ORDERING DYNAMICS AND PATH TO CONSENSUS IN THE MULTI-STATE VOTER MODEL

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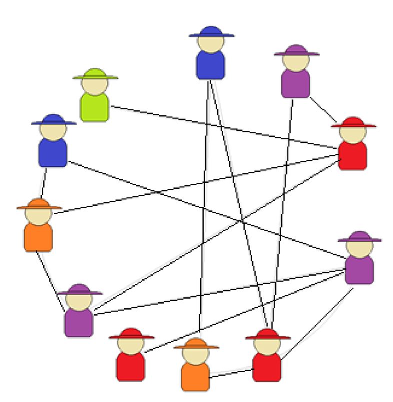
Multi-State Voter Model

We investigate how **multi-state voter models (MSVM)** [1,2] approach consensus states with a focus on the coarsening process.



N interacting agents, placed in the nodes of a network.

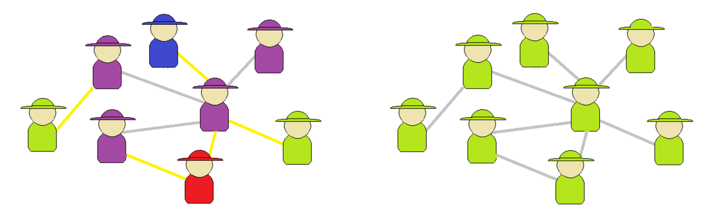
M possible opinion-states All states are equivalent and agents can switch between opinion-states without restrictions.

In each time step, randomly chosen agent **copies the state** of one of its neighbors.

The **coarsening process** can be studied from the time evolution of the **density of active links**, ρ

$$\rho(t) = \frac{links\ connecting\ nodes\ with\ different\ opinion-states}{total\ quantity\ of\ links}$$

The decrease of ρ(t) is an indicator of coarsening



… and from the **entropy**, S

$$S = -\sum_\alpha x_\alpha \ln x_\alpha$$

x_α fraction of agents in the opinion-state α

purely determined by the fractions of agents in the different opinion-states

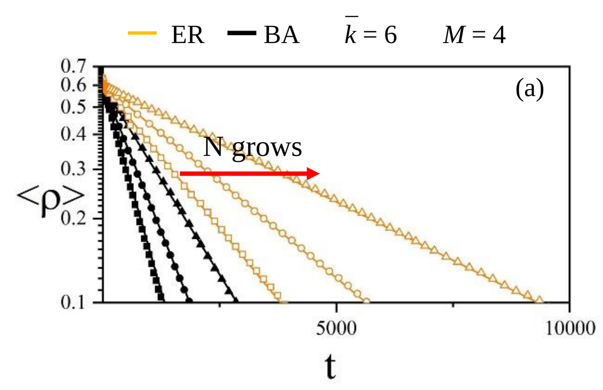
Ensemble Average

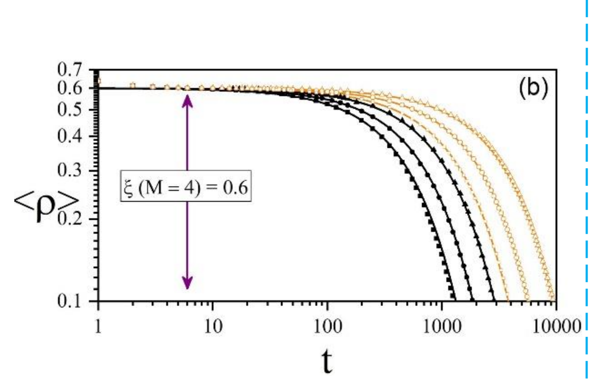
Density of active links description: Uncorrelated networks (Scale Free and Erdös Renyi)

Pair approximation for the Multi-state VM

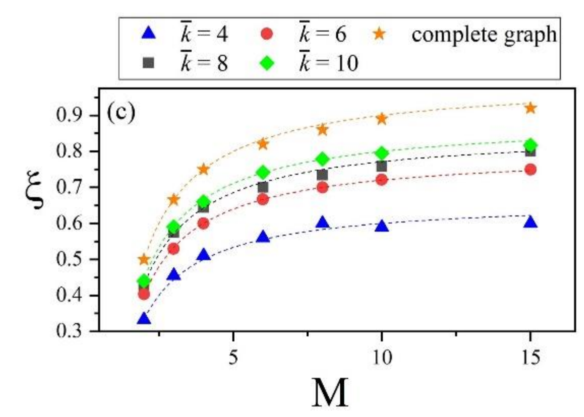
$$\langle\rho(t)\rangle = \langle\rho^{cg}[\mathbf{n}(0)]\rangle \frac{\bar{k}-2}{\bar{k}-1} e^{-t/\tau}, \quad \tau = \frac{(\bar{k}-1)\bar{k}^2 N}{2(\bar{k}-2)\bar{k^2}}.$$

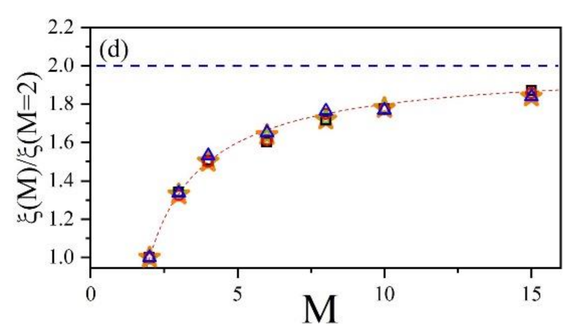
$$\xi = \frac{(\bar{k}-2)}{(\bar{k}-1)}\left(1-\frac{1}{M}\right)$$ Homogeneous Initial conditions





- $<\rho>$ has an exponentially decreasing behavior with *t*, Fig(a).
- The log-log plot of $<\rho>$, Fig(b), exhibits a plateau that indicates a metastable state, ξ, in which there is no coarsening.
- As in the binary model, ξ does not depend on the structure of the network [3,4] and grows with *k*, Fig(c).
- The plateau value has a growing behavior *M*, that can be clearly seen in the scaling shown in Fig(d) .





Entropy description

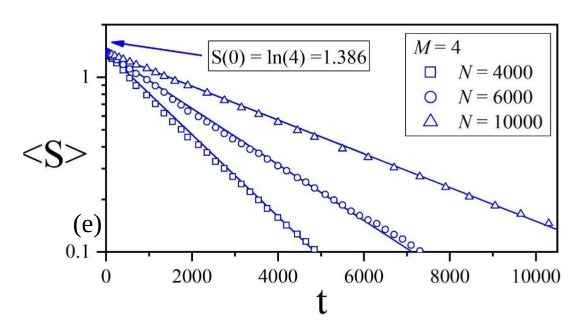
$$S = -\sum_\alpha x_\alpha \ln x_\alpha$$

$$\langle S(t)\rangle = \xi_S\, e^{-t/\tau_S}$$

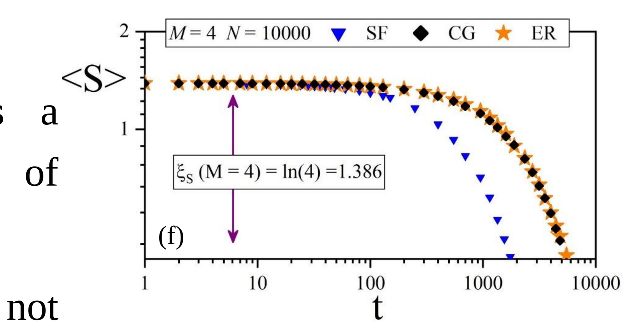
$\tau_S \sim N$ for ER and CG; $\tau_S \sim N \ln N$ for SF

Homogeneous Initial conditions $x_\alpha = 1/M$

$$\xi_S = \ln M$$



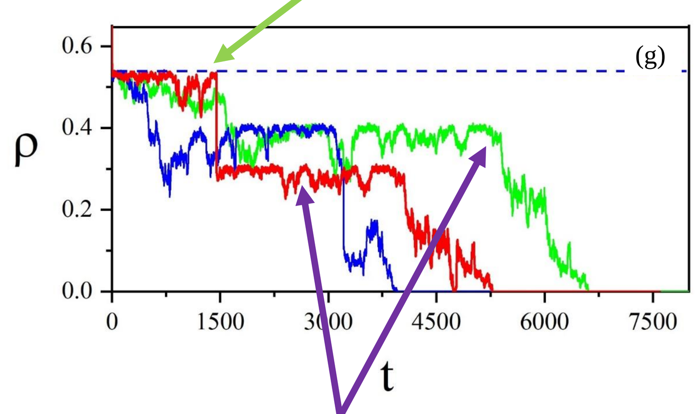
- $<S>$ has an exponentially decreasing behavior with *t*, Fig(e).
- The plateau in Fig(f) indicates a metastable state: average fraction of agents in each op-state is constant.
- The plateau grows with M and does not depend on the topology, Fig(f).



Individual Realisations

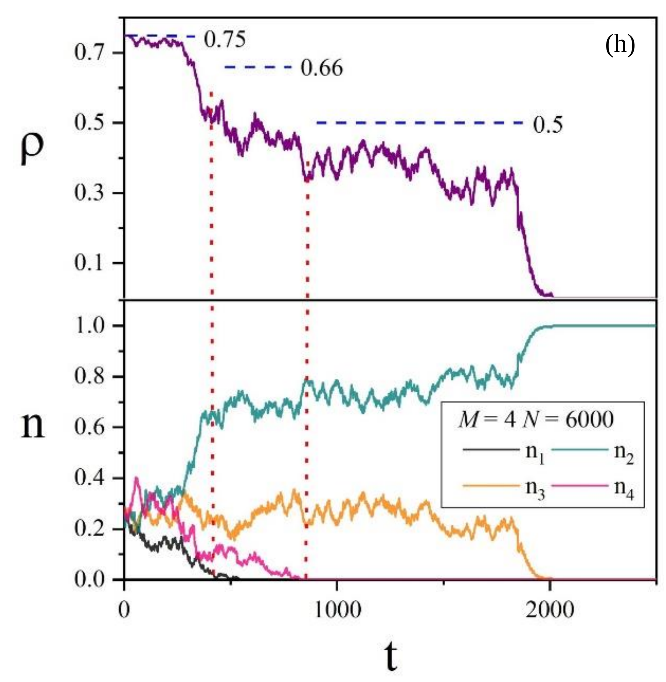
Evolution of individual realisations: intermediate plateaux

The individual realizations fluctuate around the plateau value ξ



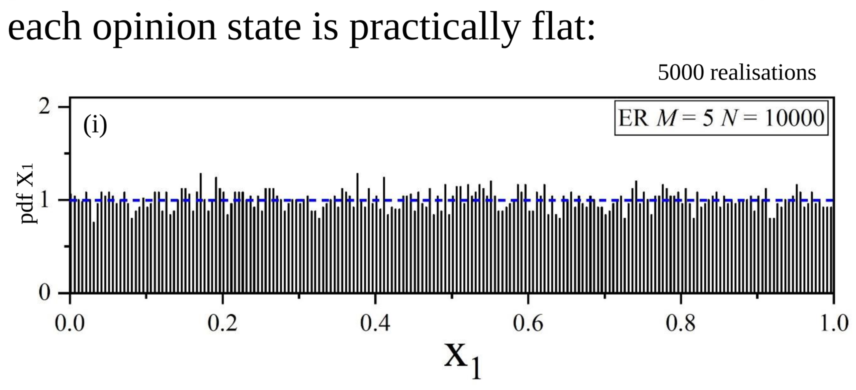
There are intermediate plateau before going to consensus

When an opinion-state disappears, individual realisations jump to a different plateau



The intermediate plateaux depend on the distribution of agents in the surviving opinion-states.

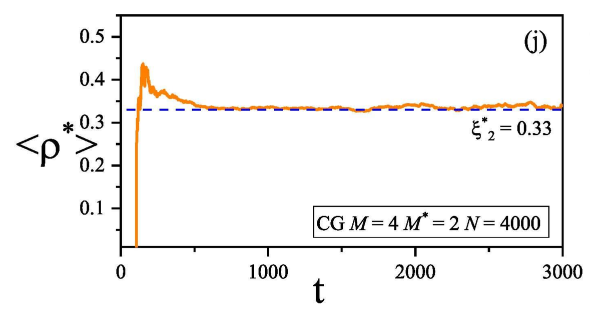
When there are only 2 opinion-states left, the probability distribution of the fraction of agents in each opinion state is practically flat:



The MSVM evolves to a 2-state VM with random initial conditions.

The intermediate plateau are not shown in the ensemble average.

We try to recover them by averaging at each time over realisations that only have M*=2 opinion-states left:

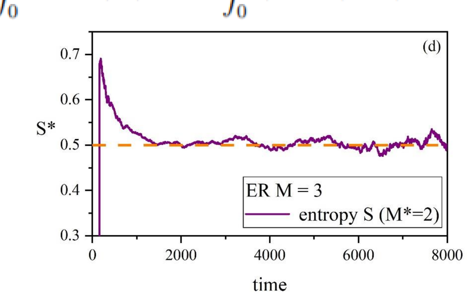


Considering the practically flat distribution for x_1 when M*=2:

$\rho = 2x_1x_2 = 2x_1(1-x_1)$

$$\xi^*_2 = 2\int_0^1 x_1(1-x_1)\,dx_1 = 0.33$$

$$S^*_2 = -\int_0^1 x_1\log(x_1)\,dx_1 - \int_0^1 (1-x_1)\log(1-x_1)\,dx_1 = 0.5$$



Conclusions

- <ρ> and <S> have an exponential decay
- The system initially stays in a metastable state: No coarsening and average fraction of agents in each state constant
- The plateaus observed in <ρ> and <S> take bigger values as M grows and, for homogeneous initial conditions:

$$\xi = \frac{(\bar{k}-2)}{(\bar{k}-1)}\left(1-\frac{1}{M}\right) \quad \xi_S = \ln M$$

- Individual realisations show intermediate plateaux before going to consensus.
- When an opinion-state disappears, the individual realisations jump to a new intermediate plateau
- The intermediate plateau depends on the distribution of agents among the remaining opinion-states
- A multi-state VM leads to a 2 state VM with random initial conditions
- We can recover the intermediate plateaux by averaging over realisations that only have a given quantity M* of opinion-states left

References

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Acknowledgements

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