Quantum memories for squeezed and coherent superpositions in a driven-dissipative nonlinear oscillator

ADRIÀ LABAY-MORA,

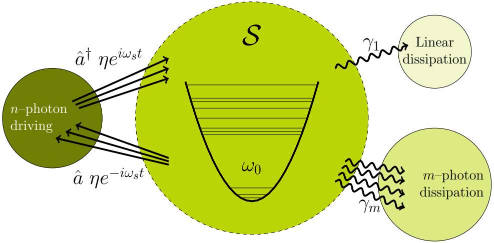
ROBERTA ZAMBRINI AND

GIAN LUCA GIORGI

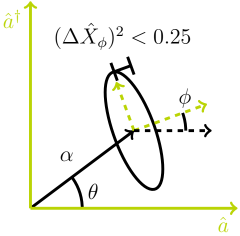
I-Link Quantum Workshop – June 06, 2023

EXCELENCIA
MARÍA
DE MAEZTU
2023 - 2027

I. The oscillator

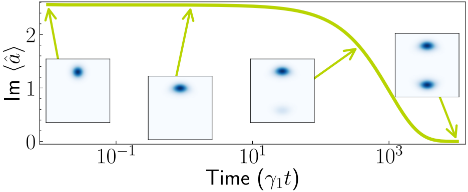


II. The lobes

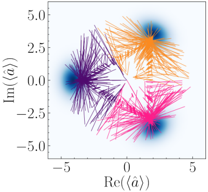


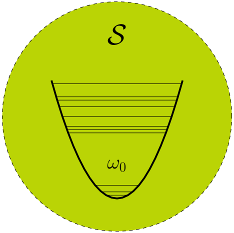
III. The applications

Storage



Associative Memory



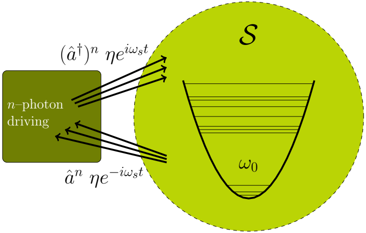


GKLS master equation

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho = -i[\hat{\mathrm{H}}, \rho]$$

Hamiltonian: $\hat{\mathrm{H}} = \omega_0 \hat{a}^\dagger \hat{a}$

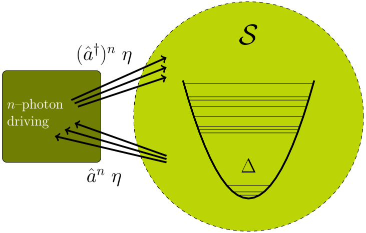
The oscillator



GKLS master equation

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho = -i[\hat{\mathrm{H}}, \rho]$$

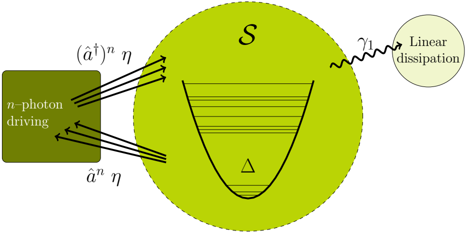
Hamiltonian: $\hat{\mathrm{H}} = \omega_0 \hat{a}^\dagger \hat{a} + i\eta[\hat{a}^n e^{-i\omega_s t} - (\hat{a}^\dagger)^n e^{i\omega_s t}]$



GKLS master equation

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho = -i[\hat{\mathrm{H}}, \rho]$$

Hamiltonian: $\hat{\mathrm{H}} = \Delta \hat{a}^\dagger \hat{a} + i\eta[\hat{a}^n - (\hat{a}^\dagger)^n]$



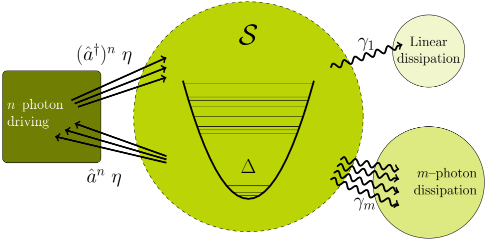
GKLS master equation

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho = -i[\hat{\mathrm{H}}, \rho] + \gamma_1 \mathcal{D}\left[\hat{a}\right]\rho$$

Hamiltonian: $\hat{\mathrm{H}} = \Delta \hat{a}^\dagger \hat{a} + i\eta[\hat{a}^n - (\hat{a}^\dagger)^n]$

Lindblad operator: $\mathcal{D}\left[J\right]\rho = J\rho J^\dagger - \frac{1}{2}J^\dagger J\rho - \frac{1}{2}\rho J^\dagger J$

The oscillator



GKLS master equation

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho = -i[\hat{\mathrm{H}}, \rho] + \gamma_1 \mathcal{D}\left[\hat{a}\right]\rho + \gamma_m \mathcal{D}\left[\hat{a}^m\right]\rho$$

Hamiltonian: $\hat{\mathrm{H}} = \Delta \hat{a}^\dagger \hat{a} + i\eta[\hat{a}^n - (\hat{a}^\dagger)^n]$

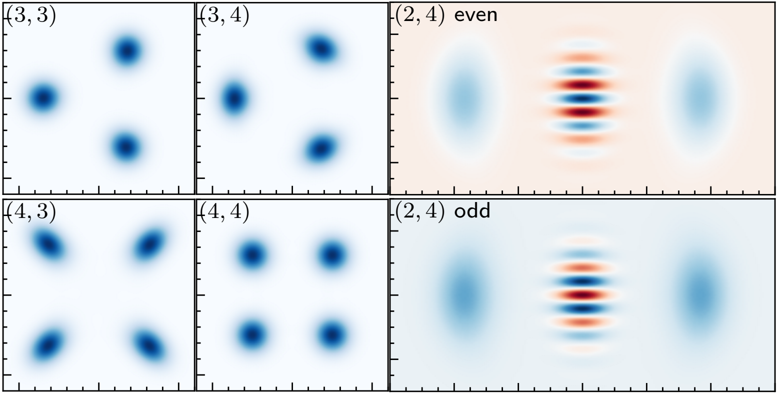
Lindblad operator: $\mathcal{D}\left[J\right]\rho = J\rho J^\dagger - \frac{1}{2}J^\dagger J\rho - \frac{1}{2}\rho J^\dagger J$

Remember $\longrightarrow \begin{cases} n & \text{driving power} \\ m & \text{dissipation power} \end{cases}$

The oscillator steady states

$$\frac{\partial \rho}{\partial t} = \eta[\hat{a}^n - (\hat{a}^\dagger)^n, \rho] + \gamma_1 \mathcal{D}[\hat{a}]\rho + \gamma_m \mathcal{D}[\hat{a}^m]\rho$$

A zoo of steady states can be generated by modifying (n, m)

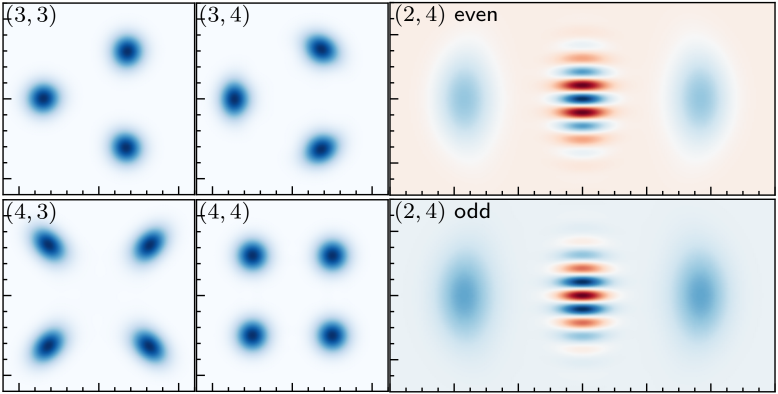


The system has a discrete symmetry:

n driving

m dissipation

$$\hat{Z}_p = \exp\left(-i2\pi\hat{a}^\dagger\hat{a}/p\right)$$



The oscillator symmetry

The system has a discrete symmetry:

n driving

m dissipation

$$\hat{Z}_p = \exp\left(-i2\pi\hat{a}^\dagger\hat{a}/p\right)$$

Weak symmetry

SS: 1 mixed state

Requirement: $[\mathcal{L}, \mathcal{Z}_n] = 0$

(essentially when not strong)

Strong symmetry

SS: p cat-states

Requirements:

1. $\gamma_1 = 0$
2. $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

The oscillator symmetry: weak

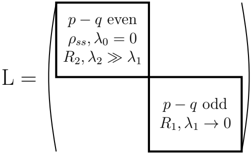
Requirement: $[\mathcal{L}, \mathcal{Z}_n] = 0$

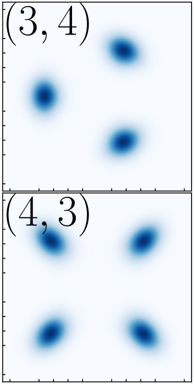
Consequences:

- The system is invariant under rotations by an angle $2\pi k/n$ ($k \in \mathbb{N}$)
- The Liouvillian can be separated into n blocks
- Only 1 steady state with n lobes

$$\rho_{ss} \approx \frac{1}{n} \sum_{k=1}^{n} \mu_k$$

where $\mu_k \approx |\psi_k\rangle\langle\psi_k|$





The oscillator symmetry: strong

Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

$$|0\rangle \quad |1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \quad |5\rangle \quad |6\rangle \quad |7\rangle \quad |8\rangle \quad |9\rangle \quad |10\rangle \quad |11\rangle$$

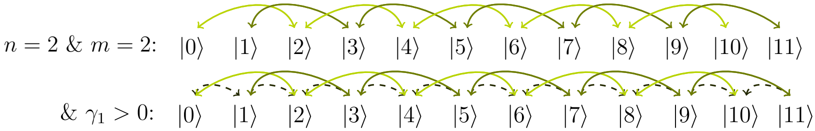
The oscillator symmetry: strong

Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

$n = 2$ & $m = 2$: $|0\rangle \quad |1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \quad |5\rangle \quad |6\rangle \quad |7\rangle \quad |8\rangle \quad |9\rangle \quad |10\rangle \quad |11\rangle$

The oscillator symmetry: strong

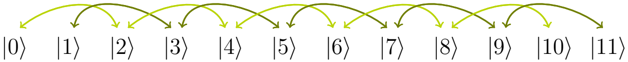
Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$



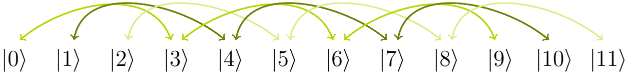
The oscillator symmetry: strong

Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

$n = 2$ & $m = 2$:

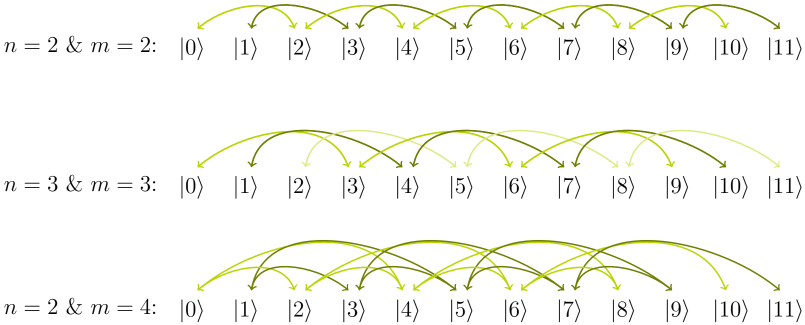


$n = 3$ & $m = 3$:



The oscillator symmetry: strong

Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$



The oscillator symmetry: strong

Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

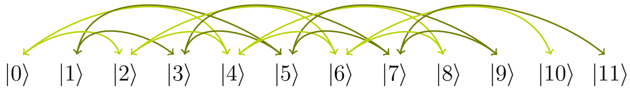
$n = 2$ & $m = 2$: $|0\rangle \quad |1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \quad |5\rangle \quad |6\rangle \quad |7\rangle \quad |8\rangle \quad |9\rangle \quad |10\rangle \quad |11\rangle$

$n = 2$ & $m = 4$: $|0\rangle \quad |1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \quad |5\rangle \quad |6\rangle \quad |7\rangle \quad |8\rangle \quad |9\rangle \quad |10\rangle \quad |11\rangle$

The oscillator symmetry: strong

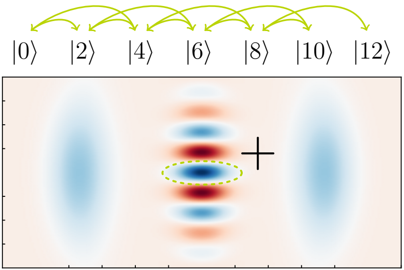
Requirements: $\gamma_1 = 0$ and $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

$n = 2$ & $m = 4$:

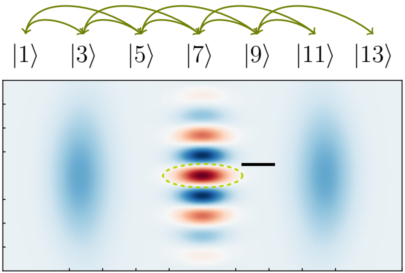


Two symmetry blocks

Even parity



Odd parity

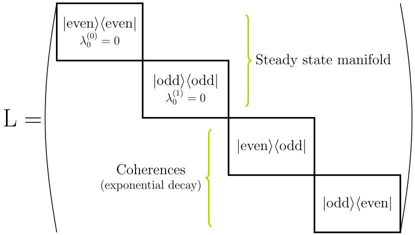


The oscillator symmetry: strong

Requirements:

1. $\gamma_1 = 0$
2. $[\hat{Z}_p, \hat{a}^n] = [\hat{Z}_p, \hat{a}^m] = 0$

   ↳ $\gcd(n, m) = p > 1$

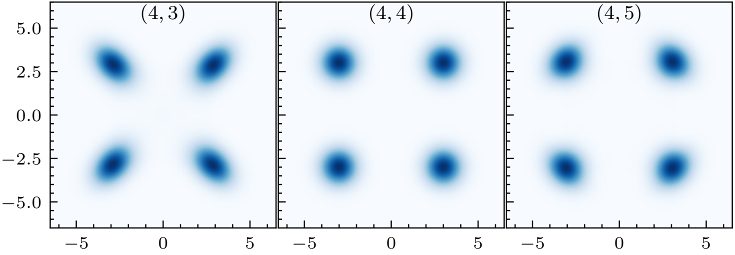


Consequences:

- Liouvillian can be separated into p^2 blocks.
- p steady states with n lobes

$$\left|\psi_{ss}^{(k)}\right\rangle \approx \frac{1}{\sqrt{n}} \sum_{k=1}^{n} c_k \left|\psi_k\right\rangle \propto \sum_{a=0}^{\infty} \left|ap + k\right\rangle$$

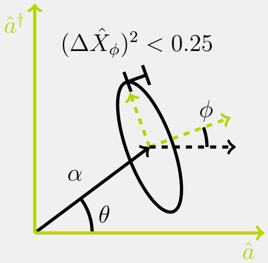
each having definite parity $k = 1, \ldots, p$



Squeezed-coherent state

$$|\alpha, \xi\rangle = |re^{i\theta}, se^{i\phi}\rangle = D(\alpha)S(\xi)\,|0\rangle$$

The quadrature variance $\left\langle (\Delta \hat{X}_\phi)^2 \right\rangle$ in the direction $X_\phi = [\hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi}]/2$ is narrower than for a coherent state



The lobes: squeezing

Mandel parameter:

$$\mathcal{Q} = \frac{\left\langle (\Delta\hat{n})^2 \right\rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle} \begin{cases} < 0 & \text{purely quantum state} \\ = 0 & \text{coherent state (Poissonian)} \\ > 0 & \text{we need more info} \end{cases}$$

Mandel parameter:

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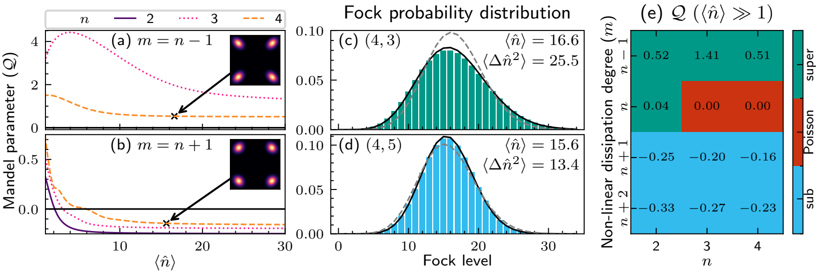

Mandel parameter:

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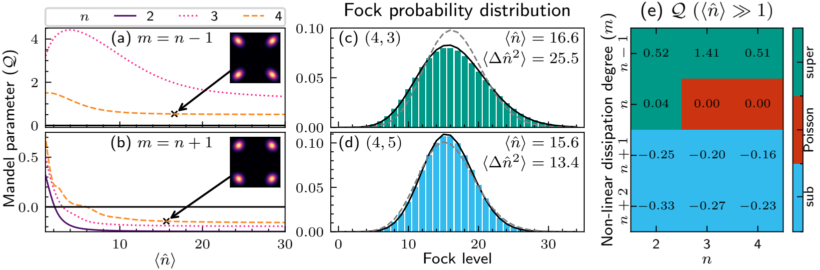
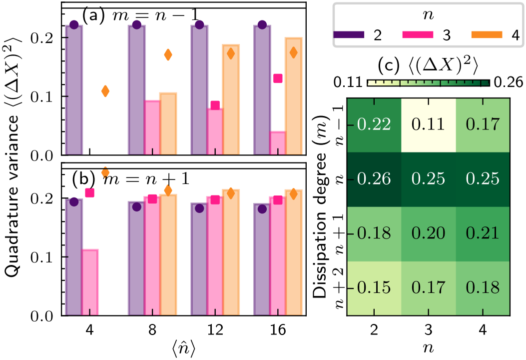


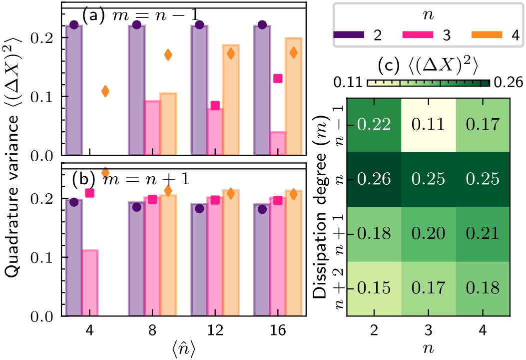
- if $n < m$, states show sub-Poissonian statistics (in fact, amplitude-squeezed)
- if $n > m$, states are super-Poissonian $\rightarrow$ we can do more

The lobes: squeezing



The variance of the quadrature operator $\hat{X}_\phi$ allows to determine the amount of squeezing and the angle.

The lobes: squeezing



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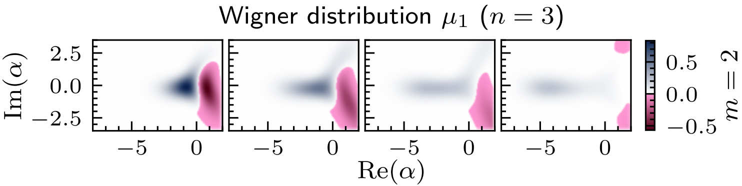

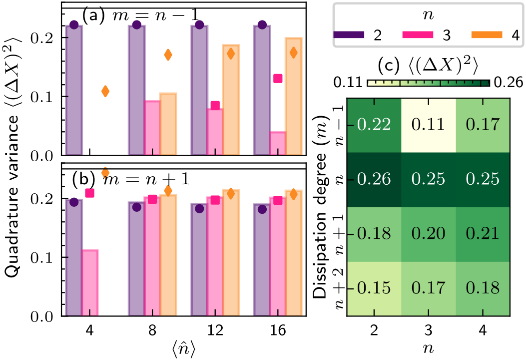

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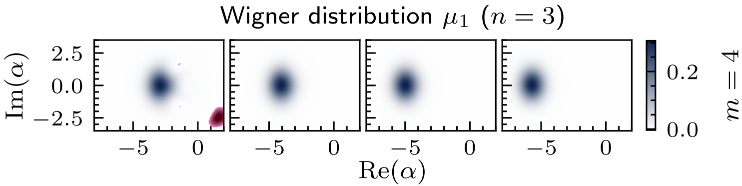
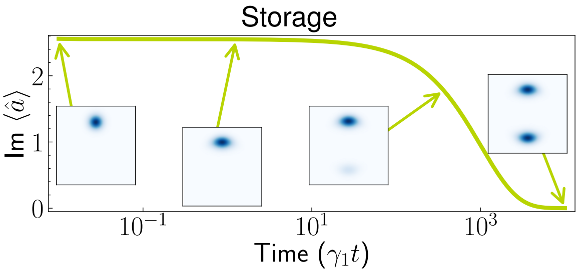
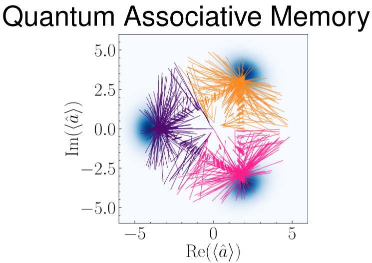


The applications



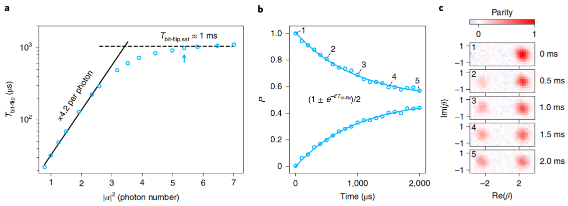


Storage

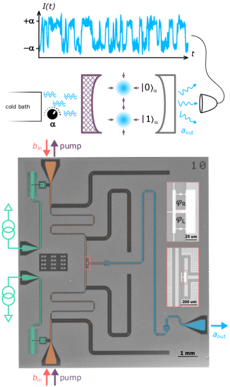
Dynamically protected cat-qubits: a new paradigm for universal quantum computation

Mazyar Mirrahimi[1,2], **Zaki Leghtas**[2], **Victor V Albert**[2,3], **Steven Touzard**[2], **Robert J Schoelkopf**[2,3], **Liang Jiang**[2,3] **and Michel H Devoret**[2,3]
[1] INRIA Paris-Rocquencourt, Domaine de Voluceau, B.P. 105, F-78153 Le Chesnay Cedex, France
[2] Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA
[3] Department of Physics, Yale University, New Haven, Connecticut 06520, USA

M. Mirrahimi et al., New Journal of Physics **16**, 045014 (2014)

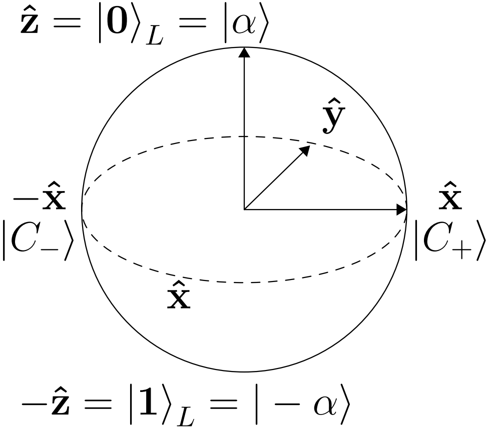


R. Lescanne et al., Nature Physics **16**, 509 (2020)

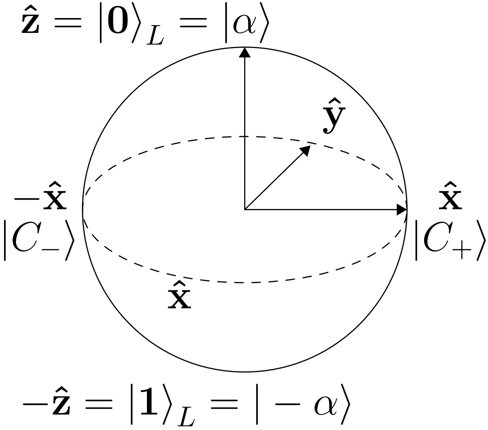


C. Berdou et al., arXiv preprint arXiv:2204.09128 (2022)

Case $n = 2$: computational basis $\{|\pm\alpha, \xi\rangle\}$ ($|\alpha| > 1$)



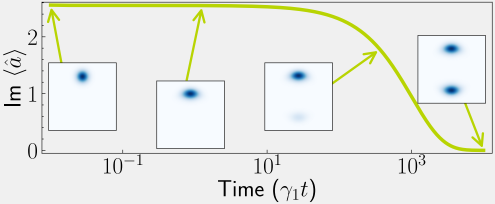
Case $n = 2$: computational basis $\{|\pm\alpha, \xi\rangle\}$ ($|\alpha| > 1$)



How long is information preserved?

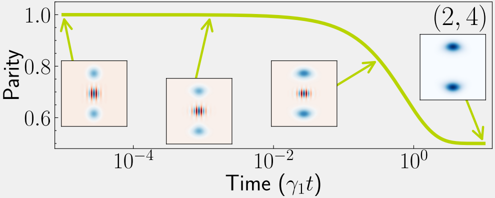
Relaxation time

Decay time to the steady state.



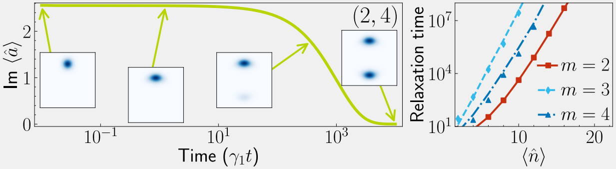
Dephasing rate

Decay of coherences between states.



Relaxation time

Decay time to the steady state.



Dephasing rate

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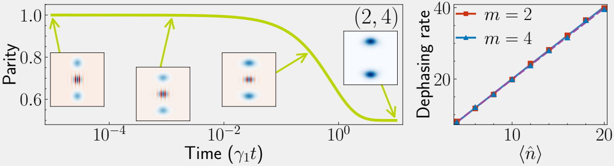
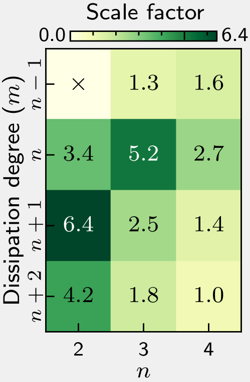


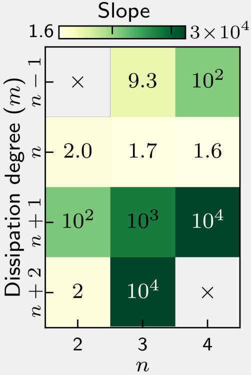
Relaxation time

- Exponential scaling
- Larger system size needed for high n



Dephasing rate

- Linear scaling
- Slope drastically larger if $\gcd(n, m) = 1$



(Quantum) Associative Memory

Neural networks and physical systems with emergent collective computational abilities

(associative memory/parallel processing/categorization/content-addressable memory/fail-soft devices)

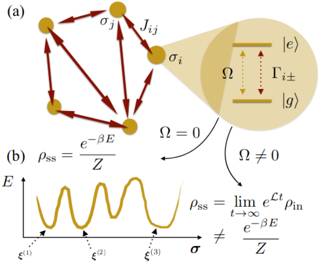
J. J. Hopfield

Division of Chemistry and Biology, California Institute of Technology, Pasadena, California 91125; and Bell Laboratories, Murray Hill, New Jersey 07974

Contributed by John J. Hopfield, January 15, 1982

J. J. Hopfield, Proceedings of the national academy of sciences **79**, 2554 (1982)

Any physical system *whose dynamics in phase space is* ***dominated*** *by a substantial number of* ***locally stable states*** *to which it is attracted can therefore be regarded as a general* ***content-addressable memory***.



P. Rotondo et al., Journal of Physics A: Mathematical and Theoretical **51**, 115301 (2018)

Quantum Associative Memory with a Single Driven-Dissipative Nonlinear Oscillator

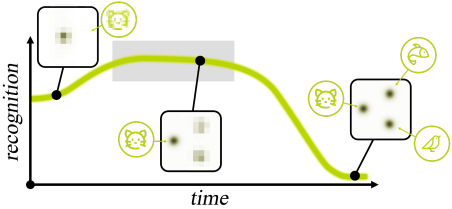
Adrià Labay-Mora,* Roberta Zambrini, and Gian Luca Giorgi
Institute for Cross Disciplinary Physics and Complex Systems (IFISC) UIB-CSIC, Campus Universitat Illes Balears, Palma de Mallorca, Spain

(Received 31 May 2022; accepted 14 April 2023; published 11 May 2023)

Algorithms for associative memory typically rely on a network of many connected units. The prototypical example is the Hopfield model, whose generalizations to the quantum realm are mainly based on open quantum Ising models. We propose a realization of associative memory with a single driven-dissipative quantum oscillator exploiting its infinite degrees of freedom in phase space. The model can improve the storage capacity of discrete neuron-based systems in a large regime and we prove successful state discrimination between n coherent states, which represent the stored patterns of the system. These can be tuned continuously by modifying the driving strength, constituting a modified learning rule. We show that the associative-memory capability is inherently related to the existence of a spectral separation in the Liouvillian superoperator, which results in a long timescale separation in the dynamics corresponding to a metastable phase.

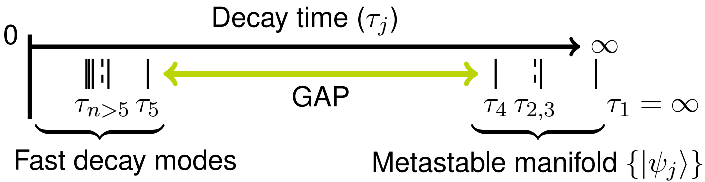
DOI: 10.1103/PhysRevLett.130.190602

A. Labay-Mora et al., Phys. Rev. Lett. **130**, 190602 (2023)



The metastability

Liouvillian spectrum $\mathcal{L}R_j = \lambda_j R_j$[1] $\tau_j = -1/\operatorname{Re}\lambda_j$



[1] K. Macieszczak et al., Physical Review Research **3**, 033047 (2021).

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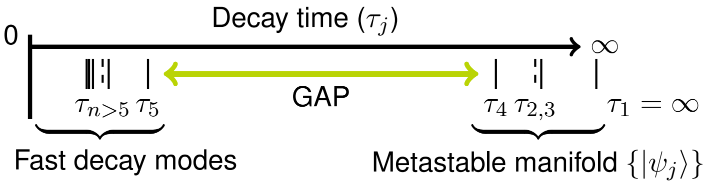

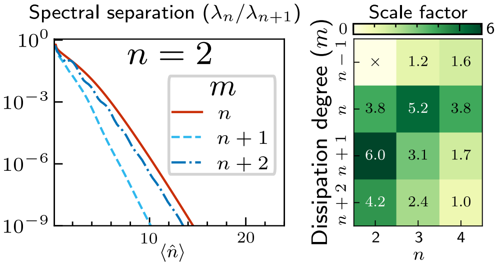

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Protocol

1. Construct the patterns by tuning the oscillator parameters.
2. Encode the initial information into a squeezed-coherent state $\left|re^{i\theta}, se^{i\phi}\right\rangle$.
3. Evolve it for at least a time $\gamma_1\tau_{n+1}$, the state will be close to one phase μ_k.
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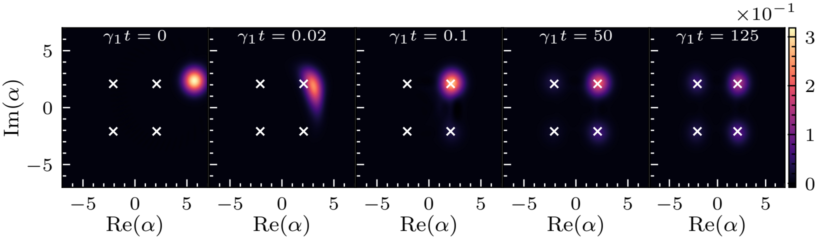

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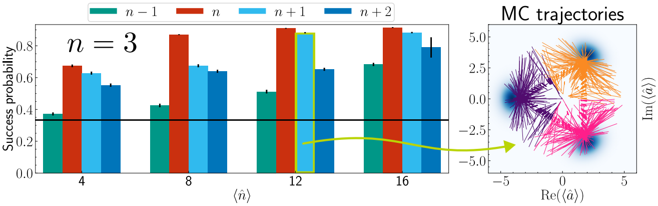
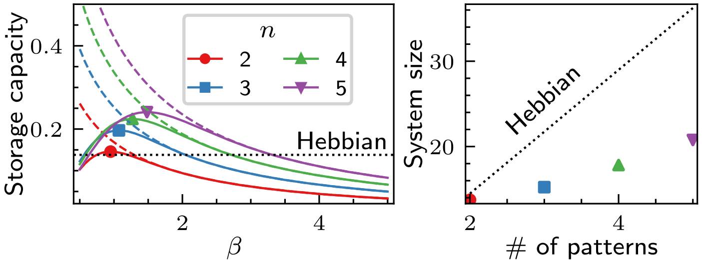


Storage capacity

$$\alpha = \frac{\text{\# of patterns}}{\text{system size}}$$

Classical limit: $\alpha_c = 0.138$[a]

[a] D. J. Amit et al., Phys. Rev. Lett. **55**, 1530 (1985).



Outlook and Future Work

- Study the generation of squeezed states
- Modifying (n, m) preserves the exponential (linear) scaling of the relaxation time (dephasing rate) with $\langle \hat{n} \rangle$.
- Squeezed lobes improve the relaxation time for $n = 2$ while maintaining the same scaling of the dephasing error rate for $m = 4$.
- Successful state discrimination in the metastable phase
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Hopefully, on arXiv soon...

THANK YOU

for your attention

BGRH-UIB-2021

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