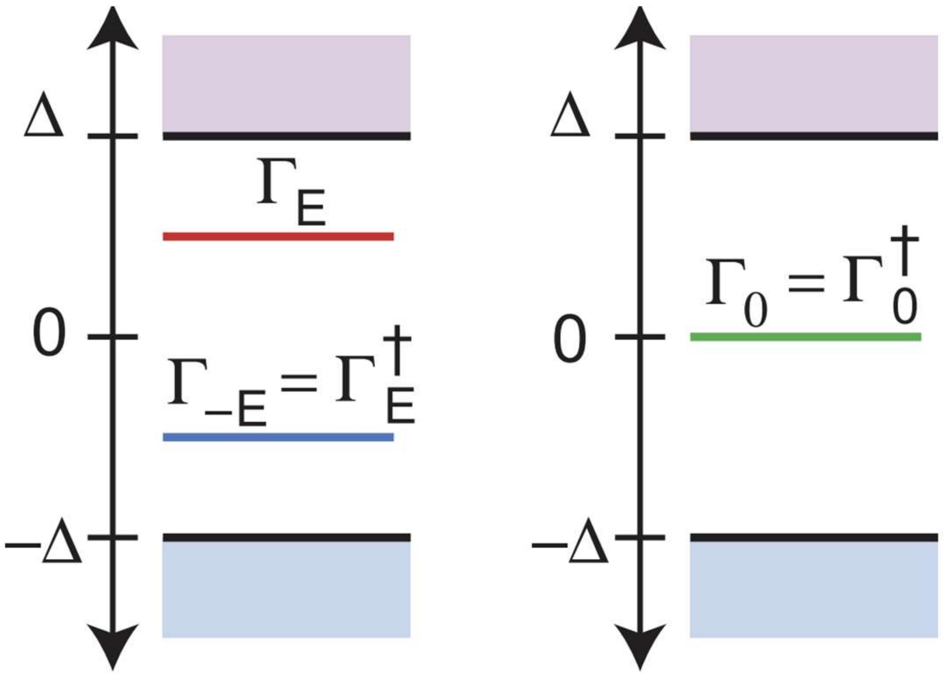


Boundary modes in topological superconductors are Majorana quasiparticles due to the presence of particle-hole symmetry

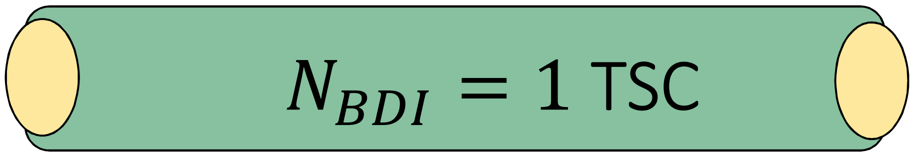
- Unpaired, zero-energy states are described by nonfermionic operators

$$\Gamma = \Gamma^{\dagger}$$

MBS edge spectrum for a $\mathrm{N_{BDI}} = 1$ SC nanowire



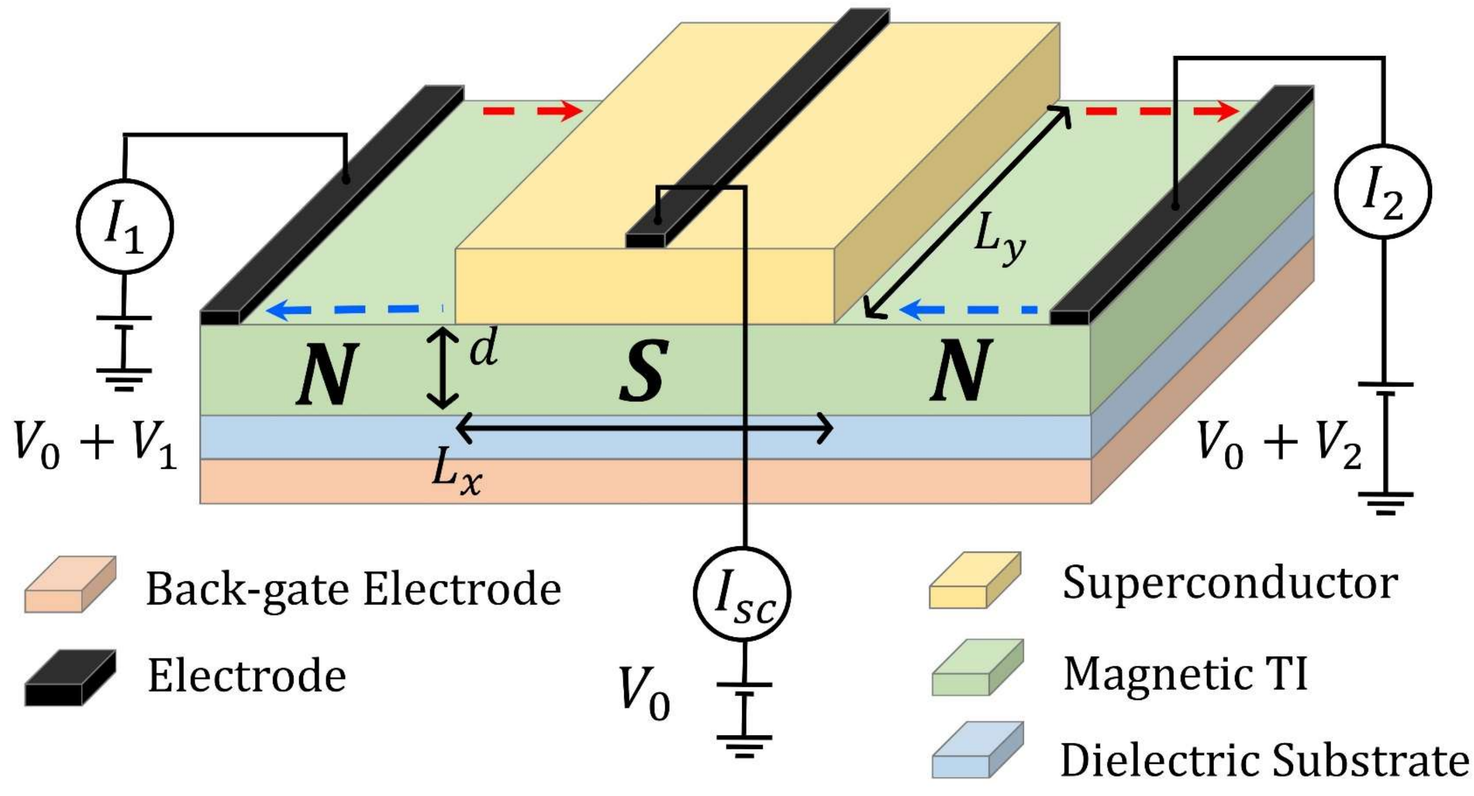
The $N_{BDI} = 1$ 1D TSC has a 0D single, unpaired Majorana bound state (MBS) on the extremities



i. **Topology in Condensed Matter Physics**
- Quantum Anomalous Hall State
  - Topological Insulators

ii. **Topological Superconductors in MTIs**
- 2D Chiral Superconductors
- 1D Topological Superconductor
- The Emergence of Majorana Modes

iii. **Detecting Majorana Excitations**
- Antisymmetric Electric Conductance
  - Numerical Results



We apply an asymmetric bias on the N leads with respect to the proximitized sector S

$$V_1 = \alpha V, \qquad V_2 = -\beta V$$

with $0 \le \alpha \le 1$ and $\beta = 1 - \alpha$ such that $V_1 - V_2 = V$

We define the differential conductance on the N terminals of the junction as

$$G_i(E) = \frac{\partial I_i}{\partial V}$$

I_i = electric current on the i=1,2 normal lead

V = total bias across the junction

The following equations can be derived using the BTK formalism

$$G_1(E) = \alpha \frac{e^2}{h}\left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h}\left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-N_2^h(\beta V) - P_{22}^{eh}(\beta V) + P_{22}^{hh}(\beta V)\right] + \alpha \frac{e^2}{h}\left[P_{21}^{he}(\alpha V) - P_{21}^{ee}(\alpha V)\right]$$

The differential conductance $G_i(E)$ is expressed in terms of the transmission amplitudess $P_{ij}^{ab}(E)$ indicating trasmission of a quasiparticle b in lead j to a quasiparticle a in lead i

$P_{ij}^{ab}(E)$: quasiparticle b, lead j → quasiparticle a, lead i

The following equations can be derived using the BTK formalism

Number of electrons

$$G_1(E) = \alpha \frac{e^2}{h}\left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h}\left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

Number of holes

$$G_2(E) = \beta \frac{e^2}{h}\left[-N_2^h(\beta V) - P_{22}^{eh}(\beta V) + P_{22}^{hh}(\beta V)\right] + \alpha \frac{e^2}{h}\left[P_{21}^{he}(\alpha V) - P_{21}^{ee}(\alpha V)\right]$$

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Normal Reflection ($P_{11}^{ee}(\alpha V)$); Andreev Reflection ($P_{11}^{he}(\alpha V)$)

$$G_2(E) = \beta \frac{e^2}{h}\left[-N_2^h(\beta V) - P_{22}^{eh}(\beta V) + P_{22}^{hh}(\beta V)\right] + \alpha \frac{e^2}{h}\left[P_{21}^{he}(\alpha V) - P_{21}^{ee}(\alpha V)\right]$$

Normal reflection ($P_{22}^{eh}(\beta V)$); Andreev reflection ($P_{22}^{hh}(\beta V)$)

The differential conductance $G_i(E)$ is expressed in terms of the transmission amplitudess $P_{ij}^{ab}(E)$ indicating trasmission of a quasiparticle b in lead j to a quasiparticle a in lead i

$P_{ij}^{ab}(E)$: quasiparticle b, lead $j \rightarrow$ quasiparticle a, lead i

The following equations can be derived using the BTK formalism

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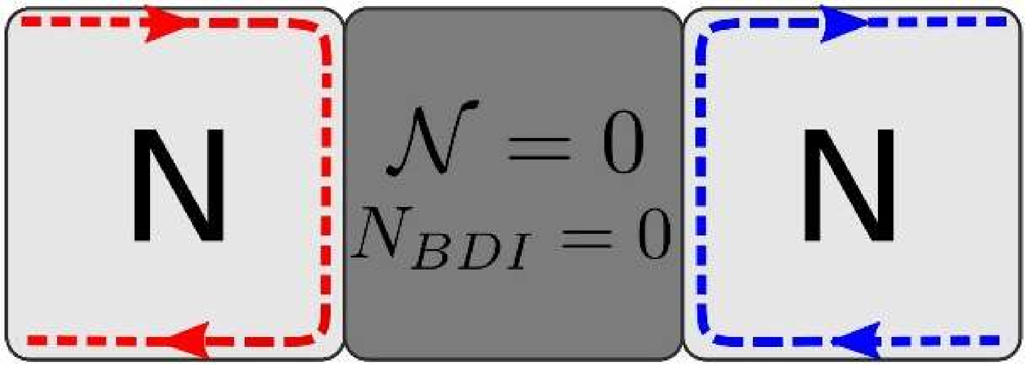
Perfect normal reflection is expected for a trivial superconductor without subgap states

$$P_{11}^{ee} = P_{22}^{hh} = 1$$



The conductance takes the following values

$$G_1(E) = \alpha \frac{e^2}{h}\left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h}\left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

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$$G_t(E) = G_1(E) + G_2(E)$$

Perfect normal reflection is expected for a **trivial superconductor** without subgap states

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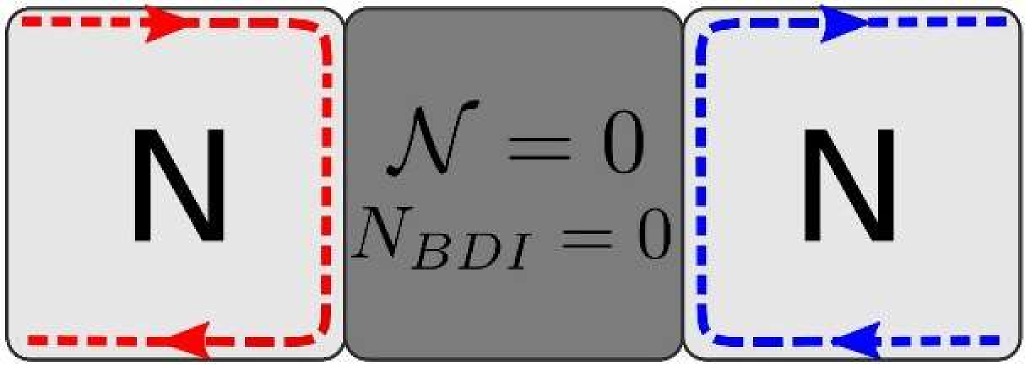


The conductance takes the following values

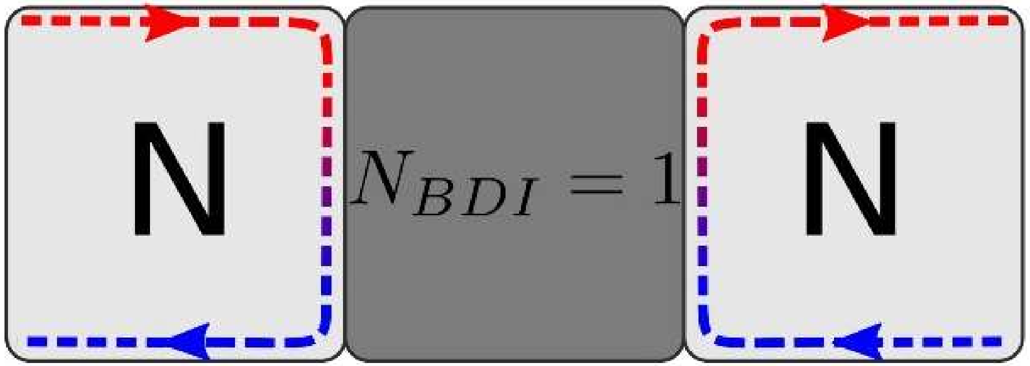
$$G_1(E) = \alpha \frac{e^2}{h}\left[\overset{=1}{N_1^e(\alpha V)} - \overset{=1}{P_{11}^{ee}(\alpha V)} + \cancel{P_{21}^{he}}(\alpha V)\right] + \beta \frac{e^2}{h}\left[\cancel{P_{12}^{hh}}(\beta V) - \cancel{P_{12}^{eh}}(\beta V)\right] = 0$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-\overset{=1}{N_2^h(\beta V)} - \overset{=1}{P_{22}^{eh}(\beta V)} + \cancel{P_{22}^{hh}}(\beta V)\right] + \alpha \frac{e^2}{h}\left[\cancel{P_{21}^{he}}(\alpha V) - \cancel{P_{21}^{ee}}(\alpha V)\right] = 0$$

$$G_t(E) = G_1(E) + G_2(E) = 0$$

Perfect Andreev reflection is expected in presence of Majorana bound states within the energy gap

$$P_{11}^{he} = P_{22}^{eh} = 1$$



The conductance takes the following values

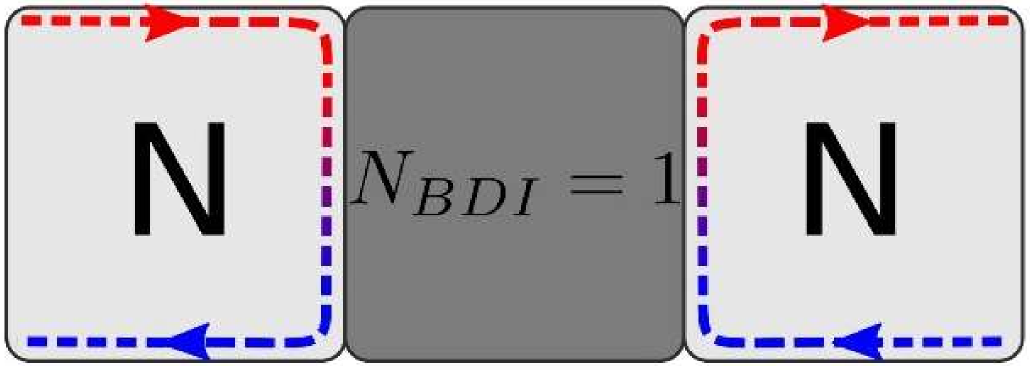
$$G_1(E) = \alpha \frac{e^2}{h}\left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h}\left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-N_2^h(\beta V) - P_{22}^{eh}(\beta V) + P_{22}^{hh}(\beta V)\right] + \alpha \frac{e^2}{h}\left[P_{21}^{he}(\alpha V) - P_{21}^{ee}(\alpha V)\right]$$

$$G_t(E) = G_1(E) + G_2(E)$$

Perfect Andreev reflection is expected in presence of Majorana bound states within the energy gap

$$P_{11}^{he} = P_{22}^{eh} = 1$$



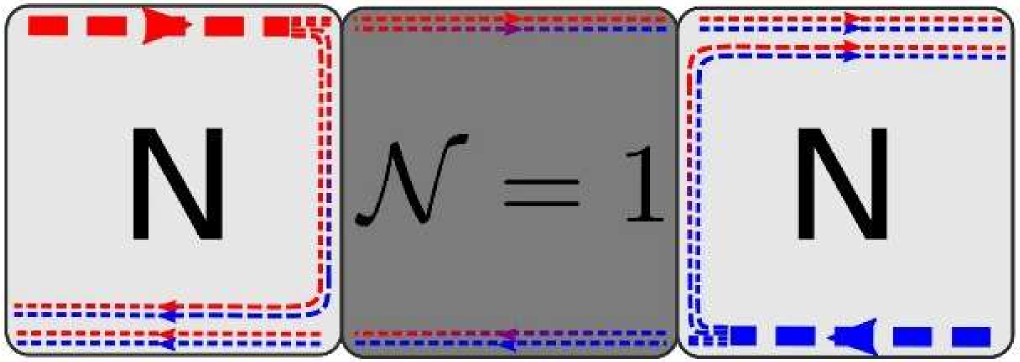
The conductance takes the following values

$$G_1(E) = \alpha \frac{e^2}{h}\left[\overset{=1}{N_1^e}(\alpha V) - \cancel{P_{11}^{ee}}(\alpha V) + \overset{=1}{P_{11}^{he}}(\alpha V)\right] + \beta \frac{e^2}{h}\left[\cancel{P_{12}^{hh}}(\beta V) - \cancel{P_{12}^{eh}}(\beta V)\right] = 2\alpha \frac{e^2}{h}$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-\overset{=1}{N_2^h}(\beta V) - \cancel{P_{22}^{eh}}(\beta V) + \overset{=1}{P_{22}^{hh}}(\beta V)\right] + \alpha \frac{e^2}{h}\left[\cancel{P_{21}^{he}}(\alpha V) - \cancel{P_{21}^{ee}}(\alpha V)\right] = -2\beta \frac{e^2}{h}$$

$$G_t(E) = G_1(E) + G_2(E) = 2(\alpha - \beta)\frac{e^2}{h} = 2(2\alpha - 1)\frac{e^2}{h}$$

In presence of Majorana chiral Edge modes all processes occur with the same probability



$$P_{11}^{ee} = P_{11}^{he} = P_{21}^{ee} = P_{21}^{he} = 0.25$$
$$P_{22}^{hh} = P_{22}^{eh} = P_{12}^{hh} = P_{12}^{eh} = 0.25$$

The conductance takes the following values

$$G_1(E) = \alpha \frac{e^2}{h}\left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h}\left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

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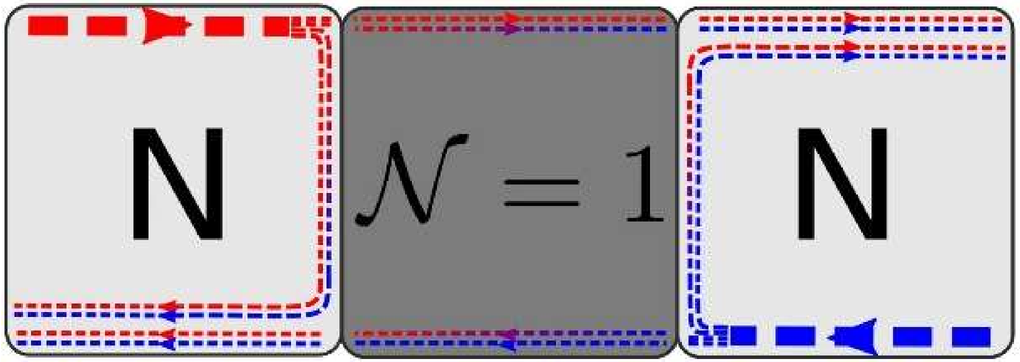

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The conductance takes the following values

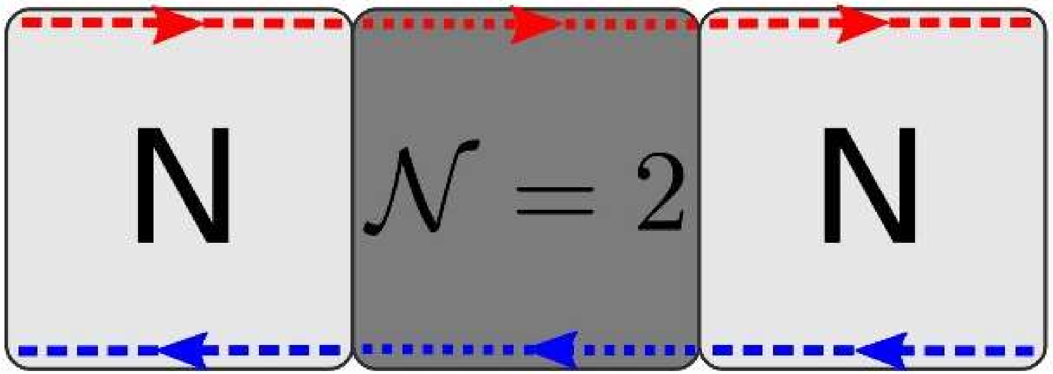
$$G_1(E) = \alpha \frac{e^2}{h}\left[\overset{=1}{N_1^e(\alpha V)} - \overset{=0.25}{P_{11}^{ee}(\alpha V)} + \overset{=0.25}{P_{11}^{he}(\alpha V)}\right] + \beta \frac{e^2}{h}\left[\overset{=0.25}{P_{12}^{hh}(\beta V)} - \overset{=0.25}{P_{12}^{eh}(\beta V)}\right] = \alpha \frac{e^2}{h}$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-\overset{=1}{N_2^h(\beta V)} - \overset{=0.25}{P_{22}^{eh}(\beta V)} + \overset{=0.25}{P_{22}^{hh}(\beta V)}\right] + \alpha \frac{e^2}{h}\left[\overset{=0.25}{P_{21}^{he}(\alpha V)} - \overset{=0.25}{P_{21}^{ee}(\alpha V)}\right] = \beta \frac{e^2}{h}$$

$$G_t(E) = G_1(E) + G_2(E) = (\alpha - \beta)\frac{e^2}{h} = (2\alpha - 1)\frac{e^2}{h}$$

In the QAH State electron and holes are perfectly transmitted across the junction

$$P_{21}^{ee} = P_{12}^{hh} = 1$$



The conductance takes the following values

$$G_1(E) = \alpha \frac{e^2}{h} \left[N_1^e(\alpha V) - P_{11}^{ee}(\alpha V) + P_{11}^{he}(\alpha V)\right] + \beta \frac{e^2}{h} \left[P_{12}^{hh}(\beta V) - P_{12}^{eh}(\beta V)\right]$$

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$$G_t(E) = G_1(E) + G_2(E)$$

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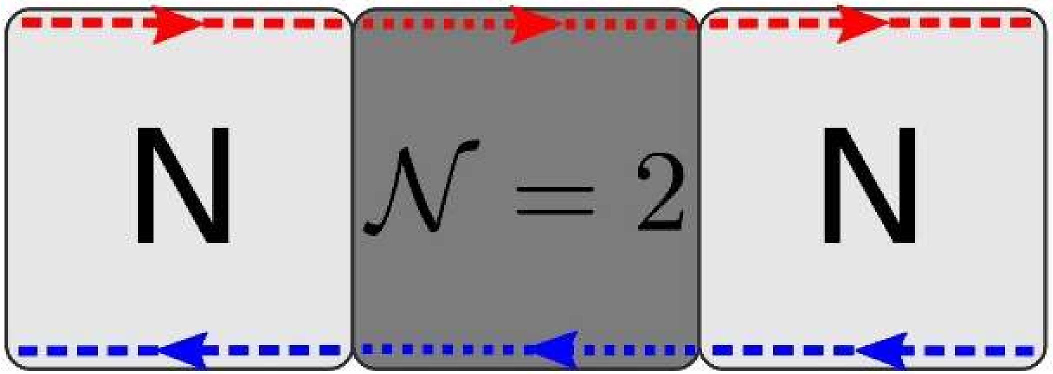

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$$G_1(E) = \alpha \frac{e^2}{h}\left[\overset{=1}{N_1^e(\alpha V)} - \cancel{P_{11}^{ee}}(\alpha V) + \cancel{P_{11}^{he}}(\alpha V)\right] + \beta \frac{e^2}{h}\left[\overset{=1}{P_{12}^{hh}(\beta V)} - \cancel{P_{12}^{eh}}(\beta V)\right] = \frac{e^2}{h}$$

$$G_2(E) = \beta \frac{e^2}{h}\left[-\overset{=1}{N_2^h(\beta V)} - \cancel{P_{22}^{eh}}(\beta V) + \cancel{P_{22}^{hh}}(\beta V)\right] + \alpha \frac{e^2}{h}\left[\overset{=1}{P_{21}^{he}(\alpha V)} - \cancel{P_{21}^{ee}}(\alpha V)\right] = -\frac{e^2}{h}$$

$$G_t(E) = G_1(E) + G_2(E) = 0$$

Conductance Summary

	G_1	G_2	$G_t = G_1 + G_2$
$N_{BDI} = 0$ *(Trivial)*	0	0	0
$N_{BDI} = 1$ *(MBS)*	$2\alpha \frac{e^2}{h}$	$2(\alpha - 1)\frac{e^2}{h}$	$2(2\alpha - 1)\frac{e^2}{h}$
$\mathcal{N} = 0$ *(Trivial)*	0	0	0
$\mathcal{N} = 1$ *(MCEM)*	$\alpha \frac{e^2}{h}$	$(\alpha - 1)\frac{e^2}{h}$	$(2\alpha - 1)\frac{e^2}{h}$
$\mathcal{N} = 2$ *(Trivial)*	0	0	0

	G_1	G_2	$G_t = G_1 + G_2$
$N_{BDI} = 0$ *(Trivial)*	0	0	0
$N_{BDI} = 1$ *(MBS)*	$2\alpha \frac{e^2}{h}$	$2(\alpha - 1)\frac{e^2}{h}$	$2(2\alpha - 1)\frac{e^2}{h}$
$\mathcal{N} = 0$ *(Trivial)*	0	0	0
$\mathcal{N} = 1$ *(MCEM)*	$\alpha \frac{e^2}{h}$	$(\alpha - 1)\frac{e^2}{h}$	$(2\alpha - 1)\frac{e^2}{h}$
$\mathcal{N} = 2$ *(Trivial)*	0	0	0

Analysis of the symmetry of G_t as a function of α

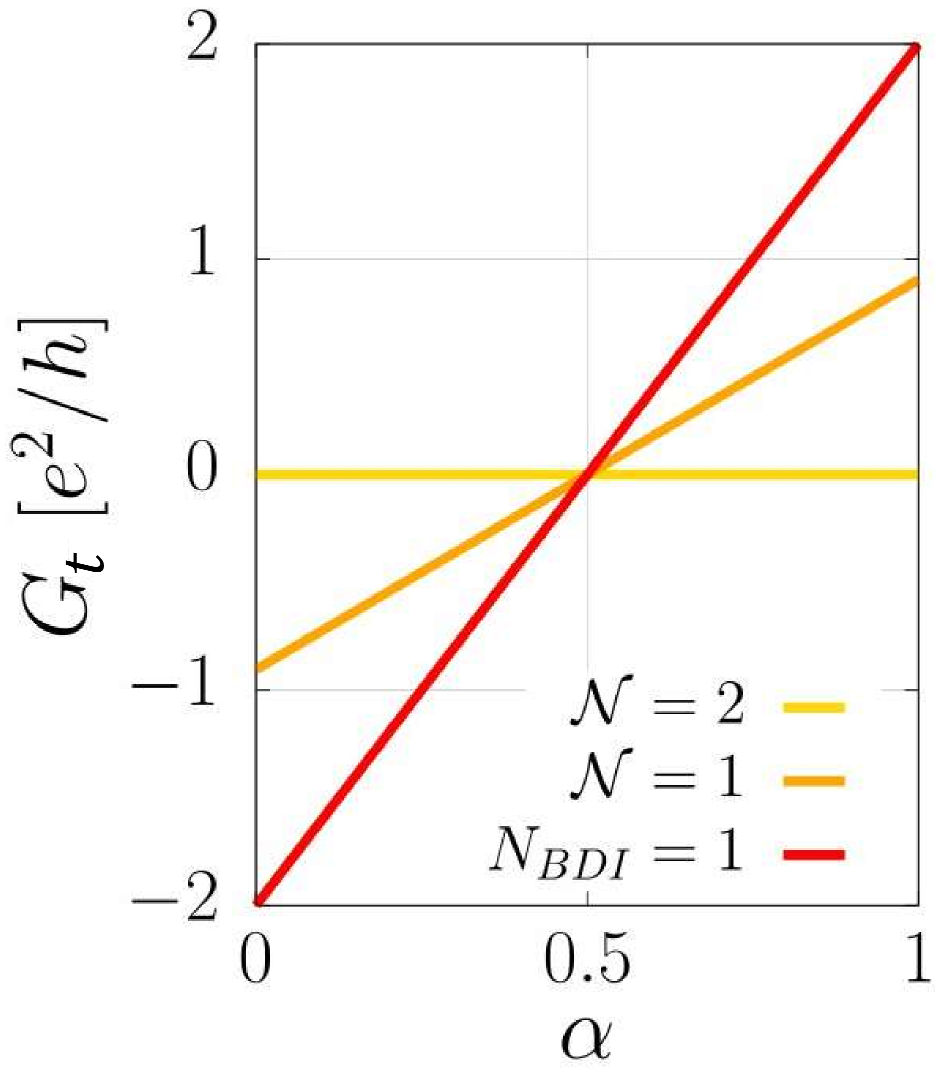
i. the **antisimmetry** around $\alpha = 0.5$ (equal bias splitting) is a necessary condition

ii. rule out electric signal produced by trivial Andreev levels

Different ratio G_t/G_0 distinguishes different Majorana excitations

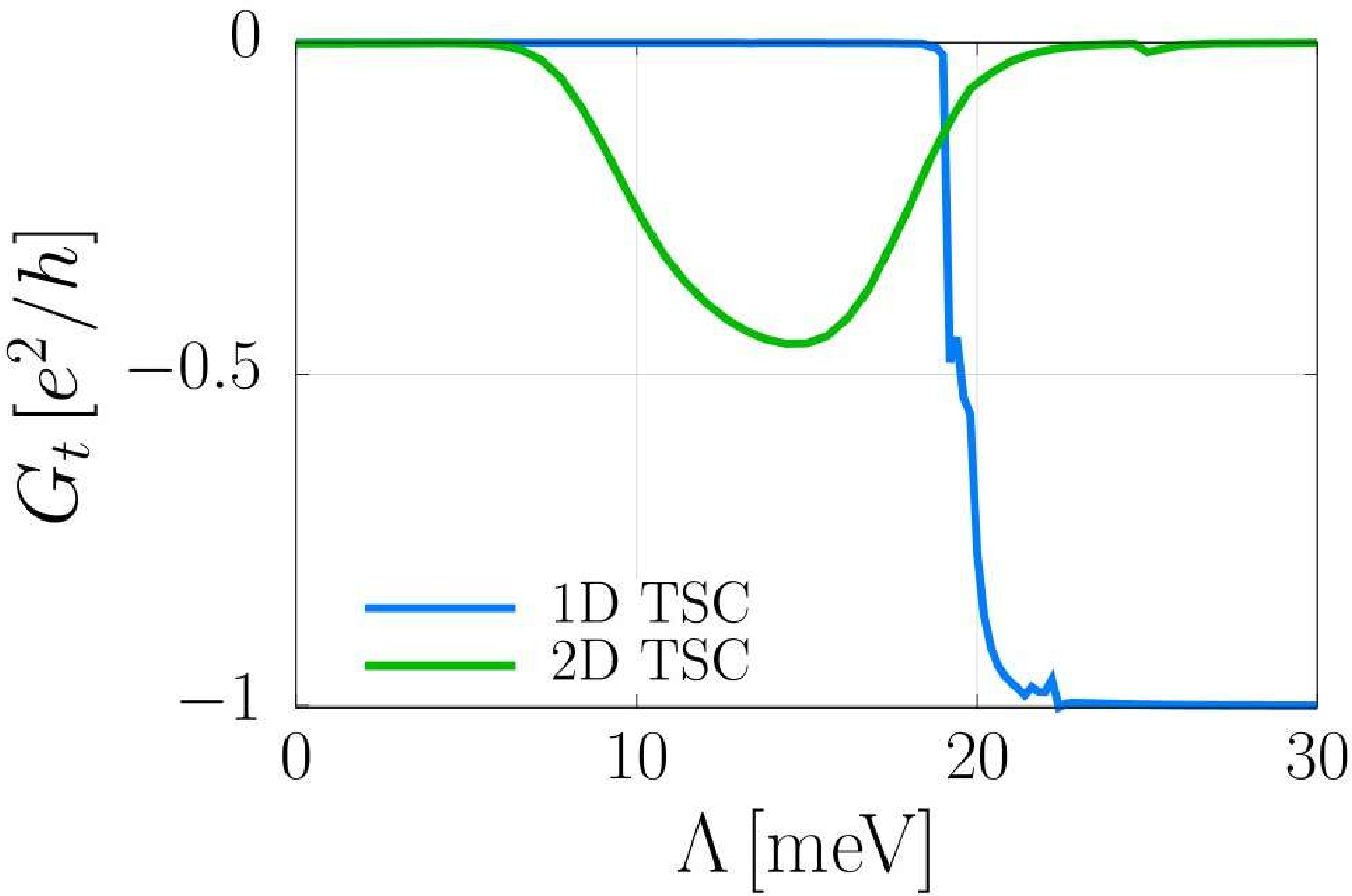
- MCEM $G_t/G_0 = (2\alpha - 1)$
- MBS $G_t/G_0 = 2(2\alpha - 1)$

$G_0 = e^2/h$ (conductance quantum)



Numerical simulations in different geometries reproduce the physics of 2D and 1D topological superconductors

Topological states with Majorana modes can be identified by $G_t \neq 0$



Conclusions

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THANK YOU

FOR YOUR ATTENTION

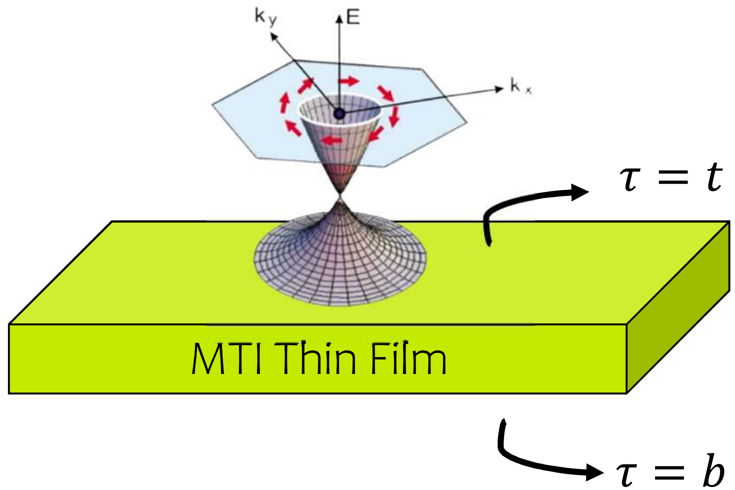
Surface Hamiltonian for the Dirac-type boundary states in top and bottom layer of a MTI thin film

$$H_0(\boldsymbol{k}) = \begin{pmatrix} \lambda & k_y + ik_x & m_0 + m_1 k_\perp^2 & 0 \\ k_y - ik_x & -\lambda & 0 & m_0 + m_1 k_\perp^2 \\ m_0 + m_1 k_\perp^2 & 0 & \lambda & -(k_y + ik_x) \\ 0 & m_0 + m_1 k_\perp^2 & -(k_y - ik_x) & -\lambda \end{pmatrix}$$

In the basis of spin $\sigma = \uparrow, \downarrow$ and layer $\tau = t, b$ eigenstates

$$(|t,\uparrow\rangle, |t,\downarrow\rangle, |b,\uparrow\rangle, |b,\downarrow\rangle)$$

Dirac cone TSSs on top and bottom surfaces



The topological state is given by th Chern invariant:

- Trivial Insulator, $\mathcal{C} = 0$ for $\lambda < m_0$
- QAH state, $\mathcal{C} = 1$ for $\lambda > m_0$

The low-energy effective Hamiltonian for 3D MTIs around the high-symmetry Dirac point Γ $\boldsymbol{k} = 0$ takes the following form

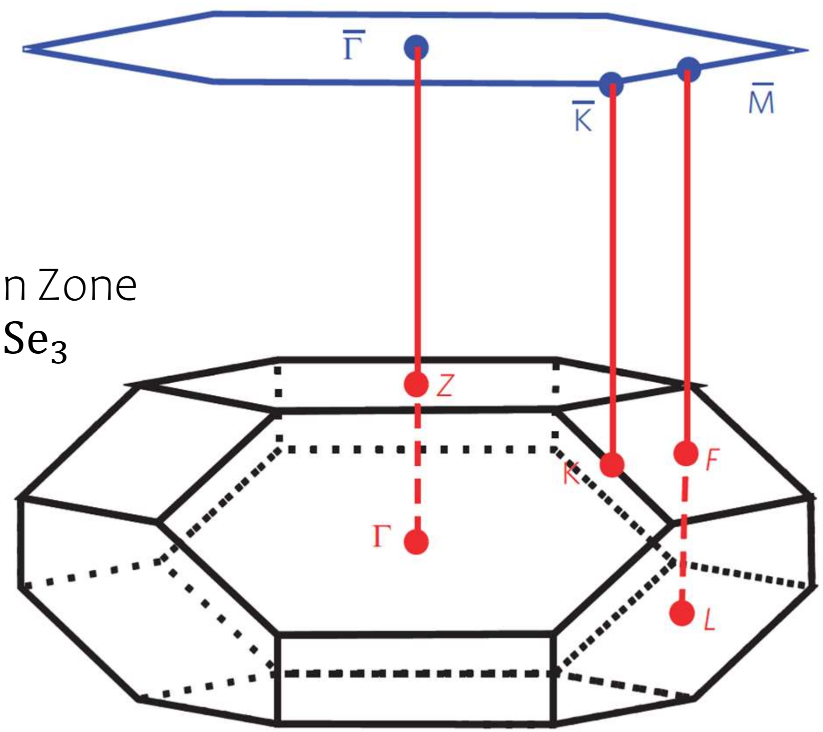
$$H_{MTI}(\boldsymbol{k}) = \epsilon_0\,(\boldsymbol{k}) I_{4\times4} + \begin{pmatrix} M(\boldsymbol{k})+\lambda & A_1 k_z & 0 & A_2 k_- \\ A_1 k_z & -M(\boldsymbol{k})+\lambda & A_2 k_- & 0 \\ 0 & A_2 k_+ & M(\boldsymbol{k})-\lambda & -A_1 k_z \\ A_2 k_+ & 0 & -A_1 k_z & -M(\boldsymbol{k})-\lambda \end{pmatrix}$$

In the basis of spin $\boldsymbol{\sigma} = \uparrow, \downarrow$ and parity $\boldsymbol{\tau} = \pm$ eigenstates

$$(|+,\uparrow\rangle, |+,\uparrow\rangle, |-,\downarrow\rangle, |-,\downarrow\rangle)$$

Dirac-cone shaped energy states are found in the boundary Brillouin zone around $\bar{\Gamma}$

Brillouin Zone for $\mathrm{Bi_2Se_3}$



The proximitized MTI thin film is described by a **Bogoliubov de Gennes Hamiltonian** that takes the form

$$H_{BdG} = \begin{pmatrix} H_0(\boldsymbol{k}) - \mu & \Delta \\ \Delta^\dagger & -H_0^*(-\boldsymbol{k}) + \mu \end{pmatrix}$$

$$\Delta = \begin{pmatrix} i\Delta_1\sigma_y & 0 \\ 0 & i\Delta_2\sigma_y \end{pmatrix}$$

Here Δ_1 and Δ_2 are the superconducting pairing amplitudes induced on the top and bottom layers, respectively.

$\sigma_{x,y,z}$ are Pauli matrices acting on the **spin subspace**

QAH insulator-superconductor heterostructure with MTI thin film