ORDERING DYNAMICS IN THE MULTISTATE VOTER MODEL

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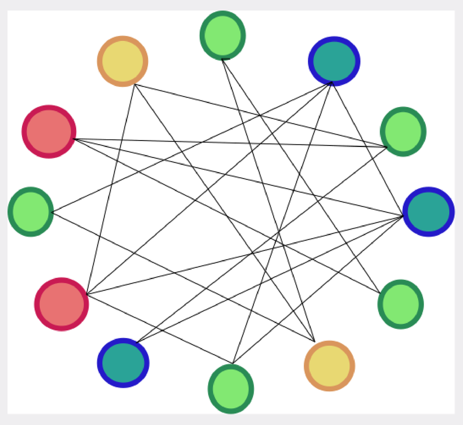
The Multistate Voter Model

The system under study consists of a set of N interacting agents placed on the nodes of a network. Two agent are called first neighbors and interact with each other if there is a link connecting them.



Agents are characterized by a variable that can be considered an opinion, state, specie, etc, and they interact with each other through an imitation process in which an agent copies the state of a randomly chosen first neighbor.

There are s possible states and all states are equivalent.

Each agent can be in only one of the s possible states and can change to a different one without restrictions.

Objectives

Evolution of the averaged active links $\langle\rho\rangle$

- Presence and value of the plateaus ξ
- Consensus time τ
- Comparison with the 2-state model

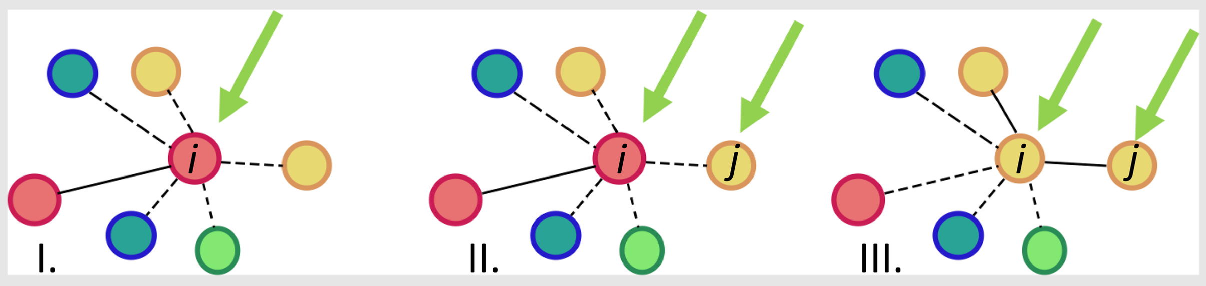
- To explore the ordering when some of the opinions disappear and near the consensus.

Imitation process : Random asynchronous node-update for node dynamics

I. An agent *i* with a given opinion/state *xi*, is randomly chosen

II. One of *i*'s neighbors, *j* with opinion/state *xj* , is chosen also at random; the first agent takes the opinion/state of its neighbor

III. Repeat



$$\rho = \frac{\#\ \text{Links between different states}}{\#\ \text{Links in the network}}$$

---- active links

—— inert links

s states $\rho = \rho_{12} + \rho_{13} + \dots \rho_{s-1,s}$

The Complete Graph

- Rate equations for s initial states

The s states are equally distributed over the N agents: $N_1 = N_2 = \dots = N_s = \frac{N}{s}$

$$\rho = \underbrace{\frac{2 N_1 N_2}{N(N-1)}}_{\rho_{12}} + \underbrace{\frac{2 N_1 N_3}{N(N-1)}}_{\rho_{13}} + \dots + \underbrace{\frac{2 N_{s-1} N_s}{N(N-1)}}_{\rho_{s-1,s}}$$

we want to write the rate equations:

$$\rho(t+\tfrac{1}{N}) = \rho(t) + P(1\to 2)\Delta\rho(1\to 2) + \dots + P(s-1\to s)\Delta\rho(s-1\to s)$$

flips

$$\left.\begin{matrix} 1\to 2 & & s\to 1 \\ 1\to 3 & \dots & s\to 2 \\ \vdots & & \vdots \\ 1\to s & & s\to s-1 \end{matrix}\right\}$$

flips are equiprobables

$$P(\text{flip}) = \frac{\rho}{s(s-1)}$$

$$\text{flip } i\to j: \quad \Delta\rho_{ij} = \frac{2(N_i - N_j - 1)}{N(N-1)}$$

Putting all together

$$\frac{1}{N}\frac{d\langle\rho(t)\rangle}{dt} = \frac{2}{N(N-1)} \cdot \frac{\rho}{s(s-1)} \cdot [-s(s-1)]$$

$$\frac{d\langle\rho(t)\rangle}{dt} = -\frac{2}{N-1}\rho$$

$$\langle\rho(t)\rangle = \langle\rho(0)\rangle e^{-\frac{t}{\tau}} \qquad \tau = \frac{N-1}{2}$$

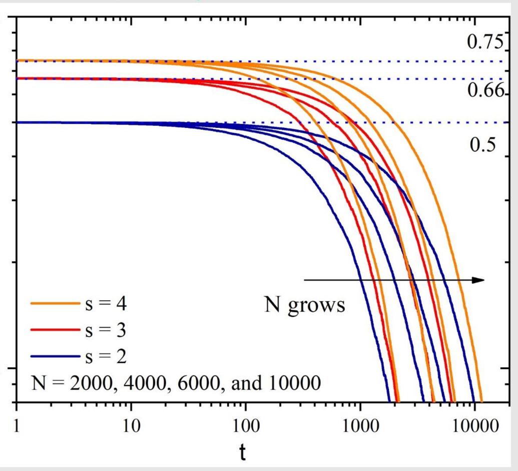


Fig. 1

- Comparison MSVM and inhomogeneous VM (2states)

For a complete graph 3states $\rho = \rho_{12} + \rho_{23} + \rho_{31}$

At final t, the absorbing state (or majority) is called "state 1"

$\rho_1 = \rho_{12} + \rho_{13}$

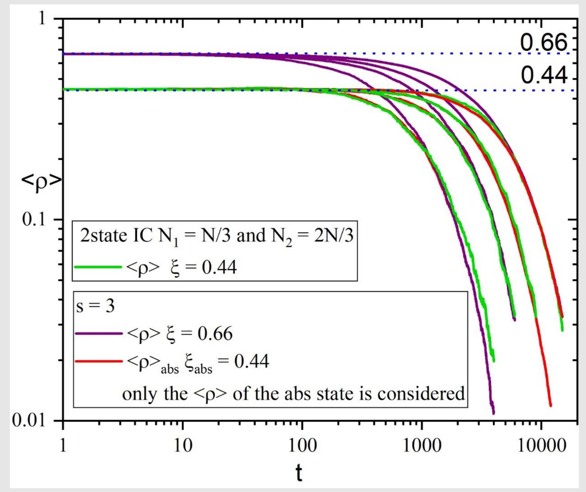


Fig. 2

Uncorrelated networks (Scale Free and Erdös Renyi)

- The evolution of $<\rho>$ (linear-log scale) for SF and ER with mean degree k= 6 and k= 8.

The system goes to an absorbing state,ρ≈0, after staying a finite time in the metastable state ,ξ. The high of ξ grows as k and s does and, as in the binary model, ξ does not depend on the structure of the network [1].

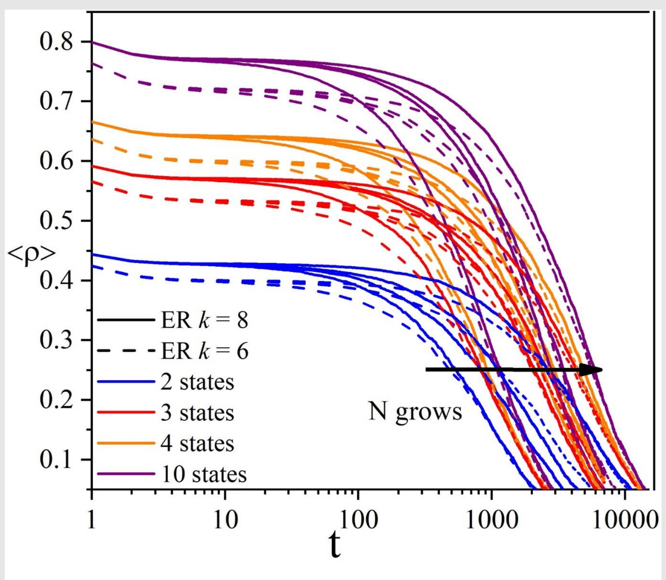


Fig. 3a

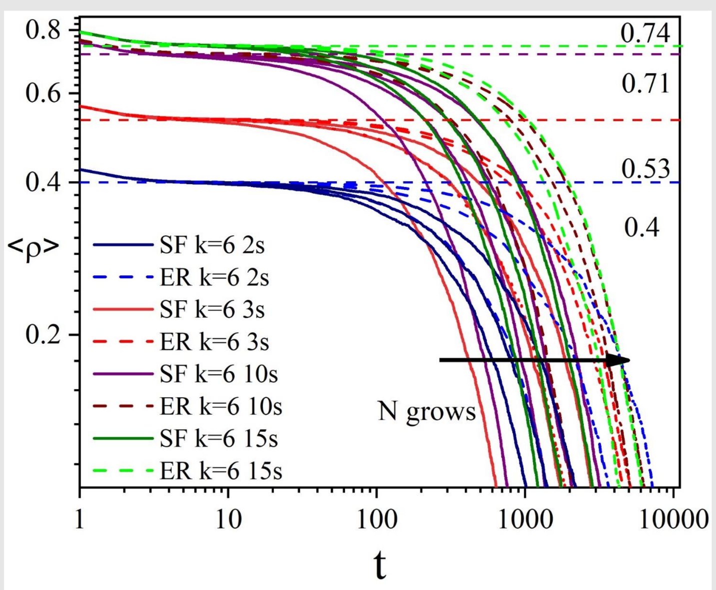


Fig. 3b

- It stands out that the radio ξ(s)=ξ(s=2) remain constant for the three networks, despite of de mean degree.

ER	2s	3s/2s	4s/2s	6s/2s	8s/2s	10s/2s	15s/2s
CG	0,5	1,33	1,5	1,64	1,74	1,78	1,85
k=4	0,333...	1,36	1,53	1,63	1,785	1,77	1,81
k=6	0,4	1,325	1,5	1,6	1,75	1,77	1,85
k=8	0,42776...	1,333	1,5	1,54	1,75	1,78	1,85
k=10	0,444...	1,34	1,5	1,59	1,74	1,79	1,84
		1,3376	1,506	1,6	1,753	1,778	1,84

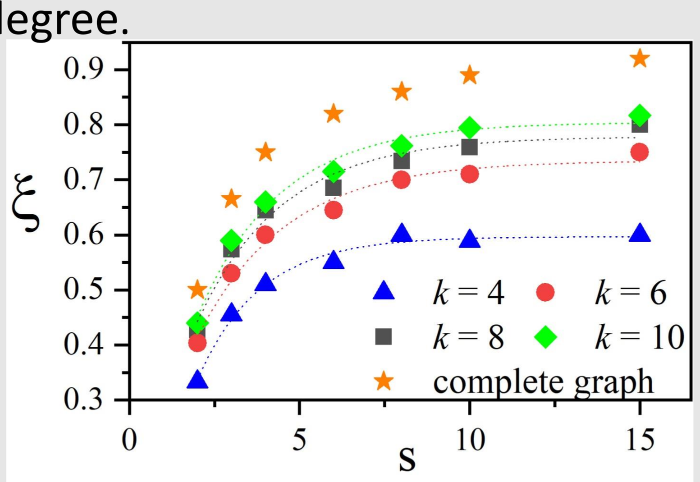


Fig. 4a

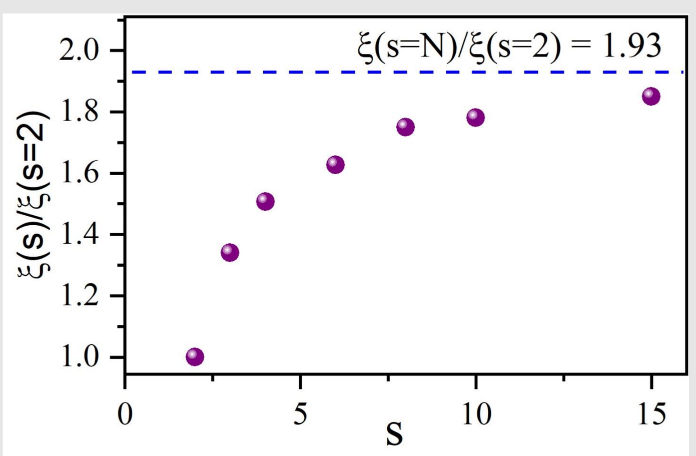


Fig. 4b

- The averaged density of interfaces has the exponential decay $<\rho> \propto e^{-t/\tau}$. τ is the consensus time of the partially ordered metastable state. For the binary model, the system scales as $\tau \sim N$ for ER and as $\tau \sim \frac{N}{\ln N}$ (0.88) for SF. In the MSVM, the consensus time for both networks scales as in the two state system [1] [2].

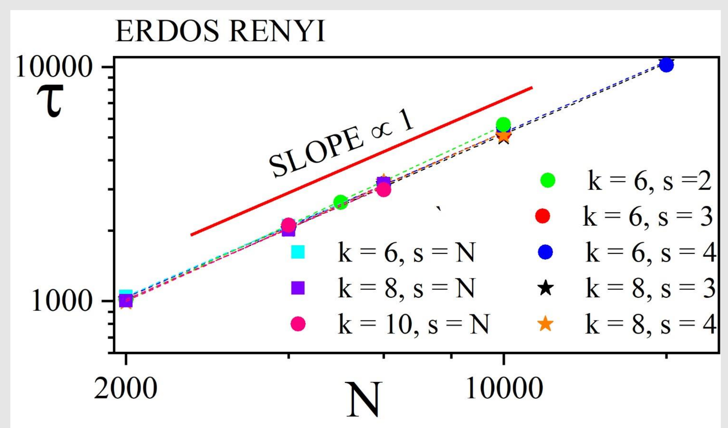


Fig. 5a

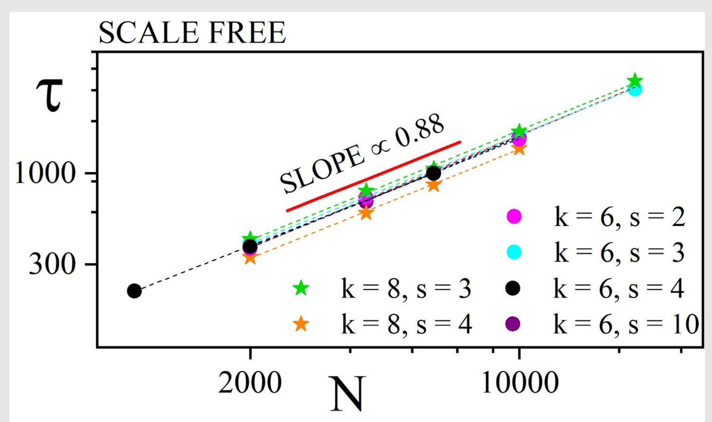


Fig. 5b

- The individual realizations seem to indicate the presence of different plateaus as some of the states disappear. The high of the observed plateaus correspond with the ξ(s) for the quantity of states that still present.

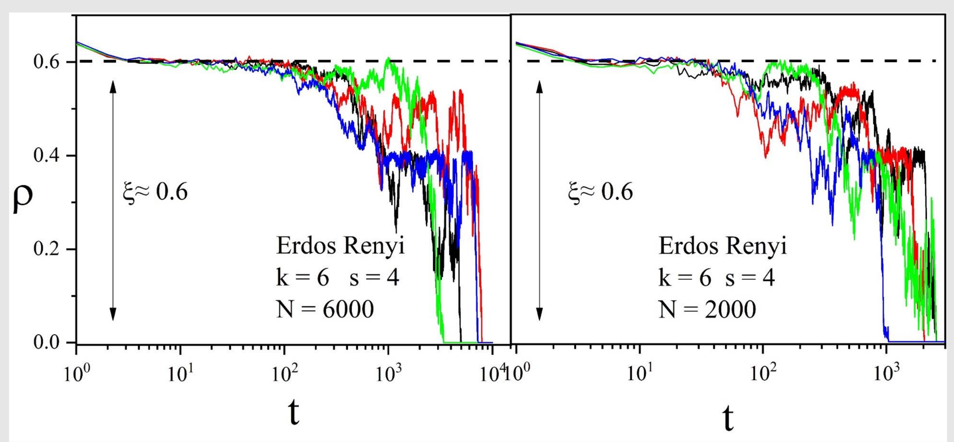


Fig. 6

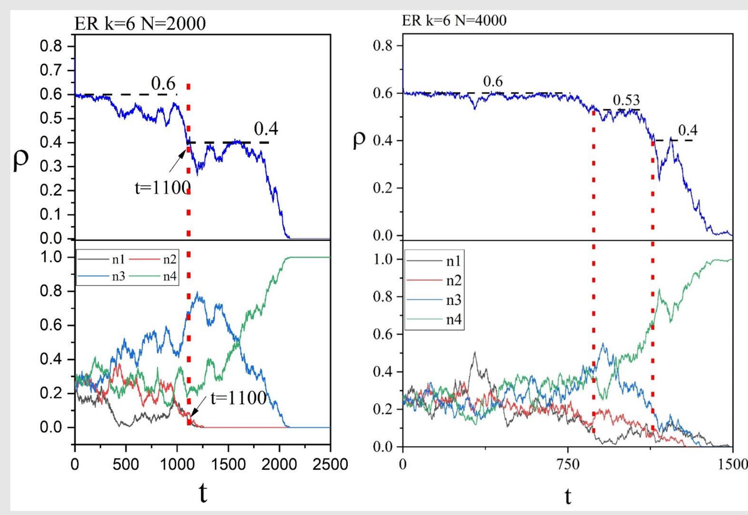


Fig. 7

To be continued…

- To obtain an analytical expression for $<\rho>$ for ER and SF that allows as to calculate the plateau value. This would helps us to understand the radios ξ(s)=ξ(s=2).
- To explore the ordering in the individual realization while some of the states disappear.

References

[1] Krzysztof Suchecki, Víctor M Eguíluz, Maxi San Miguel, Phys. Rev. E 72, 036132 (2005).

[2] Sood, V., and S. Redner, 2005, Phys. Rev. Lett. 94, 178701 (2005).