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Complex contagion with heterogeneous timing interactions

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Abstract

- We study the effects of heterogeneous timing interactions in processes of complex contagion, focusing on the threshold model [1] with aging.
- Motivation comes from the empirical evidence that social interactions do not occur at constant rate.
- Endogenous aging is considered as the property of agents to be less prone to change state the longer they have been in the current state.
- In exogenous aging, memory is lost after failed attempts to change state [2].

Threshold model with aging

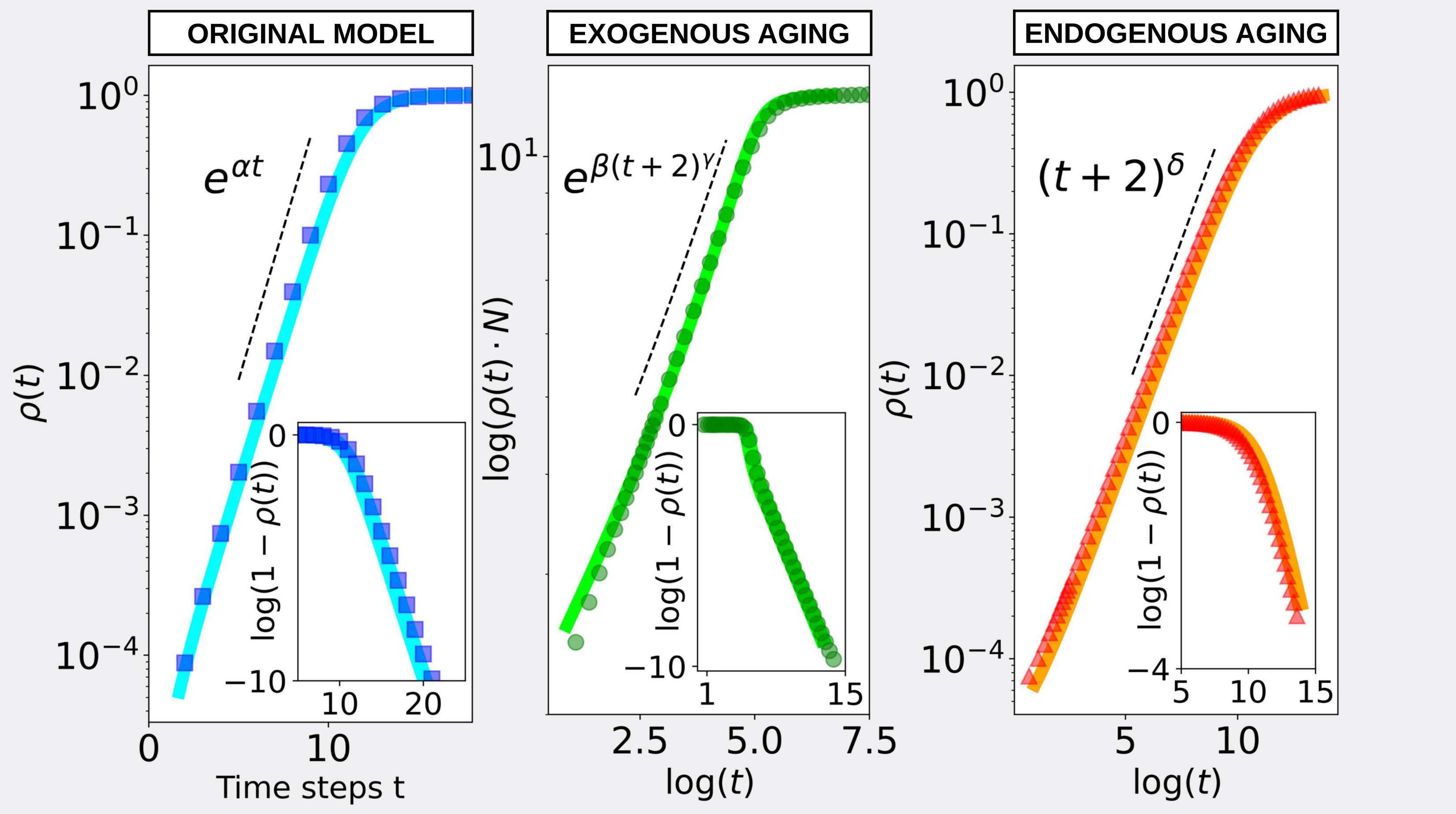
- The probability to adopt depends on the local participation of social contacts (peer pressure) and the time spent in current state (aging).
- For a node with age j and m adopted of the k neighbors, the adopting probability is:

$$F(k,m,j) = p(j)\cdot\theta\left(\frac{m}{k}-T\right) \quad \text{with } p(j) = \frac{1}{j+2}$$

- Asymmetrical model: Agents adopt the technology (or join a riot, political campaign…) but cannot come back.
- **ENDOGENOUS AGING:** The internal time is reset just when an agent gets adopted.
- **EXOGENOUS AGING:** The internal time is reset after an attempt to change state.

Complex Networks

- The exponential increase from the original model is replaced by a stretched exponential and power-law increase when we include exogenous and endogenous aging, respectively.

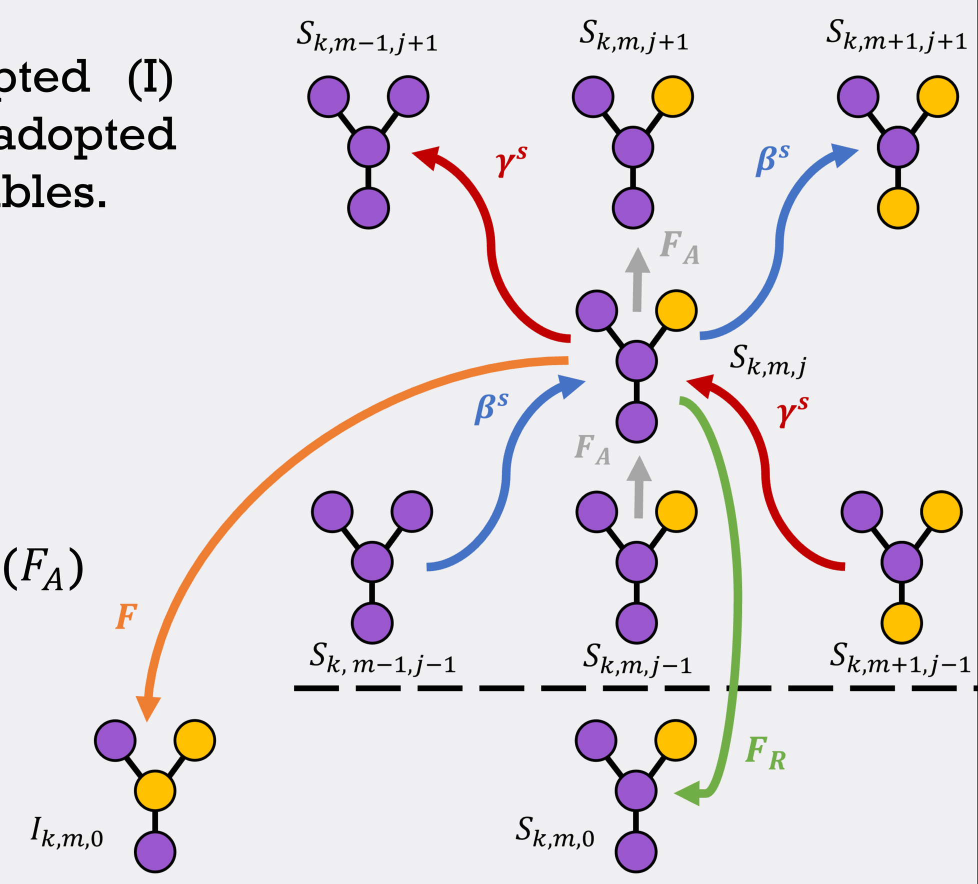


Approximate Master Equation (AME)

- We derive an approximate master equation (AME) for any network with p_k [3] reducing the dynamics to Markovian by enlarging the number of variables [4].
- Sets of non-adopted (S) and adopted (I) agents with k neighbors, m adopted neighbors and age j as different variables.
- Fraction of adopted at time t is:

$$\rho(t) = 1 - \sum_k p_k \sum_m \sum_j s_{k,m,j}$$

- Aging mechanism is set via aging (F_A) and reset (F_R) probabilities.
- The approximate master equation is:



$$\frac{ds_{k,m,0}}{dt} = -s_{k,m,0} - \beta^S(k-m)s_{k,m,0} + \sum_{l=0} F_R(k,m,l)s_{k,m,l}$$

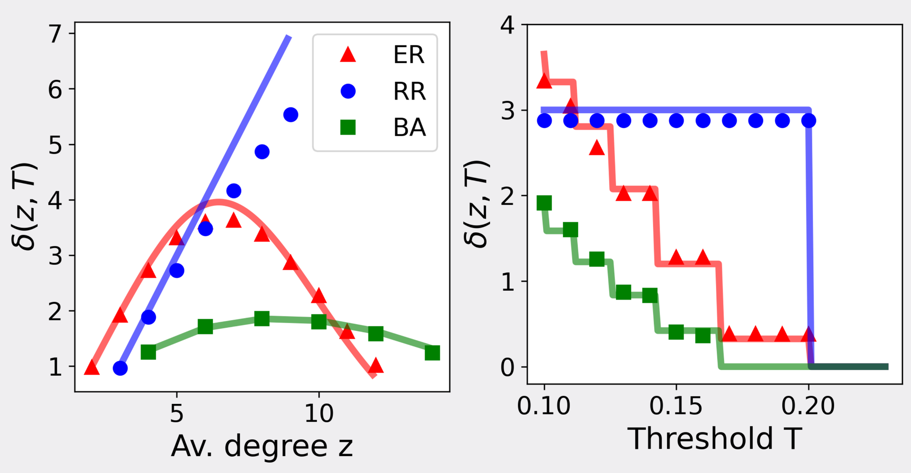
$$\frac{ds_{k,m,j}}{dt} = -s_{k,m,j} - \beta^S(k-m)s_{k,m,j} + \beta^S(k-m+1)s_{k,m-1,j-1} + F_A(k,m,j-1)\, s_{k,m,j-1}$$

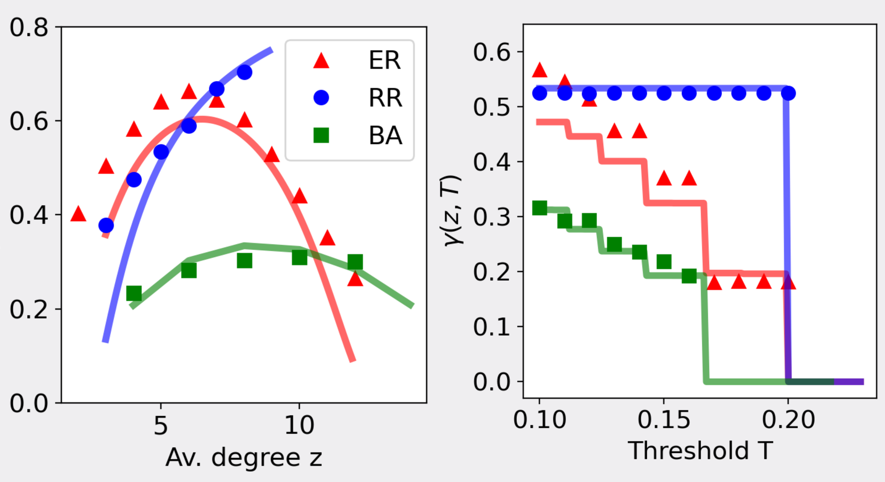
Analytical results in Complex Networks

Endogenous aging:

- Approximated expression for power-law exponent, which coincides with the exponent without aging.

$$\delta(z,T) = \alpha(z,T) = \sum_{k=0}^{[1/T]} \frac{k(k-1)}{z} p_k - 1$$



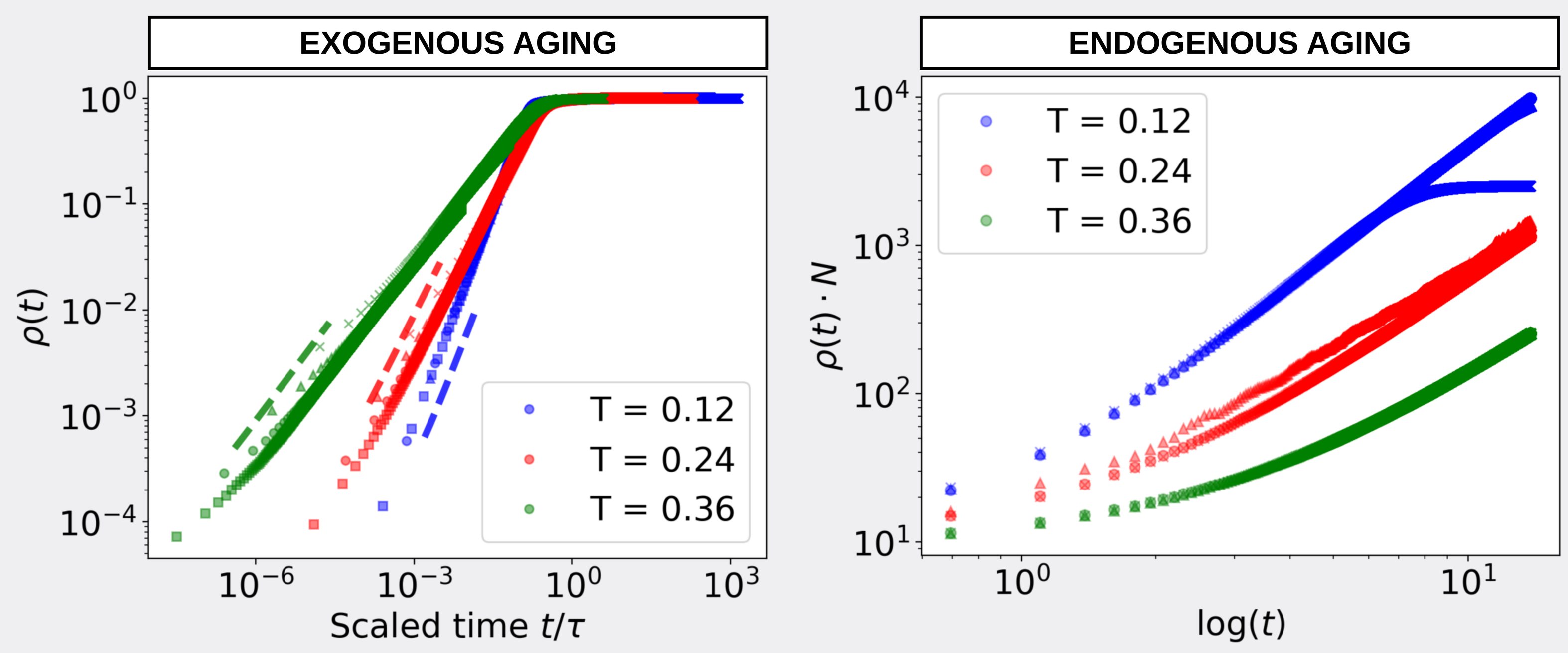


Exogenous aging:

- Reducing the AME to a simpler expression is not trivial.
- The exponents from the numerically integrated solutions are compared with the results from numerical simulations.

Lattice

- The original model $\rho(t) \sim t^2$ is replaced by a slower power-law increase $\rho(t) \sim t^{\epsilon(T)}$, which exponent depends on T when aging mechanism is exogenous.
- Endogenous aging shows a very slow increase, similar to logarithmic.



References

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Acknowledgments

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