Light-matter Correlations in Quantum Floquet Engineering

Beatriz Pérez González, Álvaro Gómez-León, Gloria Platero

5-6/06 @ IFISC (UIB-CSIC)
Novel trends topological systems and quantum thermodynamics

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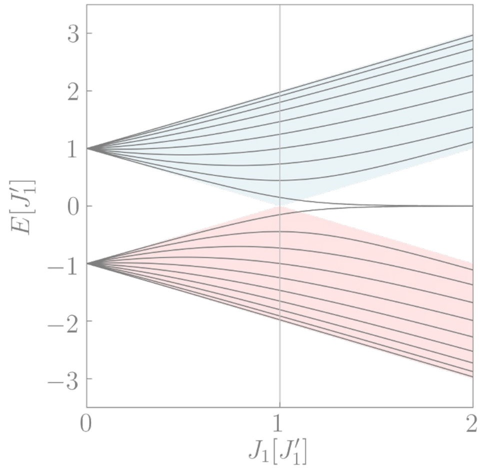
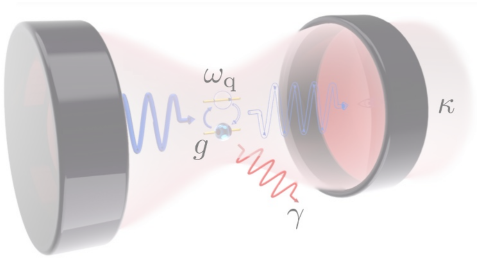
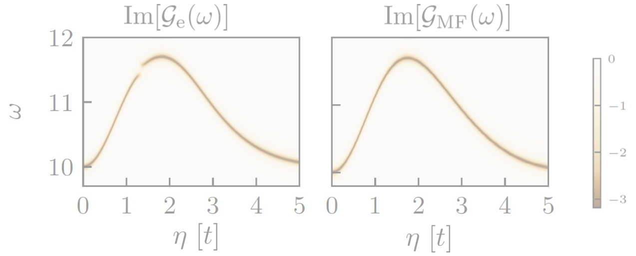


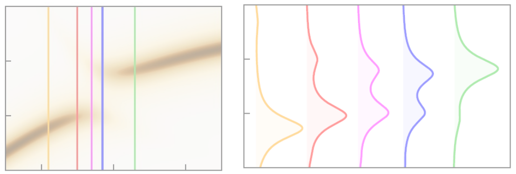
$E\,[J_1']$
3
2
1
0
−1
−2
−3
0
1
2
$J_1\,[J_1']$



ω_q
g
κ
γ



$\mathrm{Im}[\mathcal{G}_e(\omega)]$
$\mathrm{Im}[\mathcal{G}_{MF}(\omega)]$
ω
12
11
10
0 1 2 3 4 5
$\eta\ [t]$



Introduction

Light-Matter Correlations in Quantum Floquet Engineering

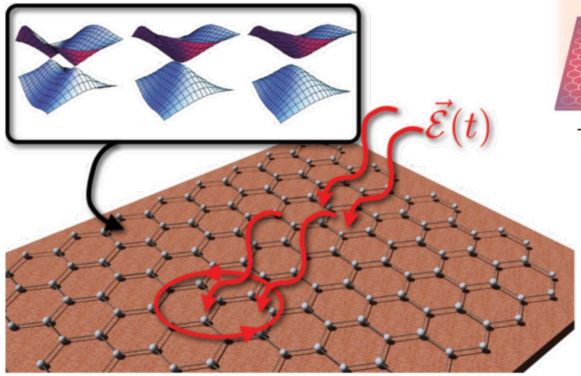
Floquet materials

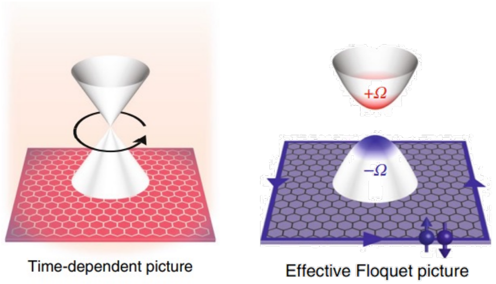
modify the properties of the system through the interaction with **classical light**

T. Oka and S. Kitamura, Ann. Rev. Cond. Matt. Phys. **10**, pp 387-408 (2019)

Floquet engineering of quantum materials

McIver *et al.*, Nature Phys. 16, 38-41 (2020)





P. Delplace *et al.*, Phys. Rev. B **88**, 245422 (2013)

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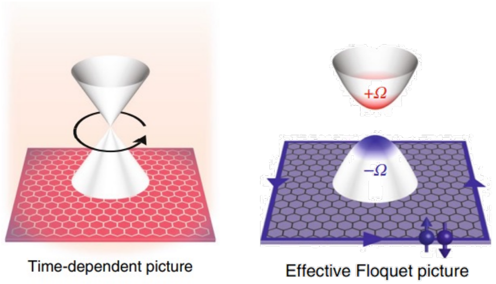
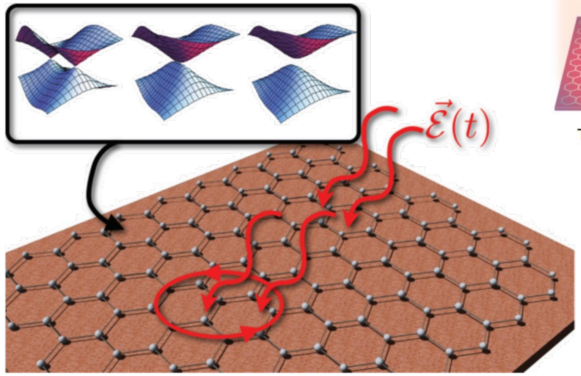


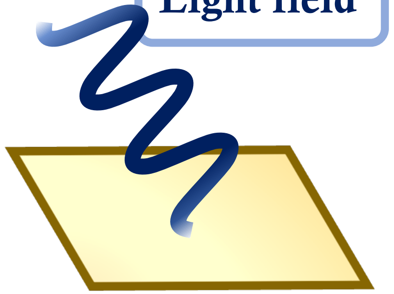


P. Delplace *et al.*,
Phys. Rev. B **88**, 245422 (2013)

- dynamics of **time-periodic** systems

$$E(t) = E_0 \sin(\omega t) \qquad \omega = \frac{2\pi}{T}$$

Light field



Quantum system

$$H_{\text{tot}}(t) = H_{\text{sys}} + H_{\text{driv}}(t)$$

$$H_{\text{driv}}(t) \propto E_0 \sin(\omega t)$$

Floquet theory

$$H_{\text{tot}}(t) = H_{\text{tot}}(t+T)$$

A. Eckardt, Rev. Mod. Phys. **89**, 011004 (2017)

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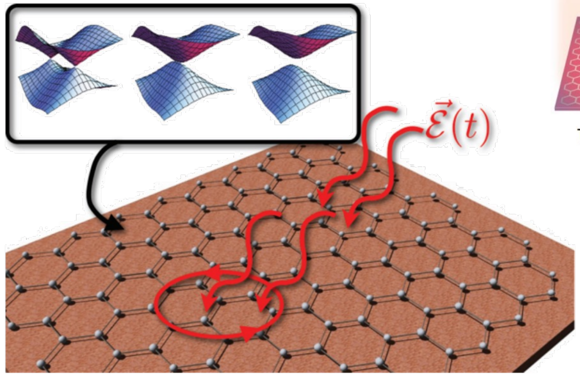
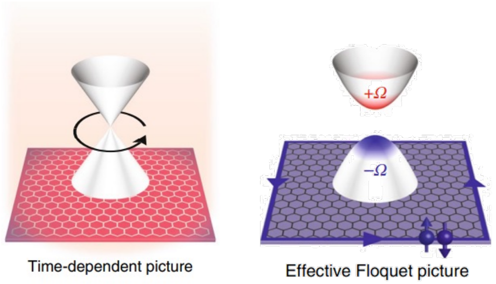
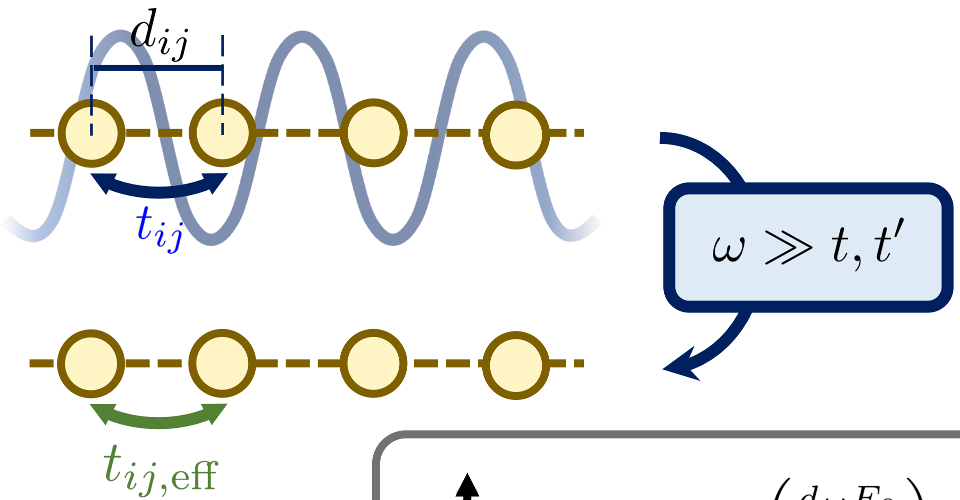
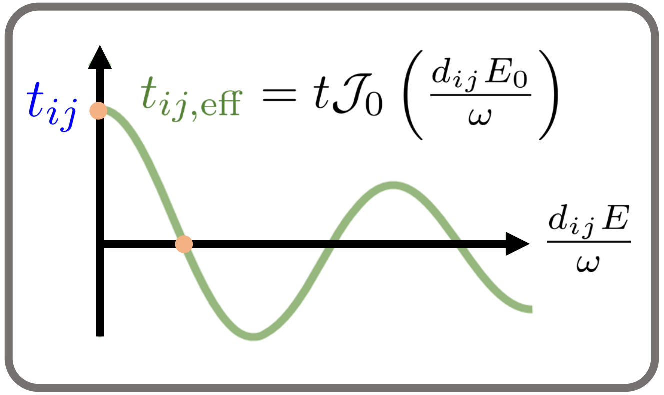




P. Delplace *et al.*, Phys. Rev. B **88**, 245422 (2013)

- **high-frequency regime**





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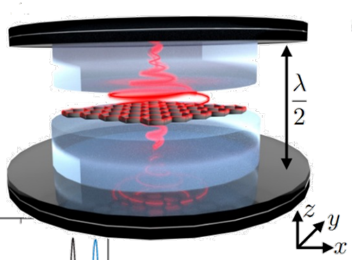
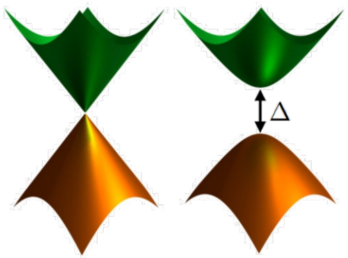
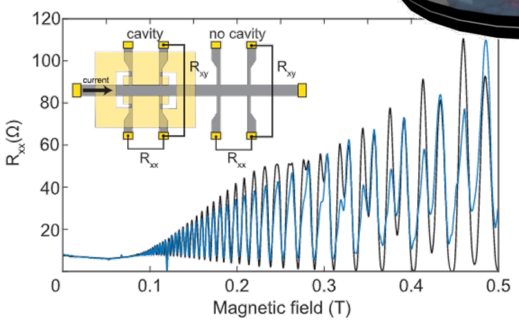
X. Wang *et al.*, Phys. Rev. B **99**, 235156 (2019)







F. Appugliese *et al.*, Science **375**, 6584 (2022)

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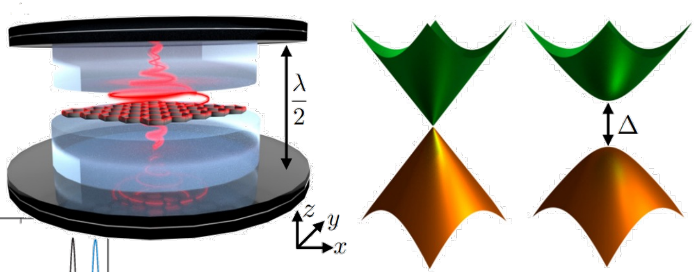

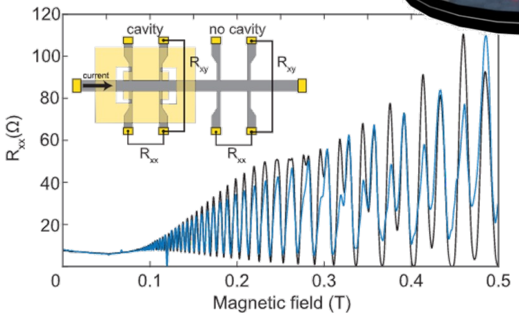

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- quantum to classical crossover
  - $E(t) \propto \sin(\Omega t) \longrightarrow E \propto d^\dagger + d$

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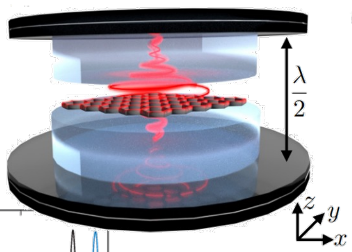

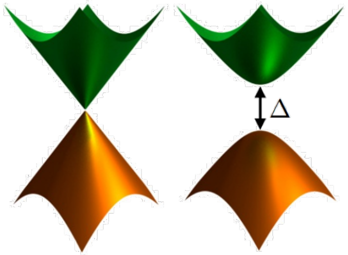

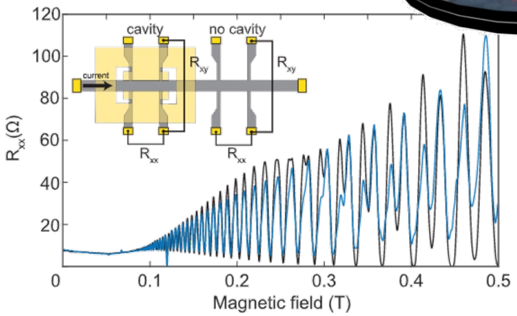

F. Appugliese *et al.*, Science **375**, 6584 (2022)

- **quantum to classical crossover**
  - $E(t) \propto \sin(\Omega t) \longrightarrow E \propto d^\dagger + d$
  - dynamics of photonic operators $d \to d e^{-i\Omega t}$

$$H_{\text{cav}} = \Omega d^\dagger d$$

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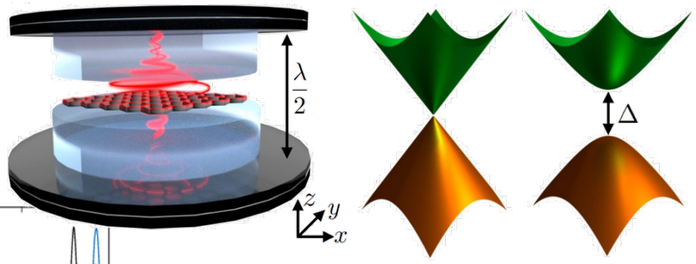
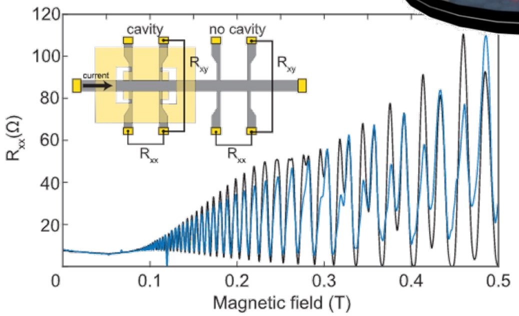




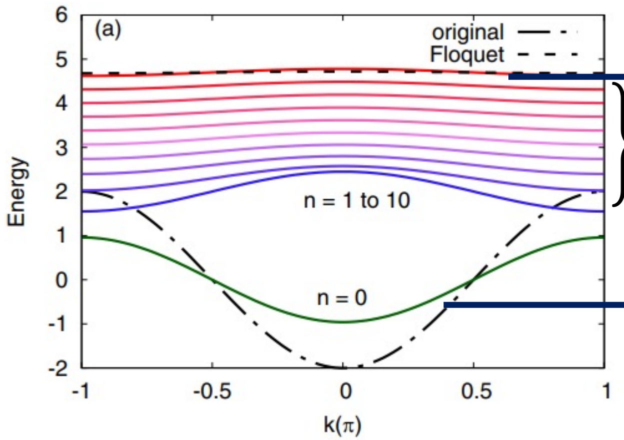
F. Appugliese *et al.*, Science **375**, 6584 (2022)

- **quantum to classical crossover**
  - $E(t) \propto \sin(\Omega t) \longrightarrow E \propto d^\dagger + d$
  - dynamics of photonic operators $\quad d \to de^{-i\Omega t}$
  - renormalization of hopping amplitudes

$$\lim_{n\to\infty} t_{ij}^{(n)} = t_{ij,\text{eff}}\left(E_0, \omega\right)$$

cQED

Floquet theory



Floquet result

Photon dressed band structure

Unperturbed fermionic system

M. A. Sentef *et al.*, Phys. Rev. Research **2**, 033033 (2020)

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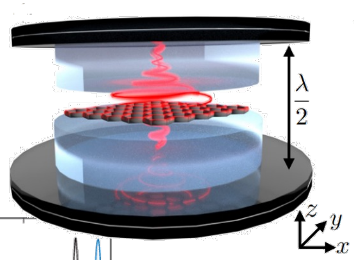

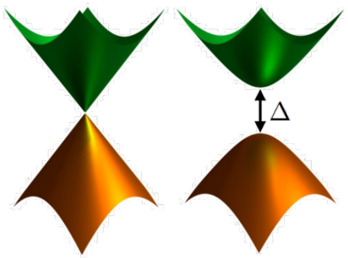

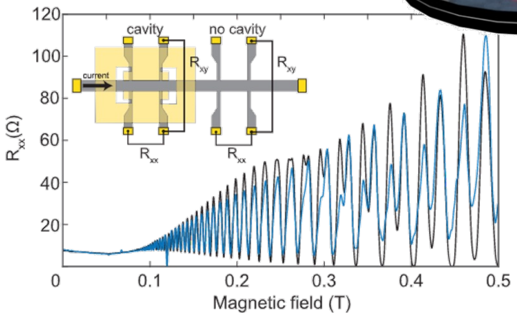

F. Appugliese *et al.*, Science **375**, 6584 (2022)

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  - $E(t) \propto \sin(\Omega t) \longrightarrow E \propto d^{\dagger} + d$
  - dynamics of photonic operators $\quad d \rightarrow d e^{-i\Omega t}$
  - renormalization of hopping amplitudes

$$\lim_{n\rightarrow\infty} t_{ij}^{(n)} = t_{ij,\text{eff}}\,(E_0, \omega)$$

  - structural similarities between the Floquet matrix and cavity Hamiltonian

C. Schäfer *et al.*, Phys. Rev. A **98**, 043801 (2018)

H. Hübener *et al.*, Nature Materials **20**, 438-442 (2021)

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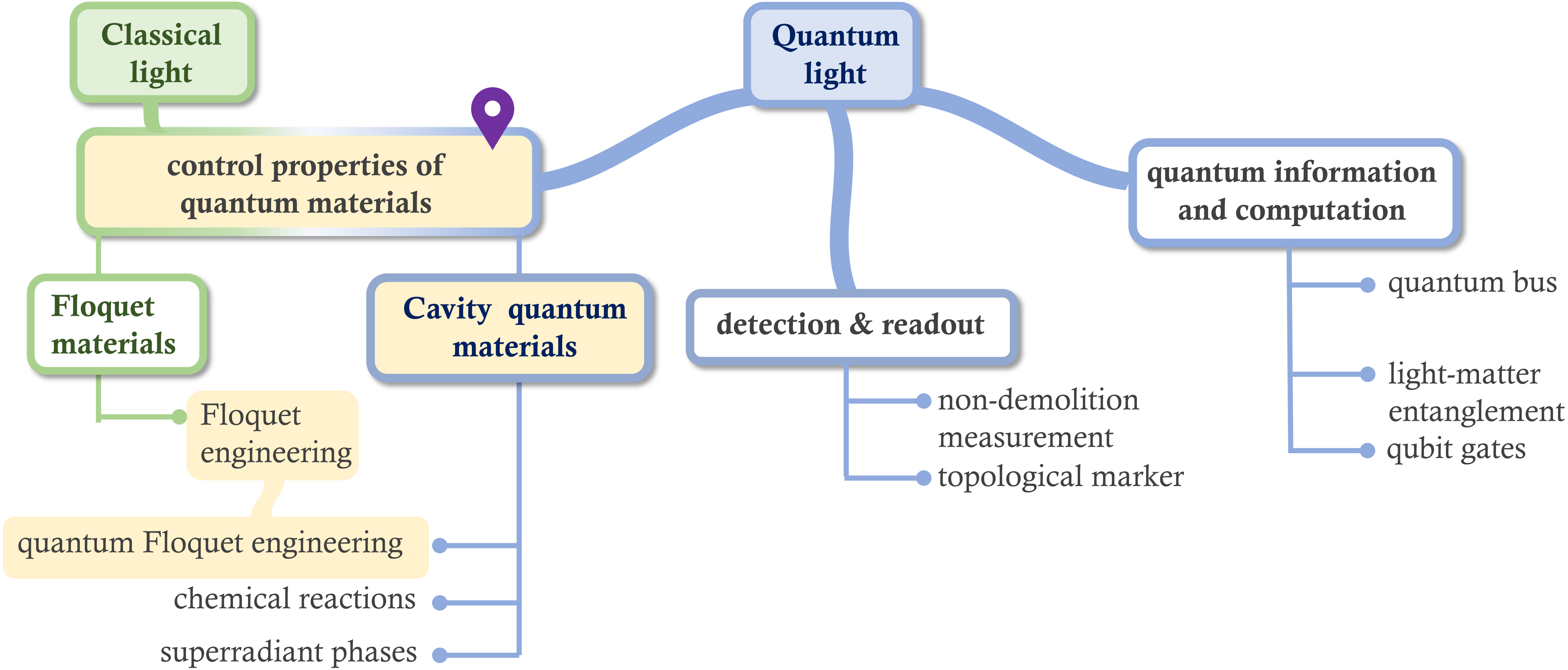
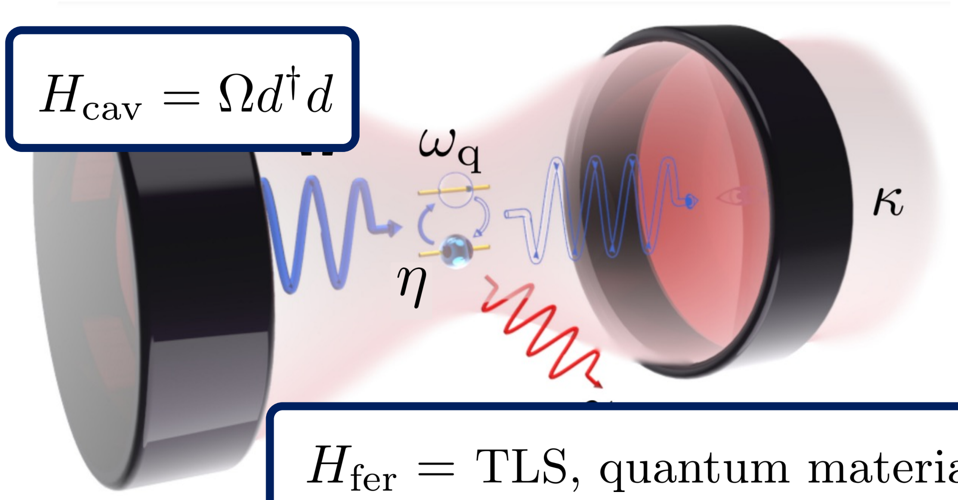


LIGHT-MATTER CORRELATIONS IN QUANTUM FLOQUET ENGINEERING

few-photon states or vacuum fluctuations trapped in small-volumen cavities…

$H_{\text{cav}} = \Omega d^\dagger d$



$H_{\text{fer}} = $ TLS, quantum material…

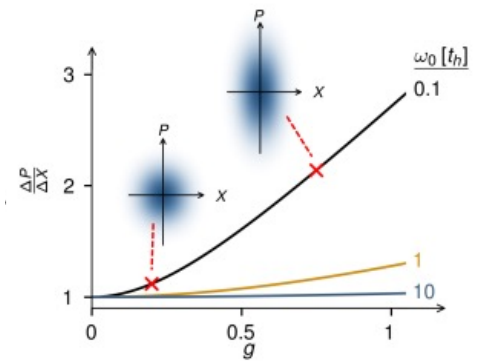
… interacting with a quantum system placed inside

A. Frisk Kockum *et al.* Nature Reviews Physics 1, 19-40 (2019)

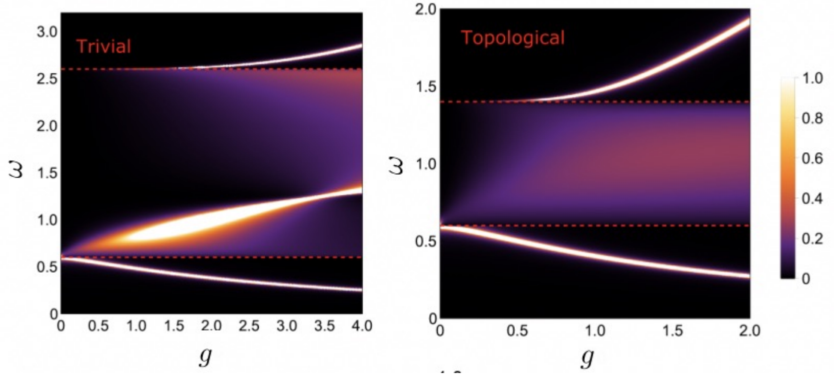
- **Disentangled light and matter (mean-field)**

$$|\psi_{\text{total}}\rangle = |\phi_{\text{phot}}\rangle \otimes |\chi_{\text{matter}}\rangle$$

  - Includes back-action between systems



C. J. Eckhardt *et al.*, Comm. Phys **5**, 122 (2022)



O. Dmytruk and M. Schirò, Comm. Phys **5**, 271 (2022)

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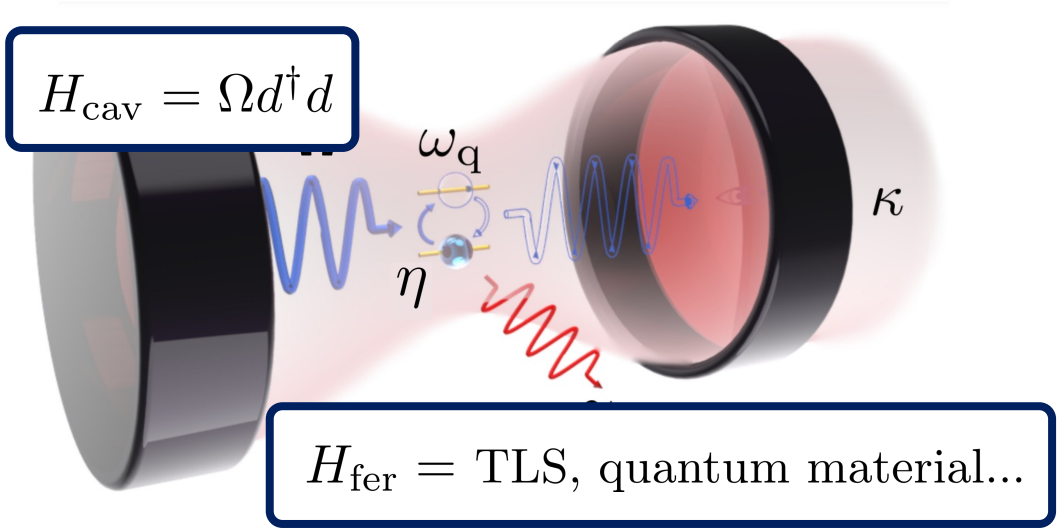


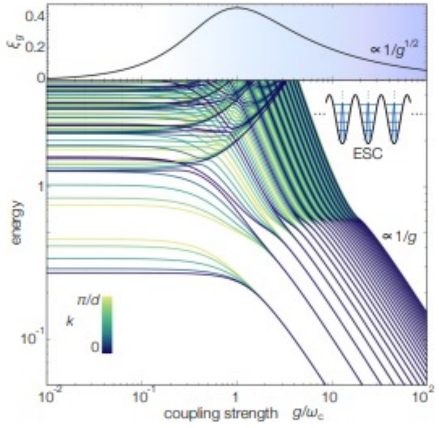
… interacting with a quantum system placed inside

A. Frisk Kockum *et al.* Nature Reviews Physics 1, 19-40 (2019)

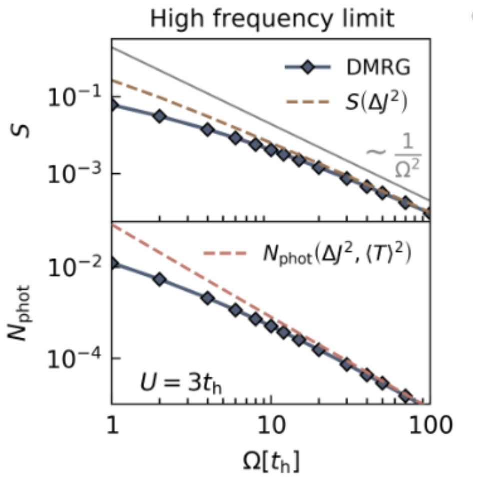
- **Light-matter correlations**

$$|\psi_{\text{total}}\rangle = |\phi_{\text{phot}}\rangle \otimes |\chi_{\text{matter}}\rangle + |\phi_{\text{corr.}}\rangle$$

  - **absent** in classical Floquet engineering
  - role in **Quantum Floquet Engineering**, and, in particular, for **topological systems**

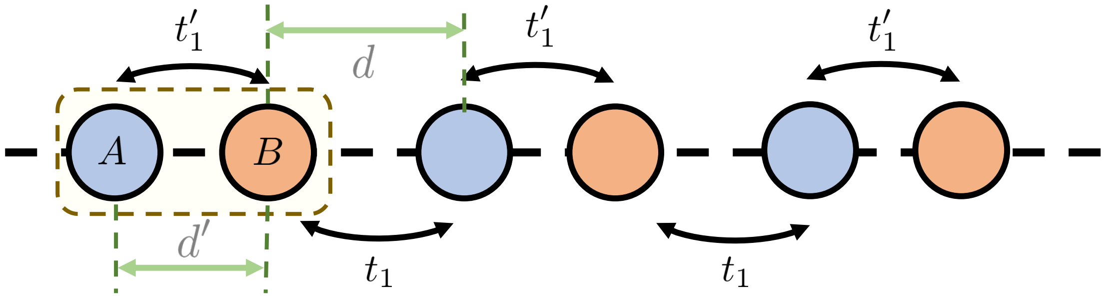


Y. Ashida et al., Phys. Rev. Lett. 126, 153603 (2021)



G. Passetti et al., arxiv:2212.03011v2

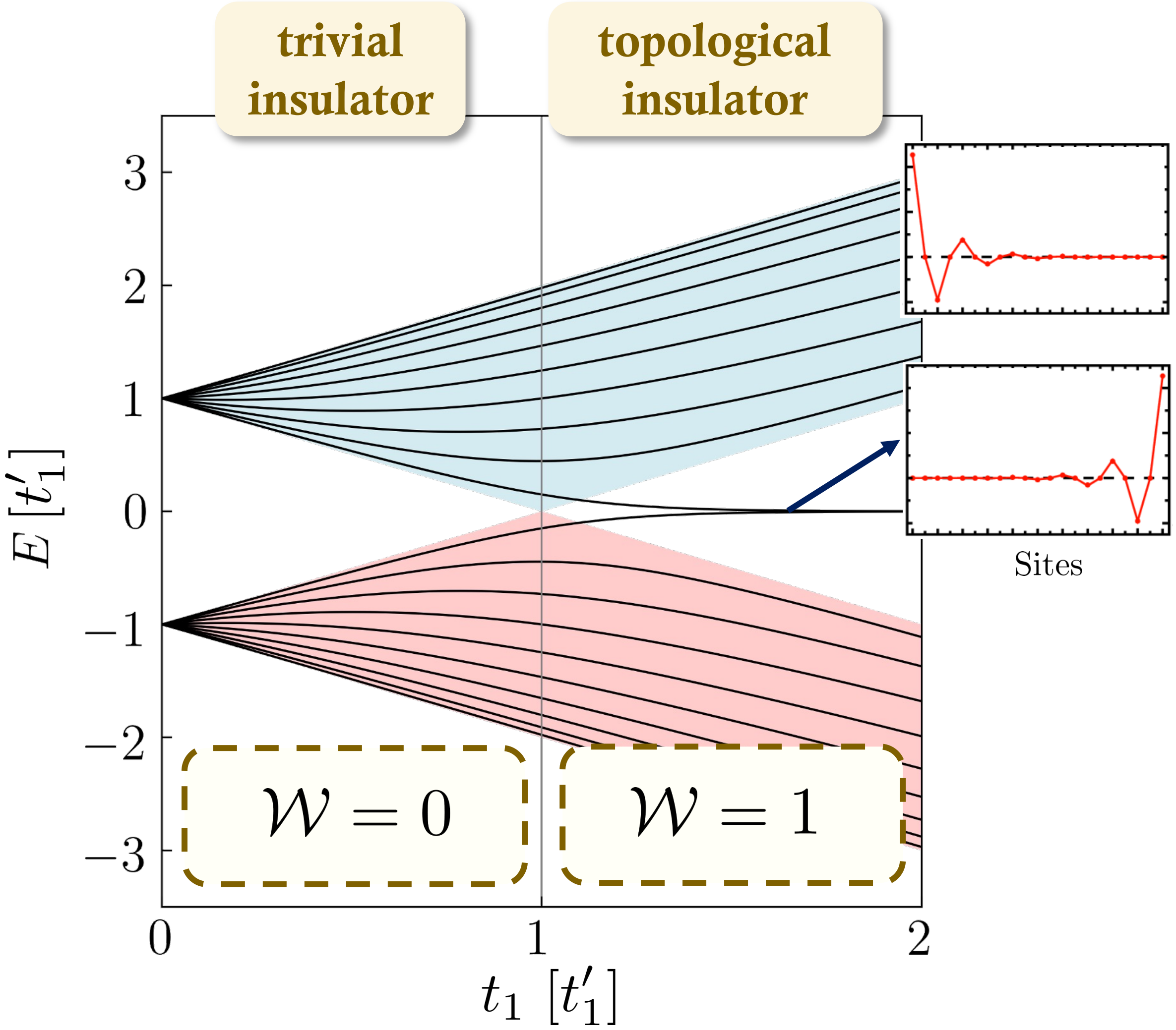
SSH model: canonical example of topological insulators (1D)



- **Alternating pattern** of hopping amplitudes

$$H_{\mathrm{SSH}} = \sum_j t'_1 a_j^\dagger b_j + t_1 b_j^\dagger a_{j+1} + \text{h.c.}$$

- ✓ topologically protected edge states by **chiral symmetry**



Light-Matter Hamiltonian

Starting Point

- **SSH Hamiltonian** interacting with a **quantized photonic field**

$$H = \Omega d^\dagger d + \sum_i t' e^{i\eta'(d^\dagger + d)} a_i^\dagger b_i + t e^{i\eta(d^\dagger + d)} b_i^\dagger a_{i+1} \quad + \text{h.c.}$$

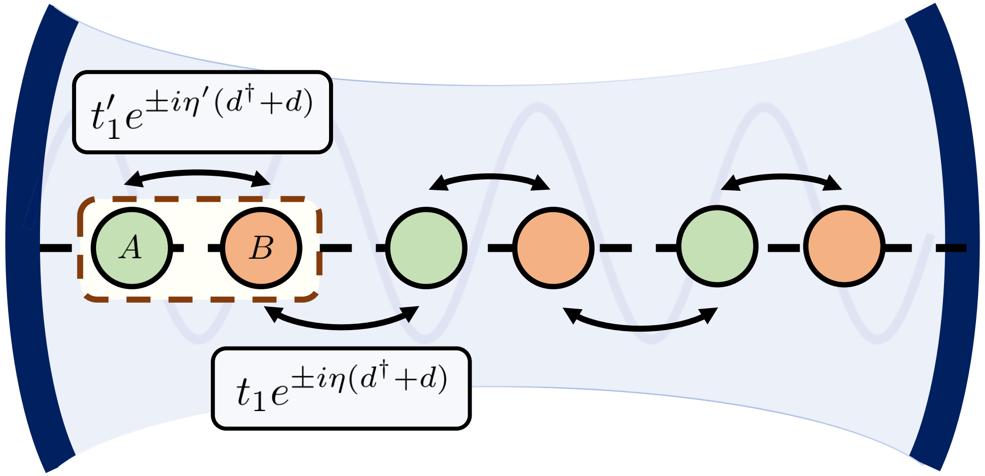
Minimal-coupling substitution in lattice models:
Peierls phase

- Gauge invariant
- Valid at **arbitrary coupling strength**
- Dipole approximation

$$t^{(\prime)} \to t^{(\prime)} e^{ied^{(\prime)}\vec{A}}, \quad \vec{A} = A_0(d^\dagger + d)\hat{u}_r$$

$$t^{(\prime)} \to t^{(\prime)} e^{i\eta^{(\prime)}(d^\dagger + d)}$$

effective coup. strength $\eta^{(\prime)} = eA_0 d^{(\prime)}$



Light-Matter Hamiltonian

Digress 1: Gauge-invariance for projected Hamiltonians

1. Implement light-matter coupling in the continuum theory

$$\vec{A} = A_0(d^\dagger + d)\hat{u}$$

Coulomb gauge

$$H^C = \frac{[\vec{p} - q\vec{A}]^2}{2m} + V(r) + \Omega d^\dagger d$$

D. de Bernardis *et al.*, Phys. Rev. A **98**, 053819 (2018)

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Coulomb gauge → (unitary transformation) → Dipole gauge

$$H^C = \frac{[\vec{p} - q\vec{A}]^2}{2m} + V(r) + \Omega d^\dagger d$$

$$H^D = \frac{\vec{p}^2}{2m} + V(r) + \Omega d^\dagger d + iqA_0\Omega(d - d^\dagger)x + \Omega q^2 A_0^2 x^2$$

D. de Bernardis *et al.*, Phys. Rev. A **98**, 053819 (2018)

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1. Implement light-matter coupling in the continuum theory

$$\vec{A} = A_0(d^\dagger + d)\hat{u}$$

2. Write in projected basis

$$H_{\text{el}} = \frac{\vec{p}^2}{2m} + V(r) = \sum \omega_n |\phi_n\rangle\langle\phi_n|$$

Coulomb gauge —unitary transformation→ Dipole gauge

$$H^C = \frac{[\vec{p} - q\vec{A}]^2}{2m} + V(r) + \Omega d^\dagger d$$

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3. Truncate to TLS

Coulomb gauge → (unitary transformation) → Dipole gauge

$$H^C = \frac{[\vec{p} - q\vec{A}]^2}{2m} + V(r) + \Omega d^\dagger d$$

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Coulomb gauge —unitary transformation→ Dipole gauge

$$H^C = \frac{[\vec{p} - q\vec{A}]^2}{2m} + V(r) + \Omega d^\dagger d$$

$$H^D = \frac{\vec{p}^2}{2m} + V(r) + \Omega d^\dagger d + iqA_0\Omega(d - d^\dagger)x + \Omega q^2 A_0^2 x^2$$

Projected Coulomb gauge —unitary transformation→ Projected dipole gauge

$$H^C_{\rm QRM} = \Omega d^\dagger d + \frac{\omega_{10}}{2}\sigma_z + g_C\sigma_y(d^\dagger + d) + D(d^\dagger + d)^2$$

$$H^D_{\rm QRM} = \Omega d^\dagger d + \frac{\omega_{10}}{2}\sigma_z + ig_D\sigma_x(d^\dagger - d)$$

D. de Bernardis *et al.*, Phys. Rev. A **98**, 053819 (2018)

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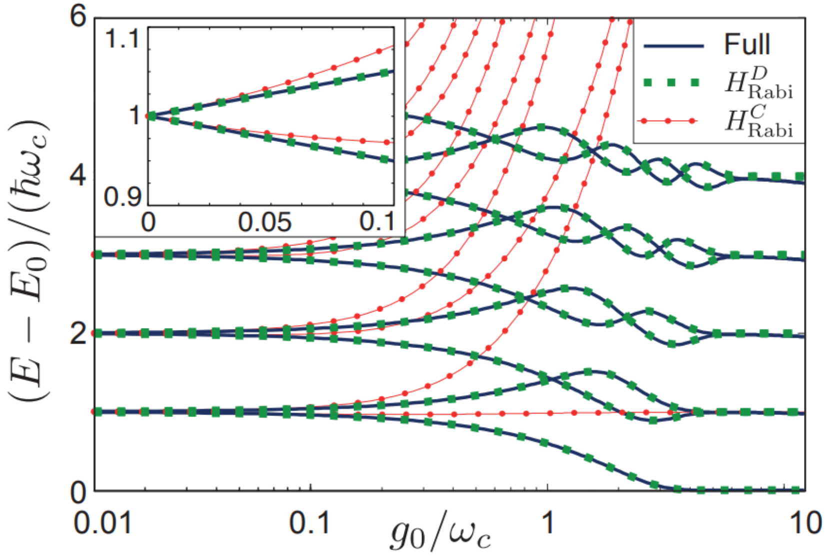
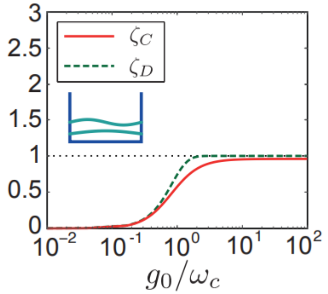
Digress 1: Gauge-invariance for projected Hamiltonians

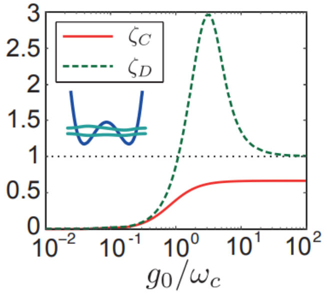
Projected Coulomb gauge → (unitary transformation) → Projected dipole gauge

$$H^C_{\mathrm{QRM}} = \Omega d^\dagger d + \frac{\omega_{10}}{2}\sigma_z + g_C \sigma_y (d^\dagger + d) + D(d^\dagger + d)^2$$

$$H^D_{\mathrm{QRM}} = \Omega d^\dagger d + \frac{\omega_{10}}{2}\sigma_z + i g_D \sigma_x (d^\dagger - d)$$







- Different energy spectrum for each gauge
- Different effective light-matter coupling strength for each gauge
- Different predictions for observables and phase transitions

D. de Bernardis *et al.*, Phys. Rev. A **98**, 053819 (2018)

Light-Matter Hamiltonian

Digress 1: Gauge-invariance for projected Hamiltonians

1. Write electronic Hamiltonian in projected basis

$$H_{\mathrm{el}} = \frac{\vec{p}^2}{2m} + V(r) = \sum \omega_n |\phi_n\rangle\langle\phi_n|$$

2. Truncate to TLS

$$H_{\mathrm{TLS}} = \frac{\omega_{10}}{2}\sigma_z$$

3. Implement light-matter coupling through **unitary transformation**

O. Di Stefano *et al.*, Nat. Phys. **15**, 803-808 (2019)

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Coulomb gauge

$$H^C = U H_{\mathrm{TLS}} U^\dagger + \Omega d^\dagger d \qquad U = \exp\{i(d^\dagger + d)\sum_{ij}\chi_{ij} c_i^\dagger c_j\}$$

$$\chi(r) = e\int_{r_0}^{r} A_0(r)\cdot dr \qquad \chi_{ij} = \langle i|\chi|j\rangle \qquad A(r) = A_0(r)(d^\dagger + d)$$

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$$\begin{aligned} H^C_{QRM} &= \Omega d^\dagger d + t|R\rangle\langle L| e^{iqaA_0(d^\dagger + d)} + \mathrm{h.c.} \\ &= \Omega d^\dagger d + \frac{\omega_{10}}{2}\left\{\cos\left[\eta(d^\dagger + d)\right]\sigma_z + \sin\left[\eta(d^\dagger + d)\right]\sigma_y\right\} \end{aligned}$$

O. Di Stefano *et al.*, Nat. Phys. **15**, 803-808 (2019)

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$$H^C_{\rm t.b.} = \Omega d^\dagger d + \sum_{ij} t_{ij} e^{ieA_0 r_{ij}(d^\dagger + d)} c_i^\dagger c_j \qquad r_{ij} = r_i - r_j$$

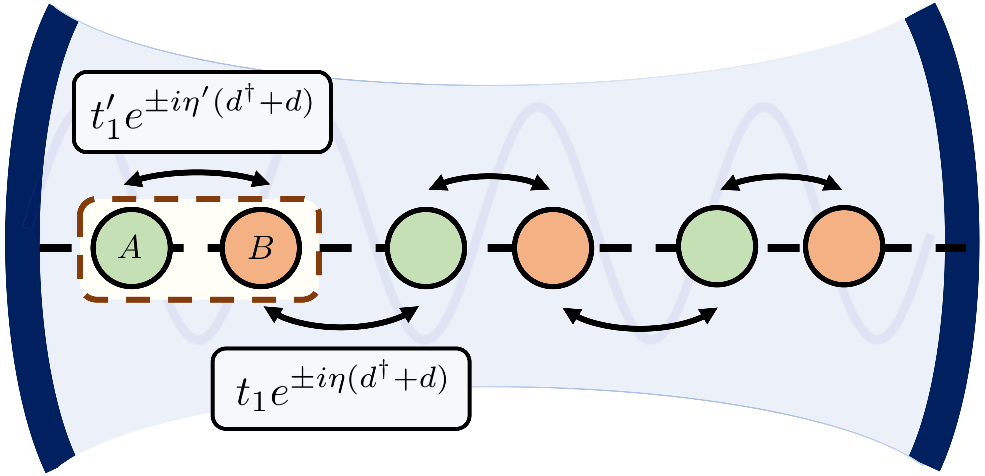
O. Di Stefano *et al.*, Nat. Phys. **15**, 803-808 (2019)

Objectives & Summary

Starting Point

- **SSH Hamiltonian** interacting with a **quantized photonic field**

$$H = \Omega d^\dagger d + \sum_i t' e^{i\eta'(d^\dagger + d)} a_i^\dagger b_i + t e^{i\eta(d^\dagger + d)} b_i^\dagger a_{i+1} \quad + \text{h.c.}$$



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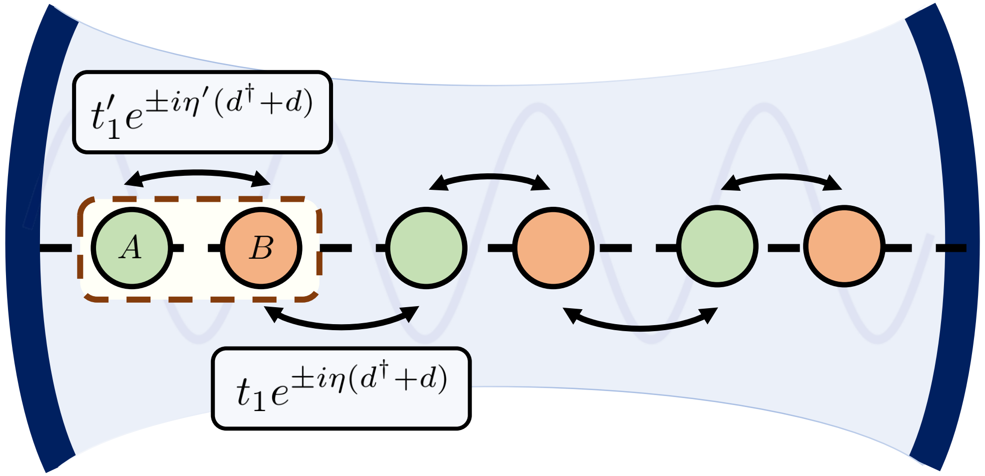
- **Our work (arXiv:2302.12290)**



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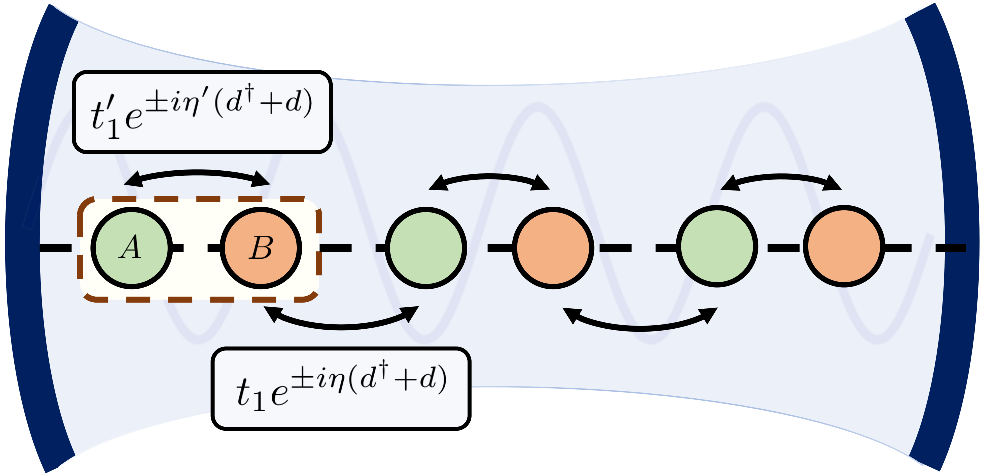
  - Find a simplified form of the Hamiltonian that
    - *i)* allows for analytical treatment,
    - *ii)* captures the relevant features of the system for arbitrary coupling strength



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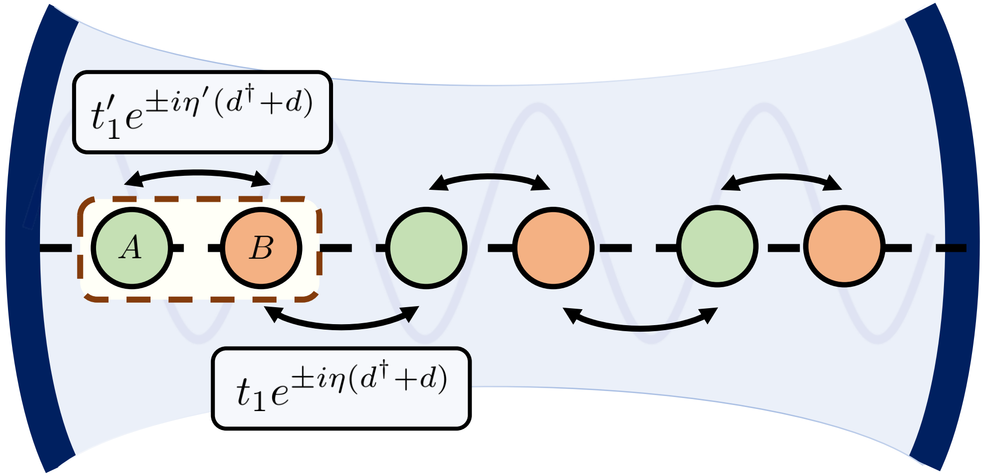
valid for arbitrary tight-binding models



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- **Our work (arXiv:2302.12290)**
  - Find a simplified form of the Hamiltonian that
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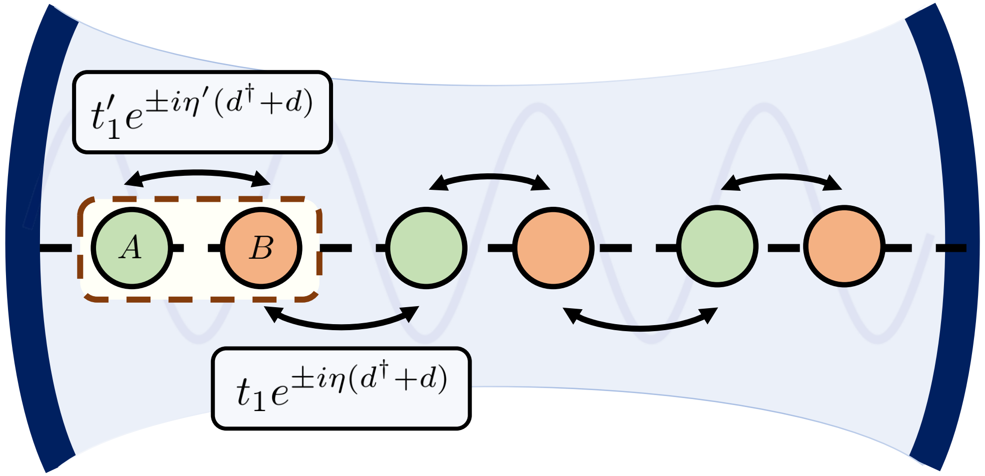
    ii) captures the relevant features of the system for arbitrary coupling strength
  - Find topological phase transitions driven by light-matter interaction (quantum Floquet engineering)
  - Identify the role of light-matter correlations



Digress 2: Floquet-Bloch Theory for the SSH chain

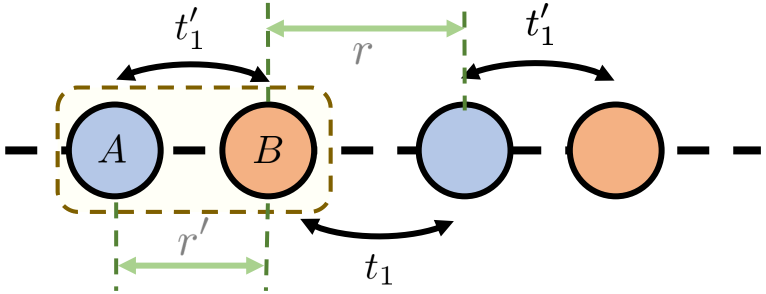
$$H_{\text{tot}}(t) = H_{\text{SSH}} + H_{\text{driv}}(t)$$

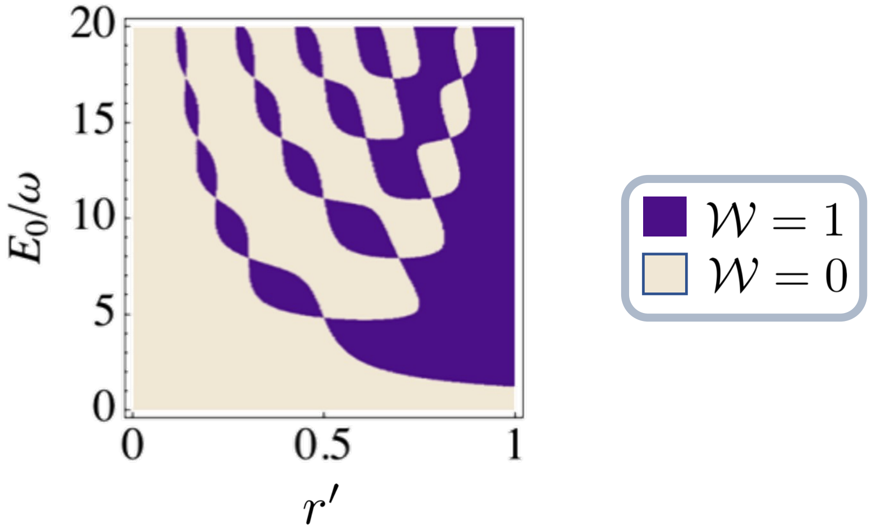
$$H_{\text{SSH}} = \sum_j t_1' a_j^\dagger b_j + t_1 b_j^\dagger a_{j+1} + \text{h.c.}$$

$$H_{\text{driv}}(t) = E(t) \sum_{i=1}^{N} x_i c_i^\dagger c_i$$

$\omega \gg t, t'$

$$H_{\text{eff}} = \sum_j \mathcal{J}_0 \left(\frac{E_0 r'}{\omega} \right) t_1' a_j^\dagger b_j + \mathcal{J}_0 \left(\frac{E_0 (1 - r')}{\omega} \right) t_1 b_j^\dagger a_{j+1} + \text{h.c.}$$





A. Gómez-León and G. Platero, Phys. Rev. Lett. 110, 200403 (2013)

Floquet materials

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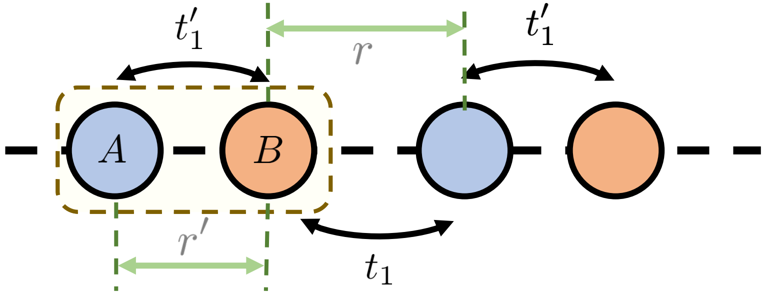

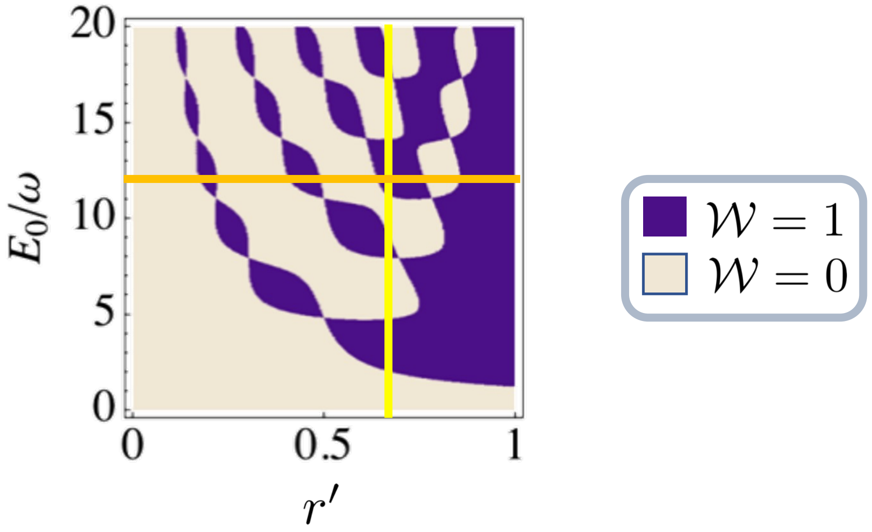

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B. Pérez-González et al.,
arxiv: 2302.12290v1 (2023)

EFFECTIVE HAMILTONIAN

- **Truncation** of the Peierls Hamiltonian

$$H = \Omega d^\dagger d + \sum_{l,j=1}^{N} t_{l,j} e^{i\eta_{l,j}(d^\dagger + d)} c_j^\dagger c_l$$

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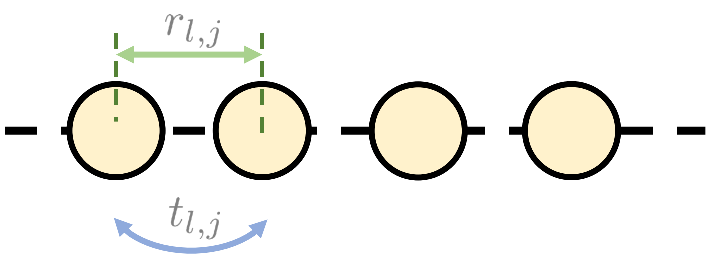
effective coupling strength

$\eta_{l,j} = eA_0 r_{l,j}$



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Quantum Floquet engineering

B. Pérez-González et al., arxiv: 2302.12290v1 (2023)

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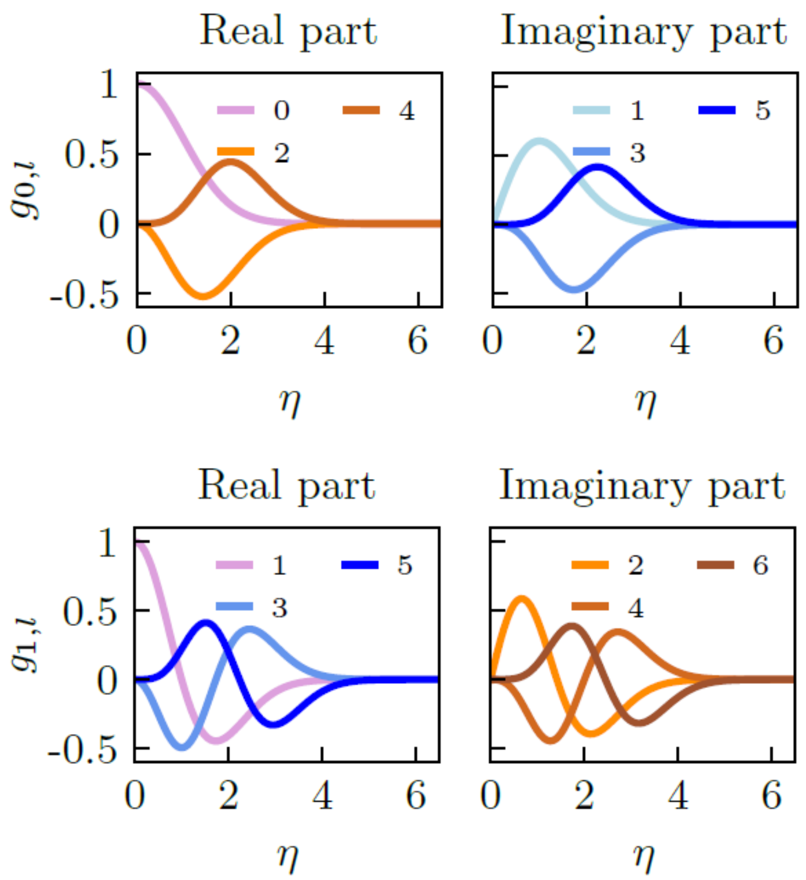
$$\sum_{n=0}^{\infty} g_{n,n}^{l,j} Y^{n,n} + \sum_{n\neq m=0}^{\infty} g_{m,n}^{l,j} Y^{m,n}$$

Photonic Hubbard operators: $Y^{m,n} = |m\rangle\langle n|$



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$$H = \begin{pmatrix} H_0 & & & \\ & H_1 & & \\ & & H_2 & \\ & & & \ddots \end{pmatrix}$$

B. Pérez-González et al., arxiv: 2302.12290v1 (2023)

Effective Hamiltonian

- **Truncation** of the Peierls Hamiltonian

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$$H = \begin{pmatrix} H_0 & H_{0\to 1} & H_{0\to 2} & \\ H_{1\to 0} & H_1 & H_{1\to 2} & \\ H_{2\to 0} & H_{2\to 1} & H_2 & \\ & & & \ddots \end{pmatrix}$$

Quantum Floquet engineering

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$$+ \sum_{m=0}^{\infty} \sum_{j,l=1}^{N} g_{m,m+1}^{l,j} t_{j,l} c_j^\dagger c_l \left(Y^{m,m+1} + Y^{m+1,m} \right)$$

(specially well-suited for the **high-frequency regime**)

$$H = \begin{pmatrix} H_0 & H_{0\to 1} & & \\ H_{1\to 0} & H_1 & H_{1\to 2} & \\ & H_{2\to 1} & H_2 & \\ & & & \ddots \end{pmatrix}$$

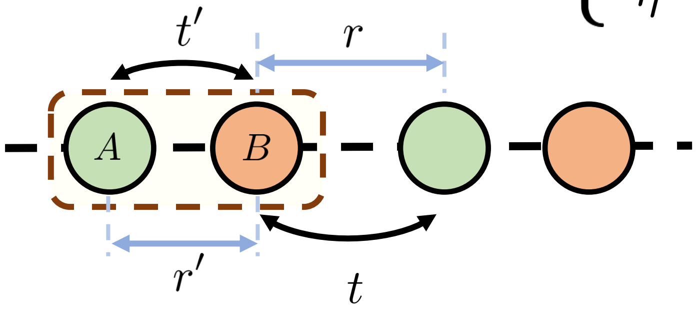
Quantum Floquet engineering

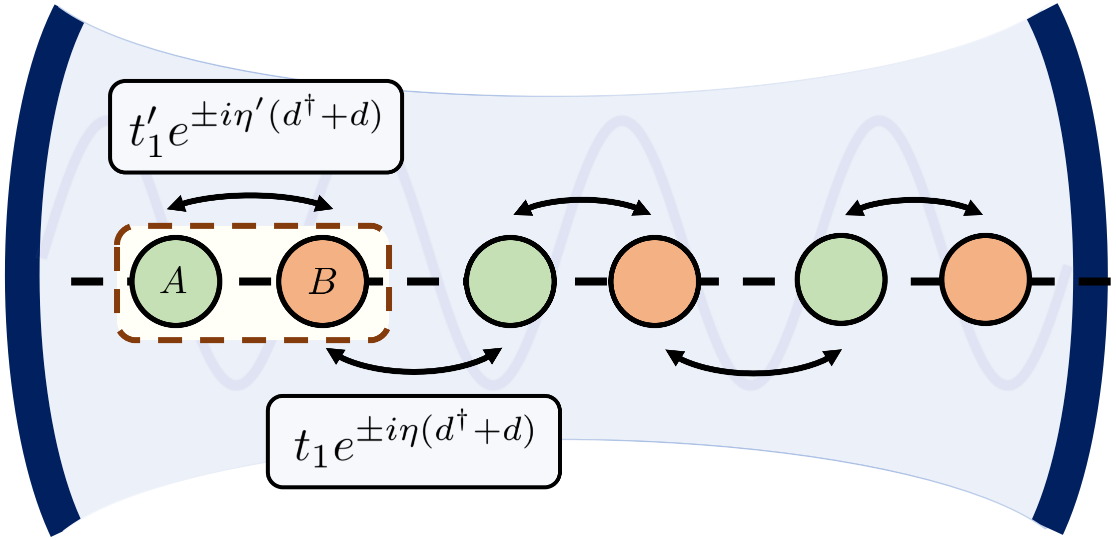
B. Pérez-González et al., arxiv: 2302.12290v1 (2023)

EFFECTIVE HAMILTONIAN

- **Truncation** of the Peierls Hamiltonian
  - **Dimerized** interaction strength

$$\eta_{j,l}\begin{cases}\eta' = eA_0 r' \\ \eta = eA_0 r\end{cases}$$

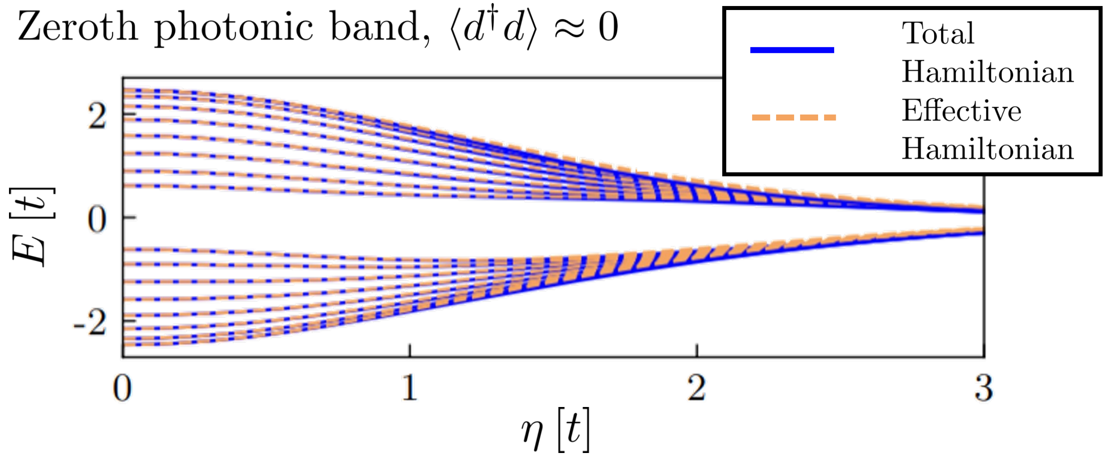




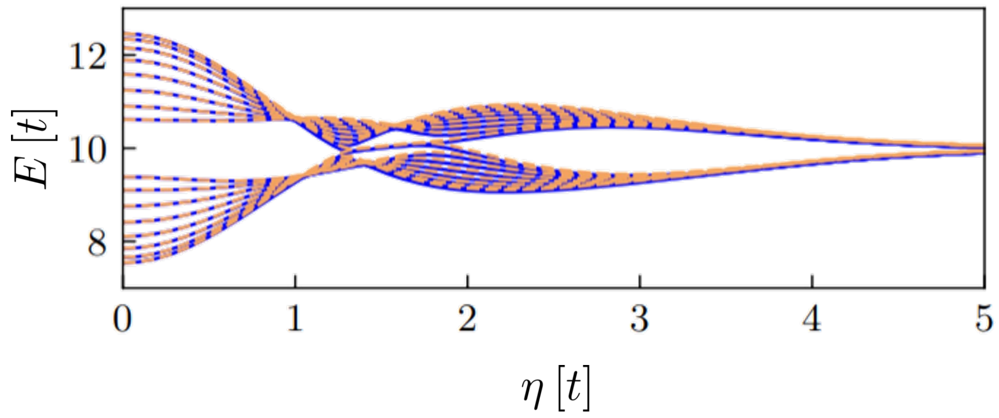
Topological phase transitions driven by light-matter interaction

Energy spectrum

a) Zeroth photonic band, $\langle d^\dagger d \rangle \approx 0$



b) First photonic band, $\langle d^\dagger d \rangle \approx 1$

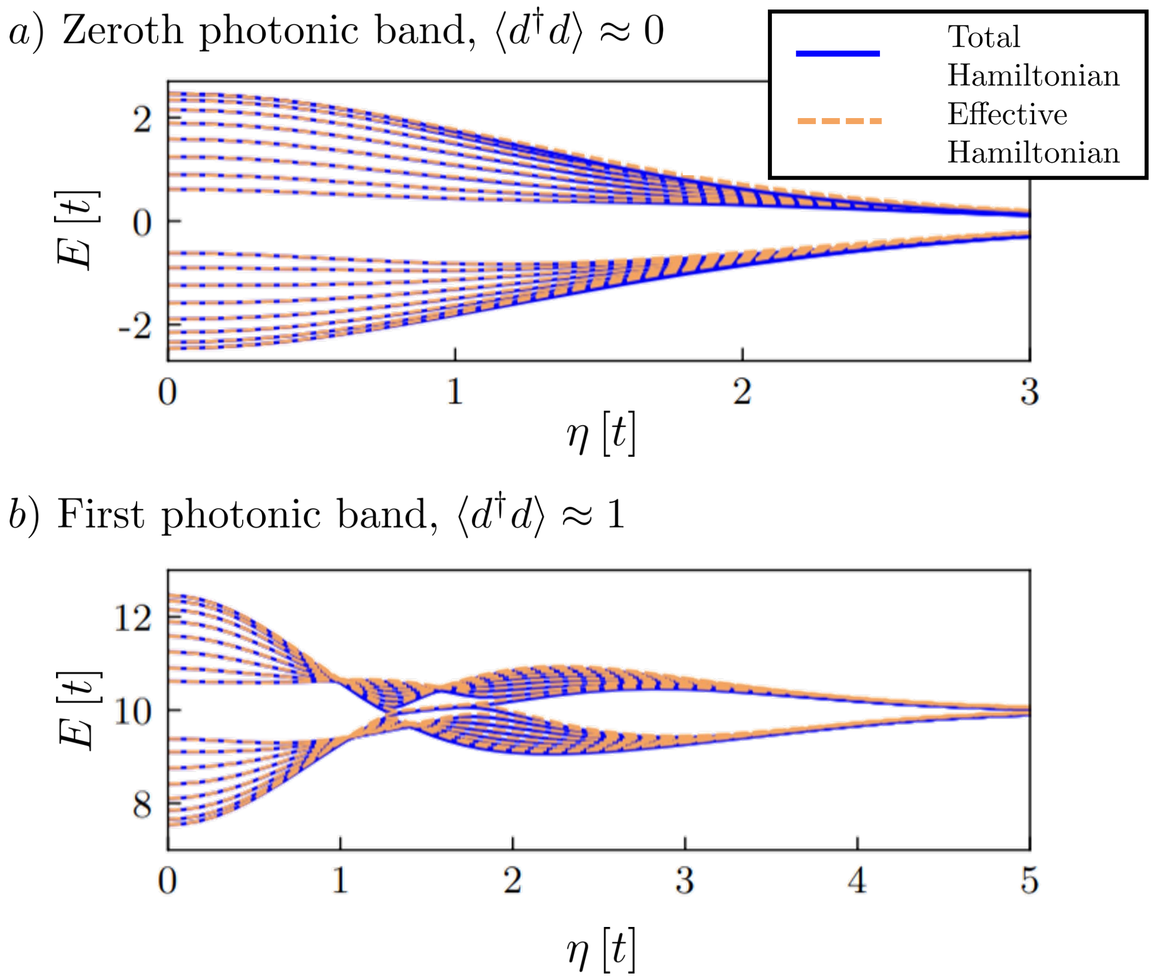


Parameter choice

- Trivial topology for the unperturbed system $t = 1,\;\; t' = 1.5$
- Highly detuned cavity $\Omega \gg t, t'$
- Coupling strength $\eta\,[t]$

Topological phase transitions driven by light-matter interaction

Energy spectrum



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- Nice agreement for the effective Hamiltonian

Quantum Floquet Engineering

arXiv:2302.12290

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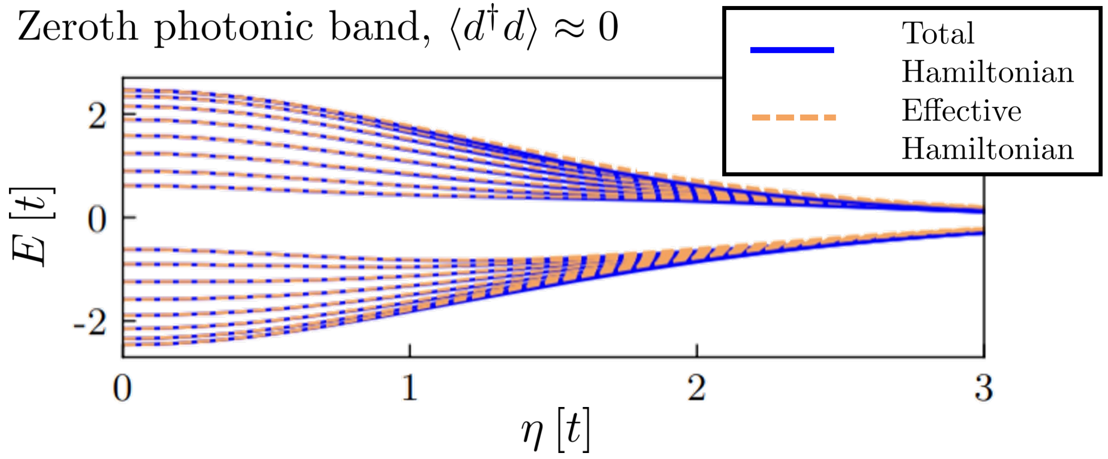

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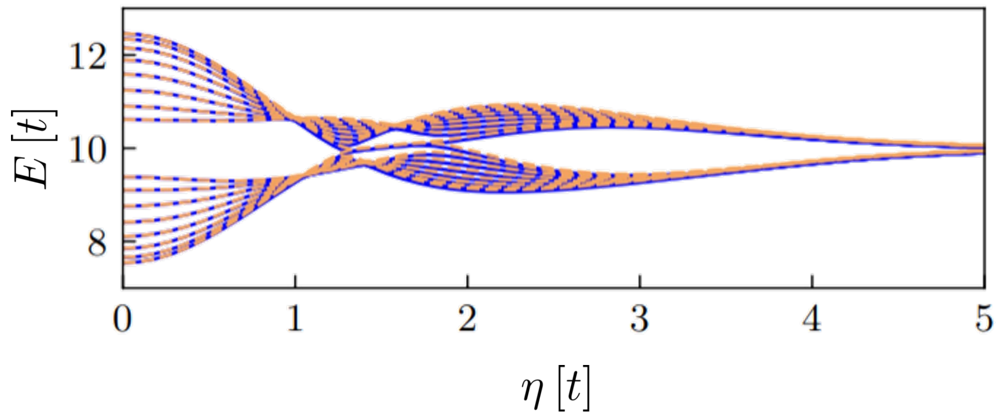

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- Different renormalization for each photonic band

Quantum Floquet Engineering

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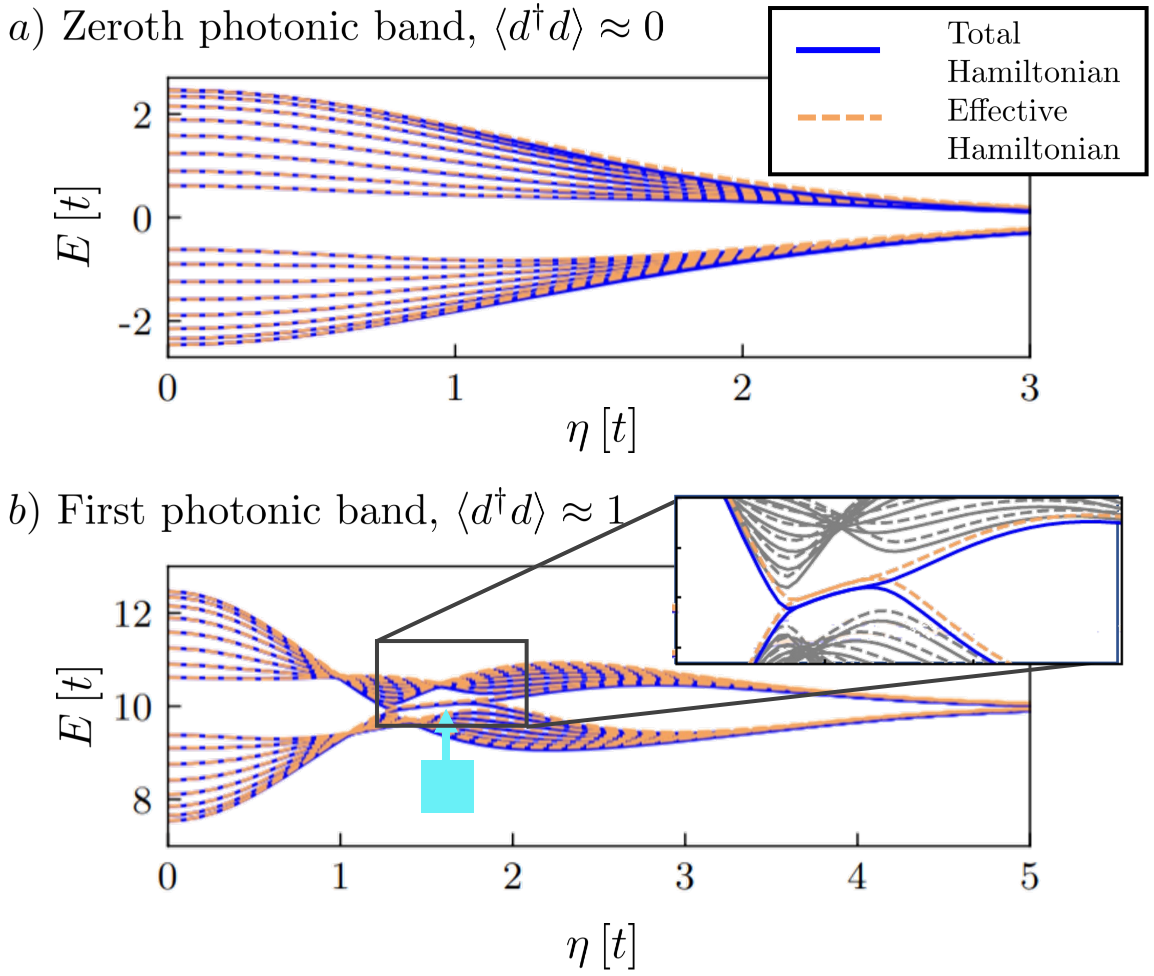
Energy spectrum



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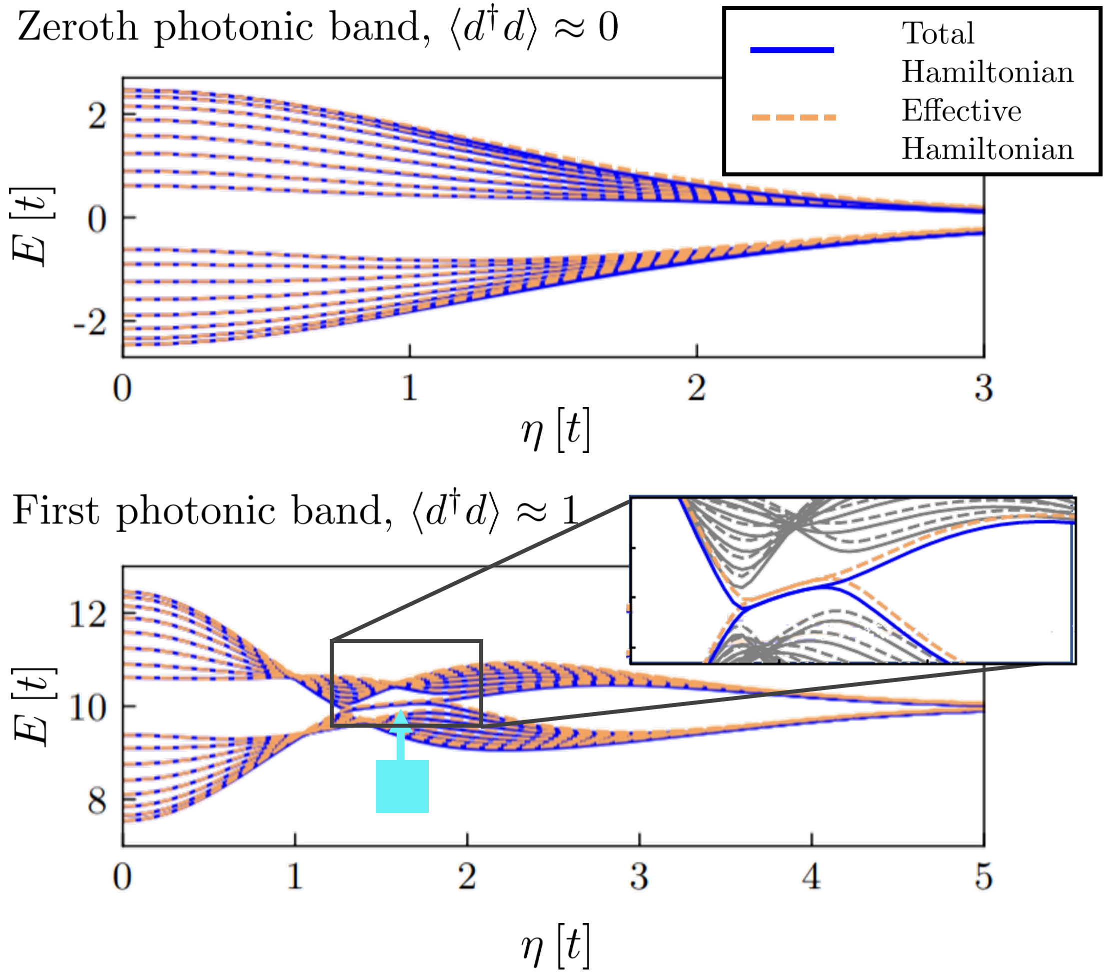
- Different renormalization for each photonic band
- Topological phase transition in the first photonic band

Topological phase transitions driven by light-matter interaction

Energy spectrum



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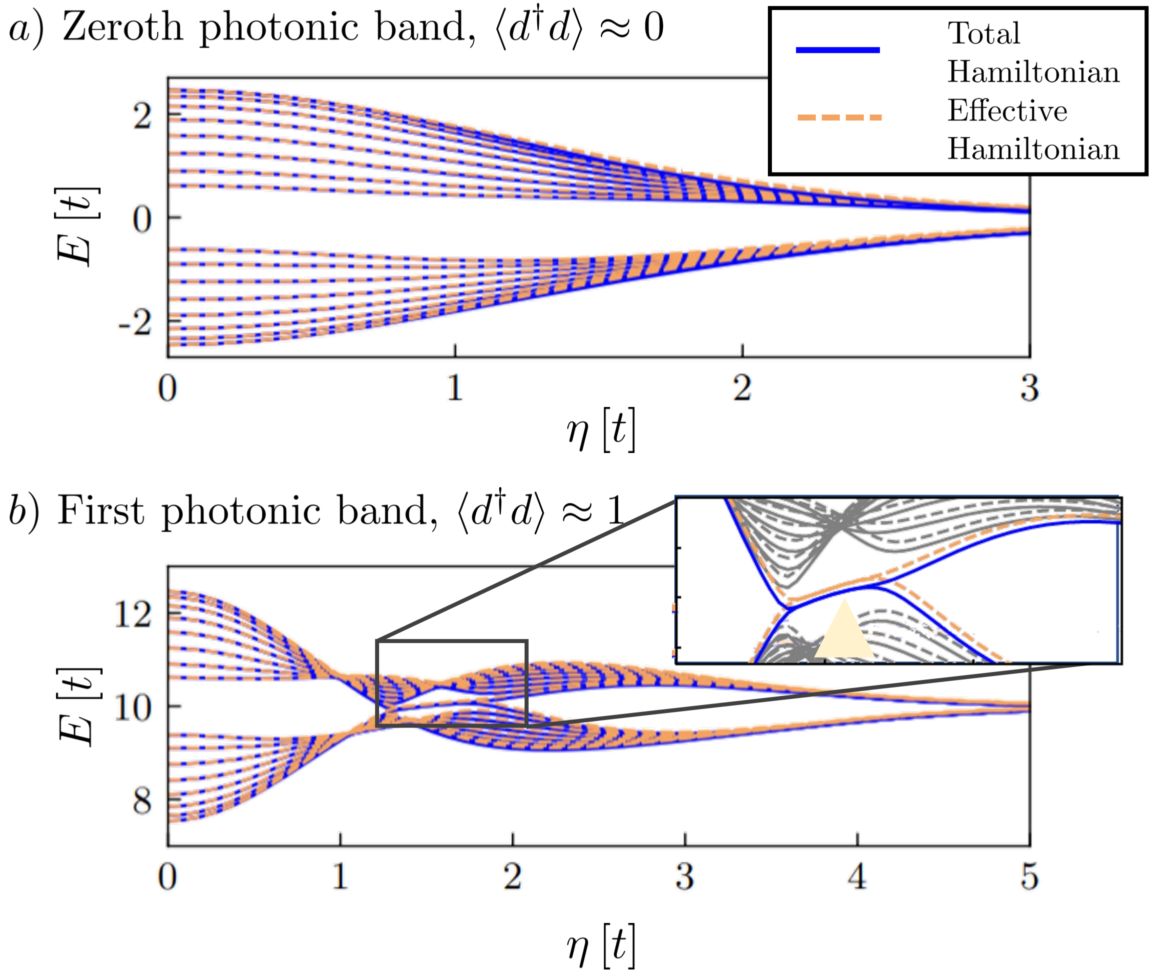
Topological properties

- **Coupling strength** (reminiscent of classical Floquet engineering)
- **Cavity state preparation** (unique to quantum Floquet engineering)

TOPOLOGICAL PHASE TRANSITIONS DRIVEN BY LIGHT-MATTER INTERACTION

Energy spectrum



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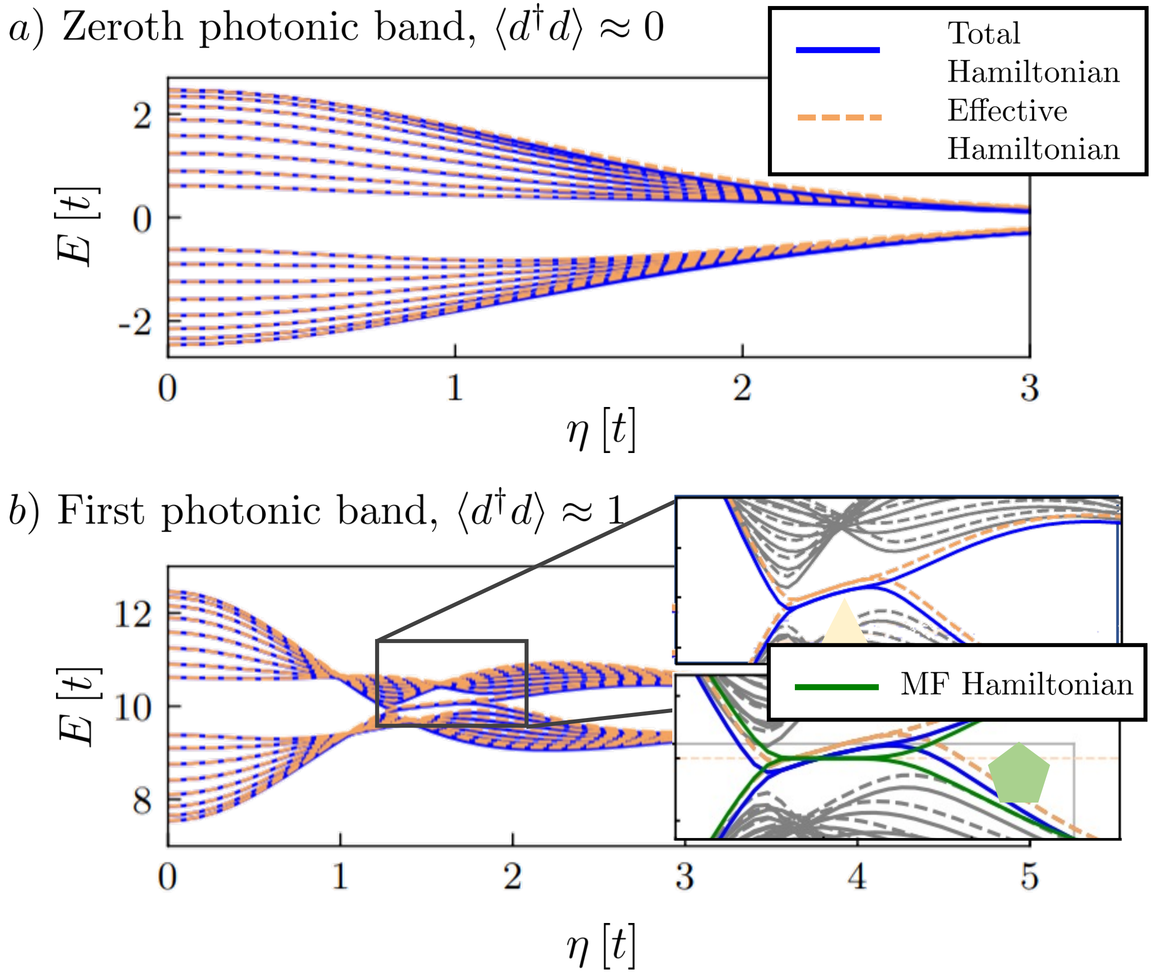
- **Chiral symmetry** breaking mechanism

Topological phase transitions driven by light-matter interaction

Energy spectrum



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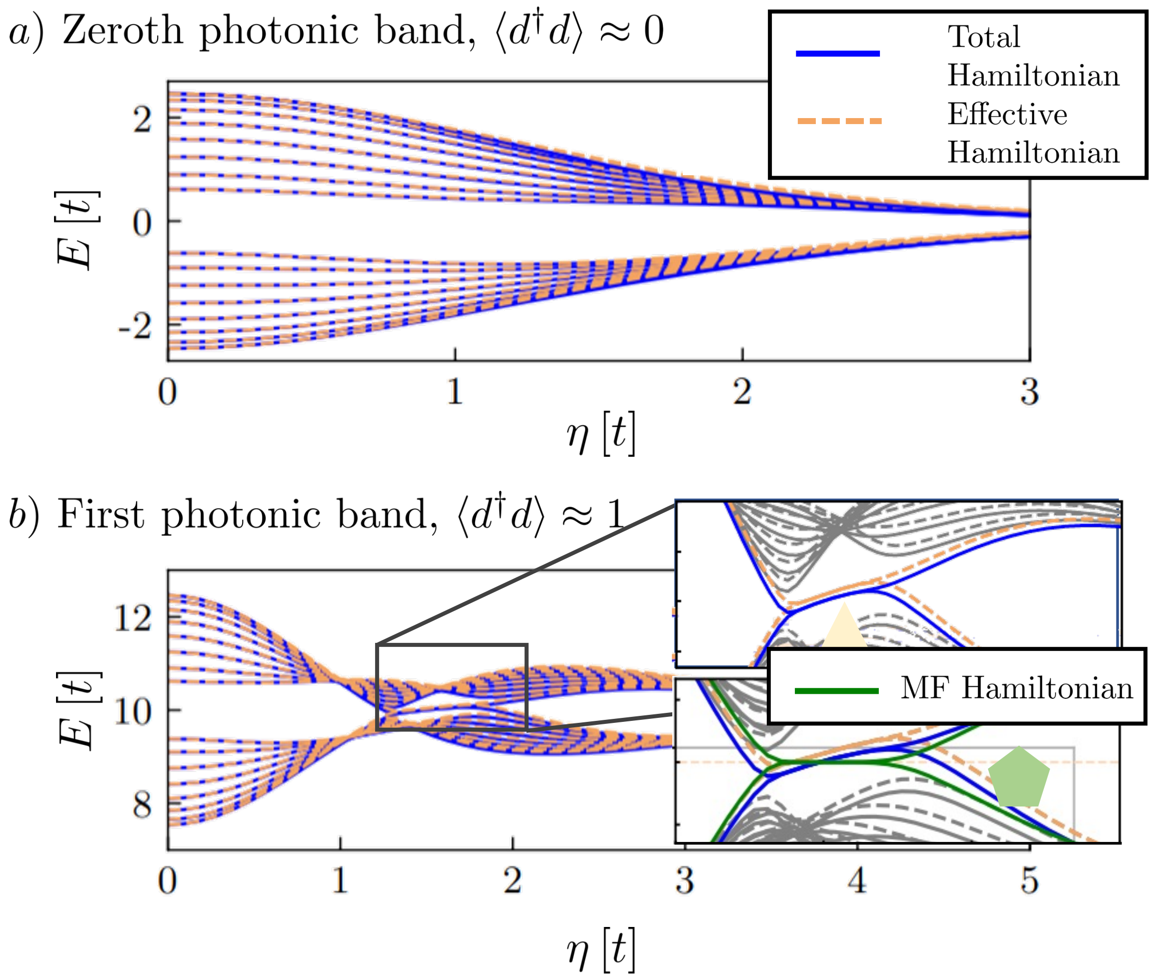
TOPOLOGICAL PHASE TRANSITIONS DRIVEN BY LIGHT-MATTER INTERACTION

Energy spectrum



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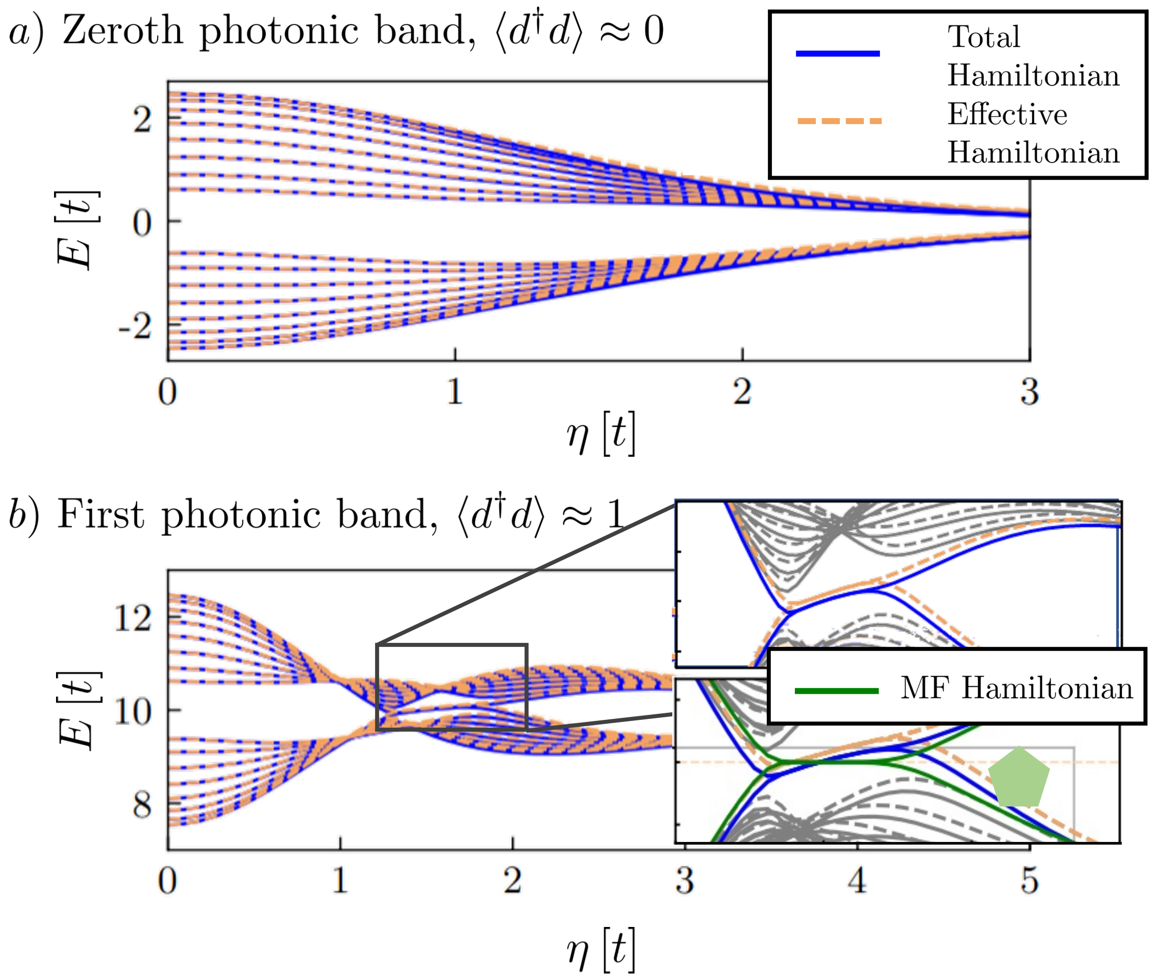
- **Chiral symmetry** breaking mechanism
- Linked to **light-matter correlations**
- Perturbative corrections, yet for topological systems it is crucial to keep them

Topological phase transitions driven by light-matter interaction

Energy spectrum

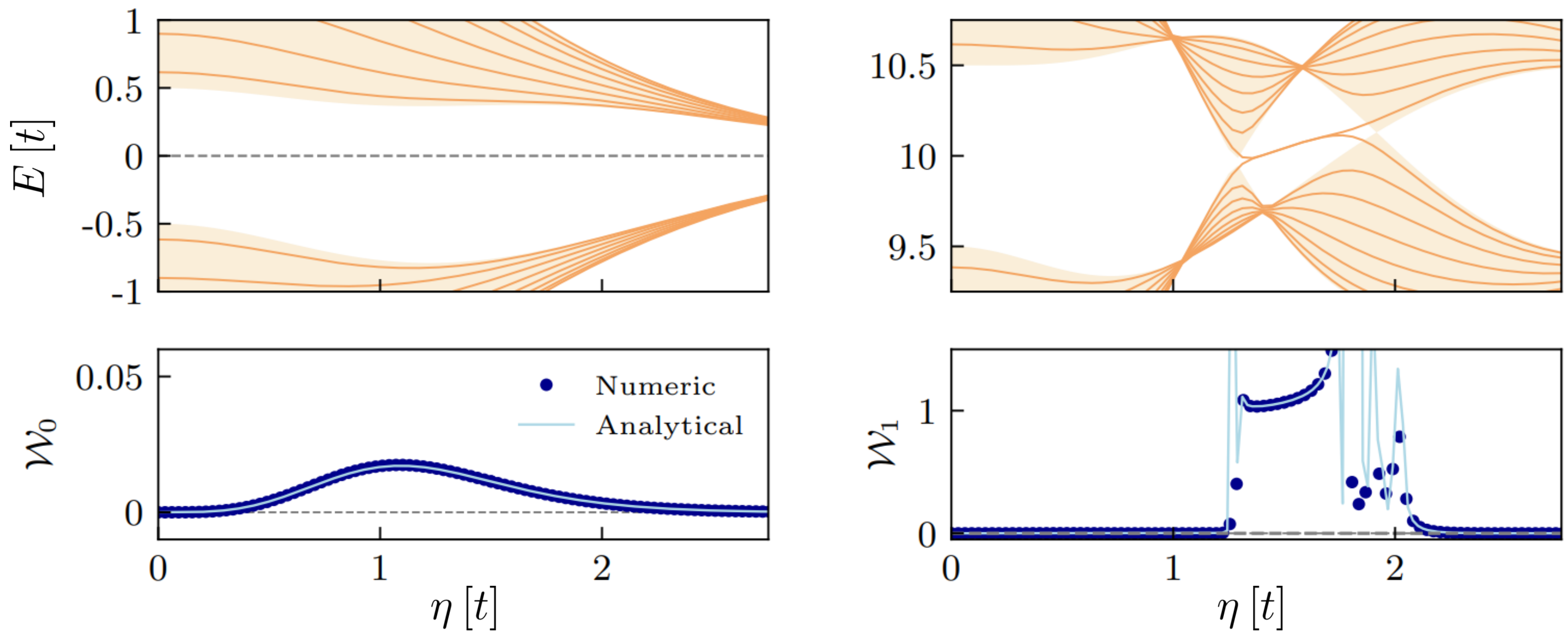


Light-matter correlations

- ✓ **Included**
  - × **broken chiral symmetry**
  - × no topological protection
  - **Total, and effective Hamiltonian**
- × **Not included**
  - ✓ preserved chiral symmetry
  - ✓ topological protection
  - **Mean-field Hamiltonian**

Analytical results for the truncated Hamiltonian

- The truncated Hamiltonian allows to solve **analytically** the calculation of…
  - … the **topological invariant**



- Topological invariant for the interacting system
- Reproduces:

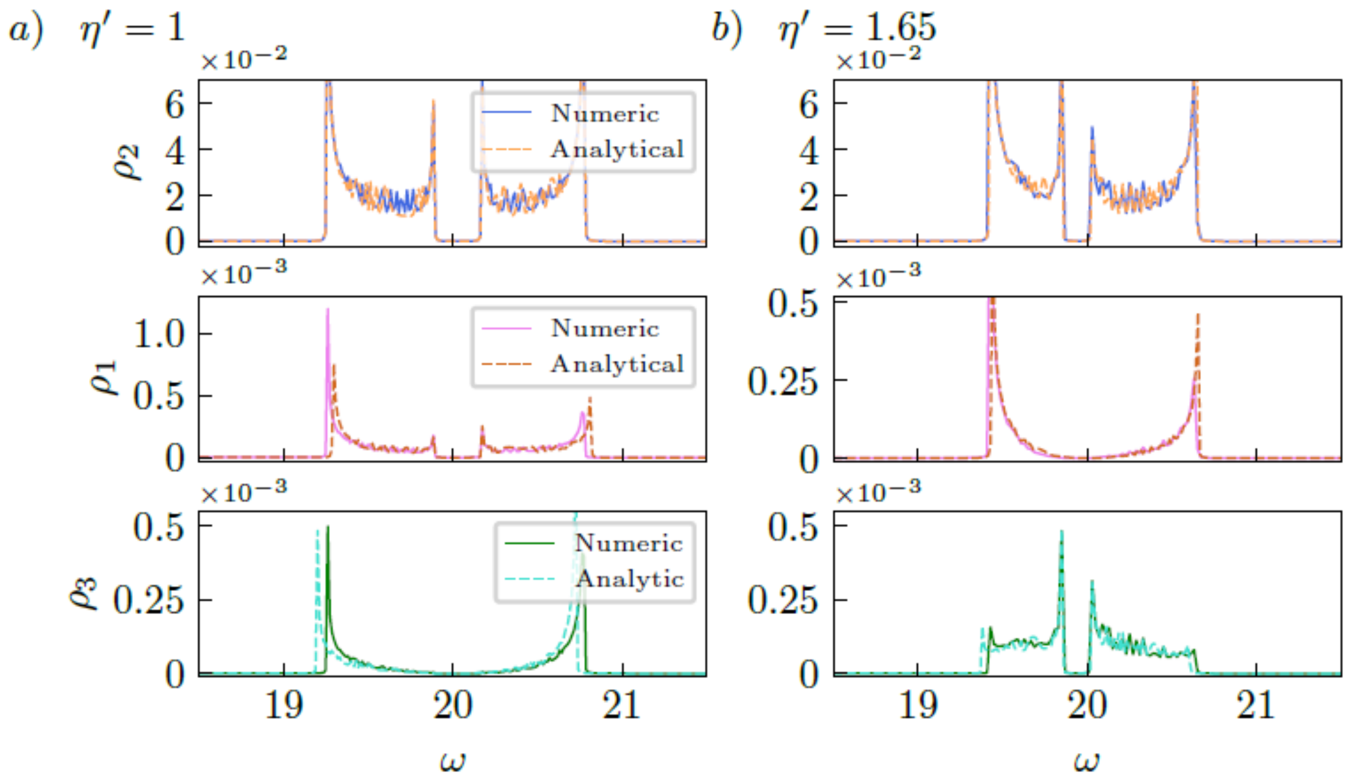
a) the phase transition in each photonic band
b) chiral-symmetry breaking due to the los of quantization

Quantum Floquet Engineering

arXiv:2302.12290

Analytical results for the truncated Hamiltonian

- The truncated Hamiltonian allows to solve **analytically** the calculation of…
  - … the **density of states**



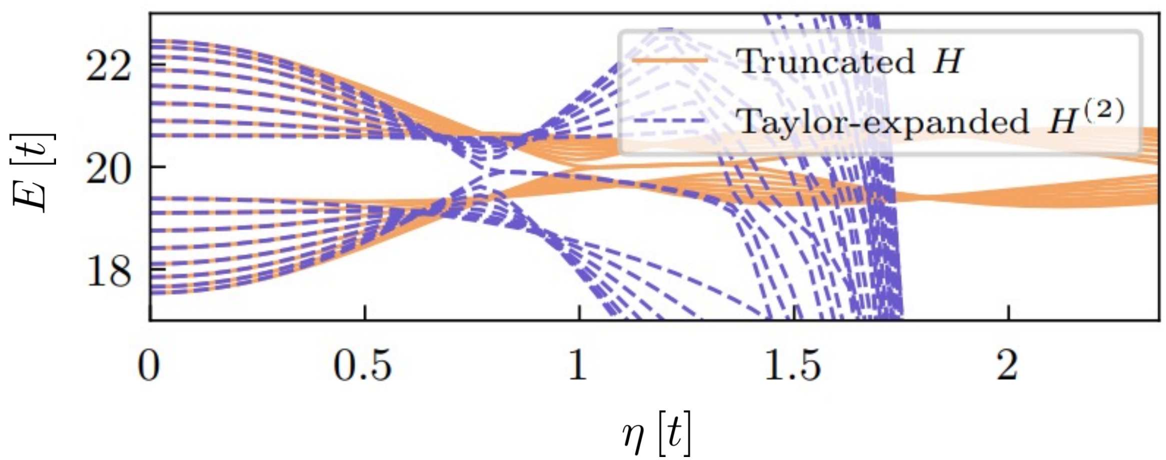
- Migration of spectral weight between photonic subspaces due to one-photon transitions
- Quantify the coupling between different photonic bands

B. Pérez-González et al., arxiv: 2302.12290v1 (2023)

Digress 3: Comparison with perturbative approaches

- Taylor-expanded Hamiltonian

$$H^{(2)} = \sum_{ij} \left[t_{ij} - \frac{\eta_{ij}^2}{2}(d^\dagger + d)^2 \right] c_l^\dagger c_j + i(d^\dagger + d) \sum_{jl} \eta_{jl} t_{lj} \left(c_j^\dagger c_l - c_l^\dagger c_j \right)$$



- Truncated Hamiltonian

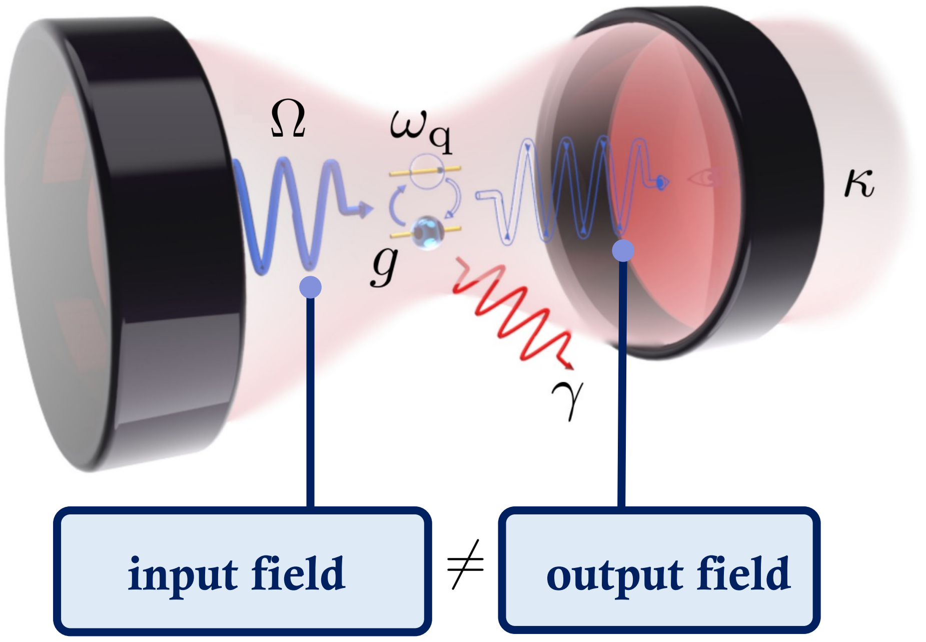
$$H = \sum_{n=0}^{\infty} \left(n\Omega + \sum_{l,j=1}^{N} g_{n,n}^{l,j} t_{j,l} c_j^\dagger c_l \right) Y^{n,n} + \sum_{m=0}^{\infty} \sum_{j,l=1}^{N} g_{m,m+1}^{l,j} t_{j,l} c_j^\dagger c_l \left(Y^{m,m+1} + Y^{m+1,m} \right)$$

Parameter choice

- Trivial topology for the unperturbed system $t = 1,\ t' = 1.5$
- Highly detuned cavity $\Omega \gg t, t'$
- Coupling strength $\eta\,[t]$

Light-Matter correlations

Experimental detection



A. Frisk Kockum *et al.* Nature Reviews Physics 1, 19-40 (2019)

- **Transmission coefficient**

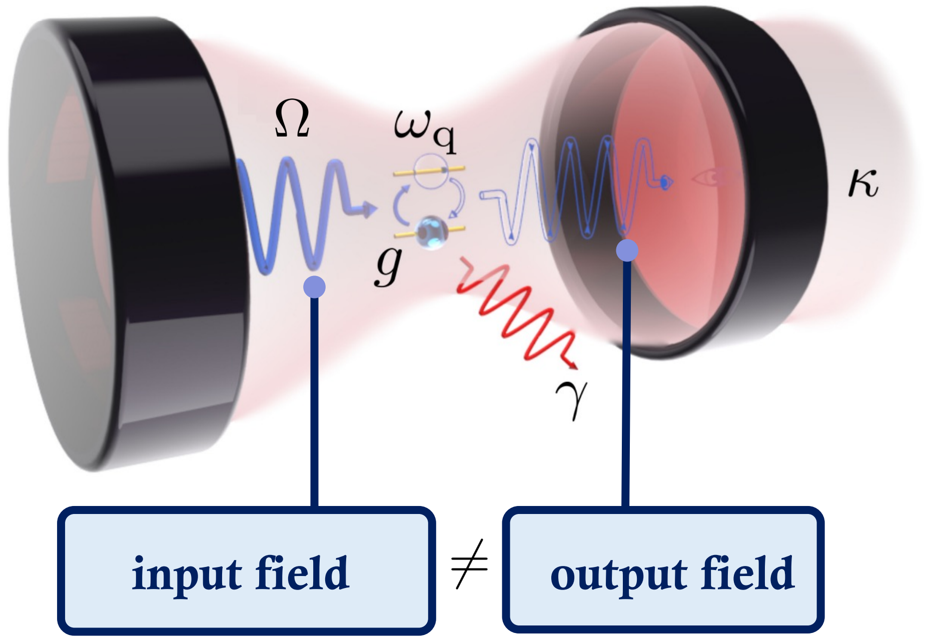
$$t_c = |t_c|e^{i\varphi} = \frac{\langle b_{\text{out}} \rangle}{\langle b_{\text{in}} \rangle}$$

Cottet et al; J. Phys.: Cond Mat., **29** 433002 (2017)
S. Kohler, Phys. Rev. A, 023849 (2018)

- b_{out} contains information about the state of the system inside the cavity
- both amplitude and phase can be detected experimentally

Light-Matter correlations

Experimental detection



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- b_{out} contains information about the state of the system inside the cavity
- both amplitude and phase can be detected experimentally
- it can also be defined in terms of the **photonic Green function**

$$t_c \propto \mathcal{G}(\omega)$$

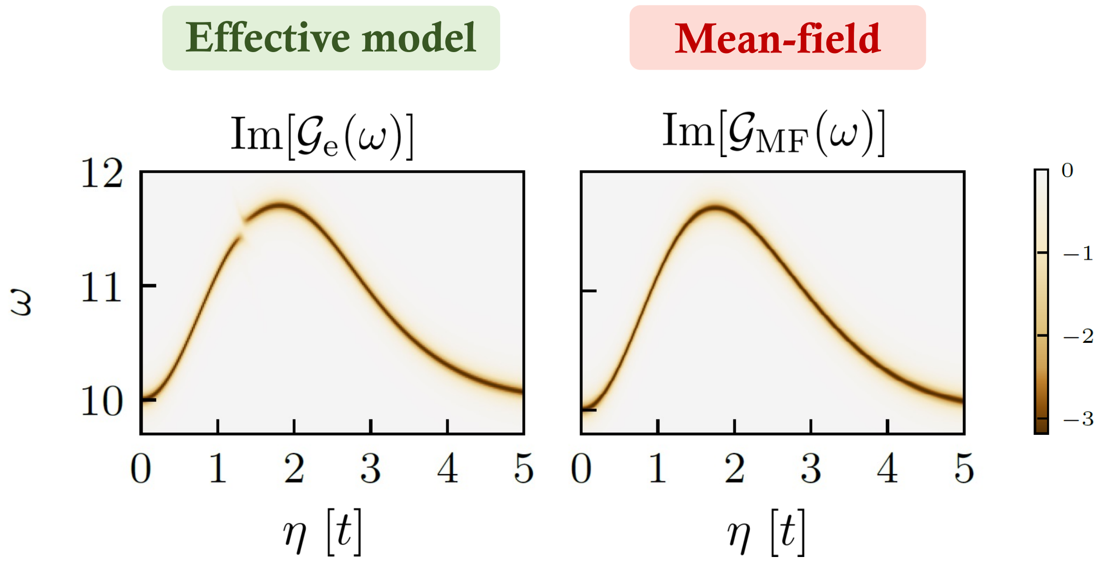
$$\mathcal{G}(t) = -i\theta(t)\langle[d(t), d^\dagger]\rangle$$
$$\mathcal{G}(\omega) = \mathcal{F}\{G(t)\}$$

B. Pérez-González et al., Phys. Chem. Chem. Phys **24**, 15860-15870 (2022)

Light-Matter correlations

arXiv:2302.12290

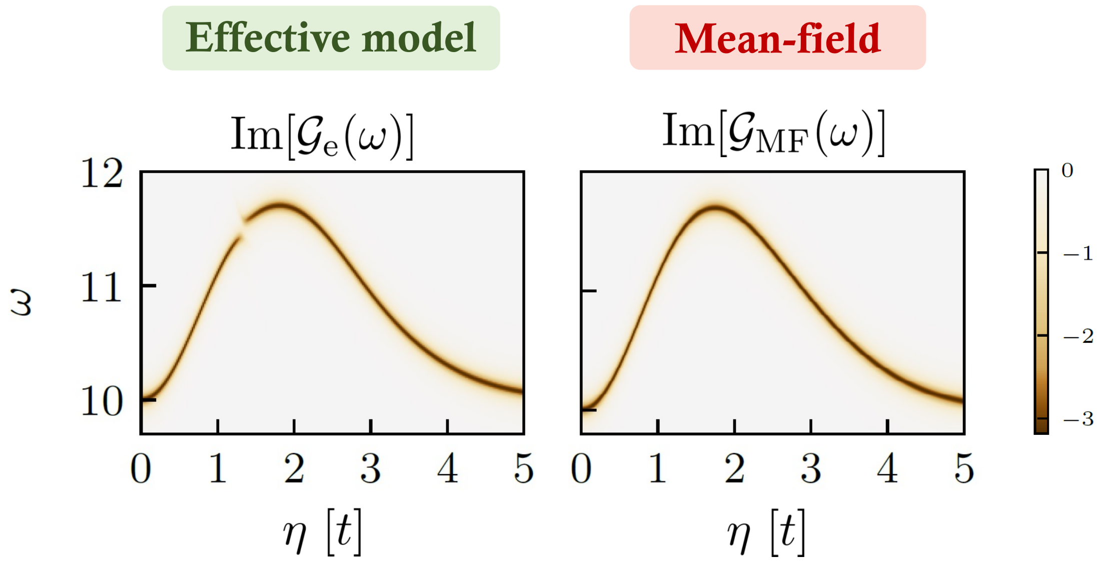
Experimental detection



Parameter choice

- $\mathcal{G}(\omega) \equiv$ photonic spectral function
- State preparation: ground state, $\langle d^\dagger d \rangle \approx 0$
- Same parameters as in previous figure
  - Trivial topology for the unperturbed system $t = 1, \; t' = 1.5$
  - Highly detuned cavity $\Omega \gg t, t'$

Experimental detection



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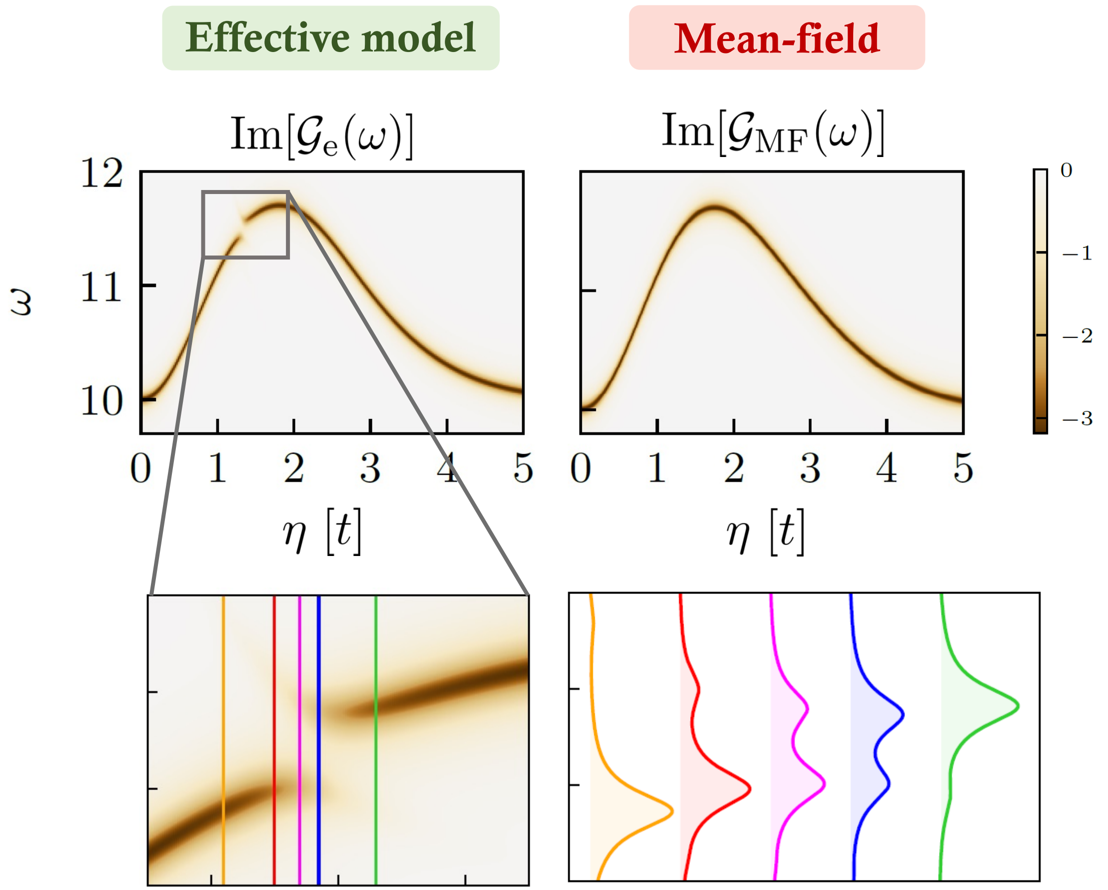
- The cavity frequency is renormalized due to the interaction with the topological system

Experimental detection



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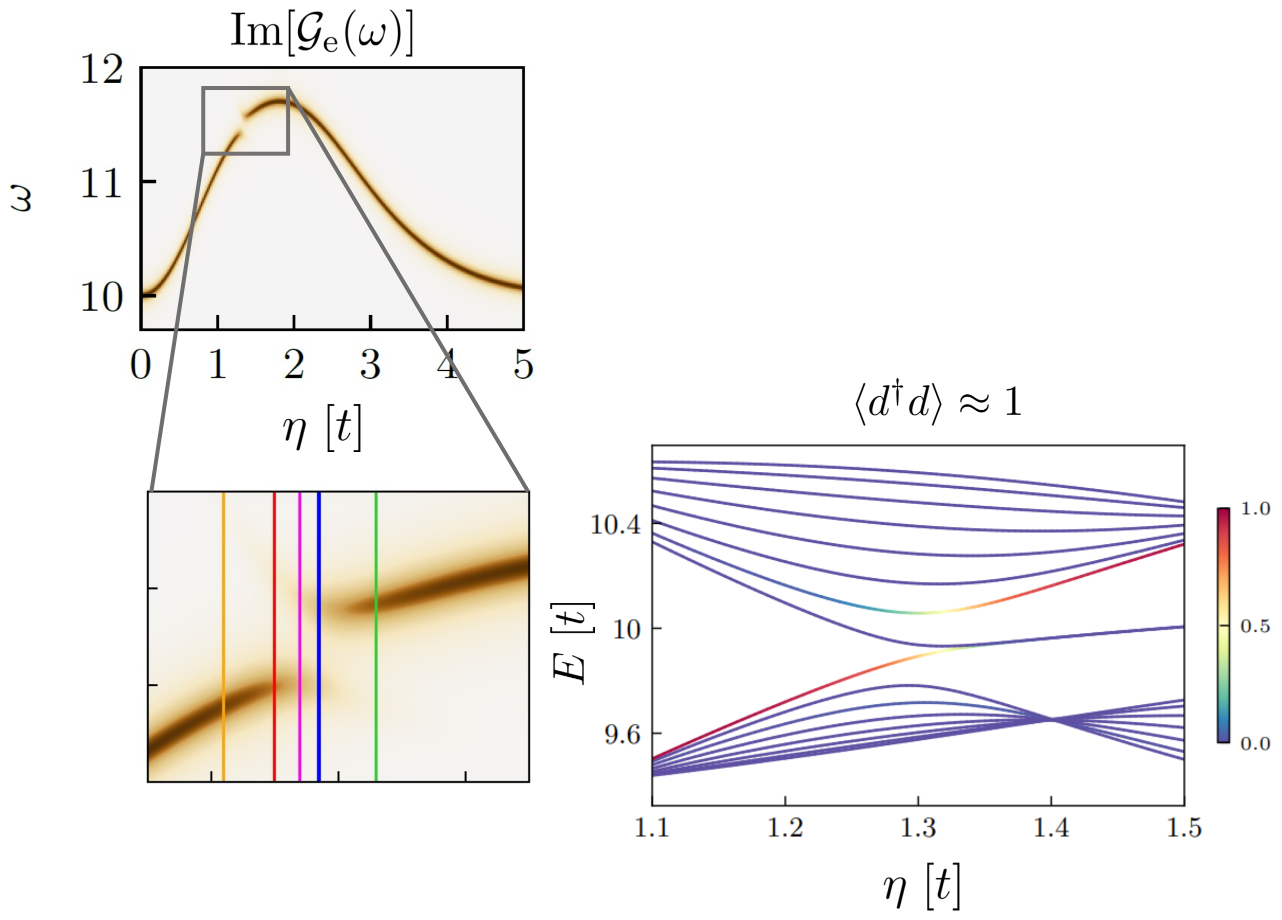
- The cavity frequency is renormalized due to the interaction with the topological system
- A splitting in the photonic branch signals that correlations are maximal at that point

Experimental detection

Effective model



Parameter choice

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  - Trivial topology for the unperturbed system $t = 1,\ t' = 1.5$
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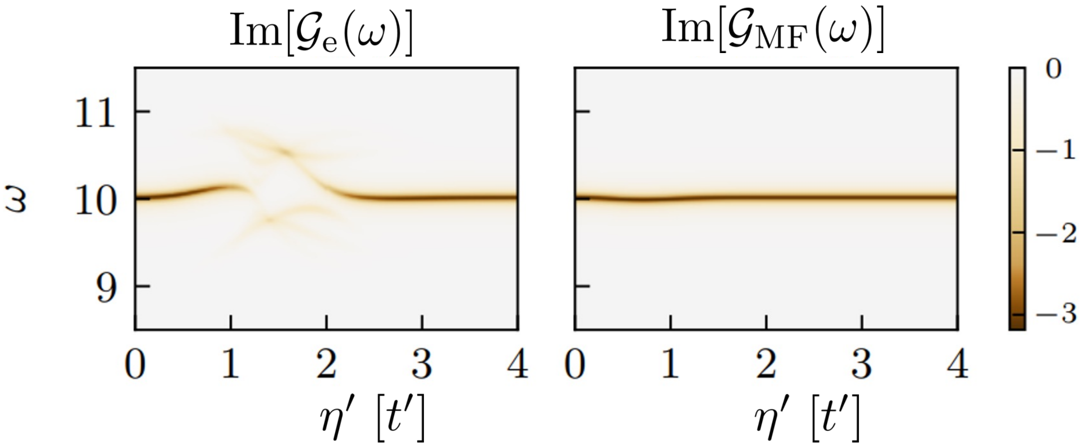
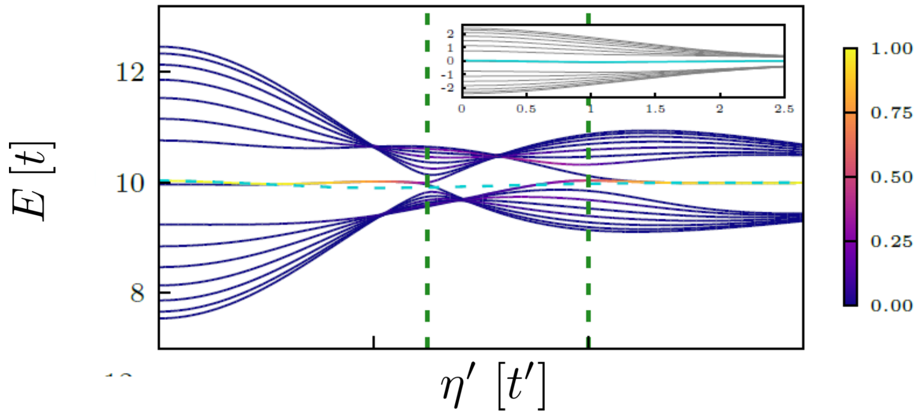
Experimental detection

Effective model **Mean-field**





Parameter choice

- $\mathcal{G}(\omega) \equiv$ photonic spectral function
- State preparation: edge state, $\langle d^\dagger d \rangle \approx 0$
- Parameters:
  - Non-trivial topology for the unperturbed system $t = 1.5, t' = 1$
  - Highly detuned cavity $\Omega \gg t, t'$

- The different coupling between bulk/edge states and the cavity maximizes the effect of light-matter correlations

Take-Home message

- effective models that capture the physics of gauge-invariant models for arbitrary light-matter coupling

- Quantum light can be used to tune the topological properties of the system, as a function of the coupling strength and the number of photons

- identify the role of light-matter correlations in the high-frequency regime and their relation to the breaking of chiral symmetry, crucial for topological systems

check the preprint!
arXiv:2302.12290