Degree-Ordered-Percolation on uncorrelated networks

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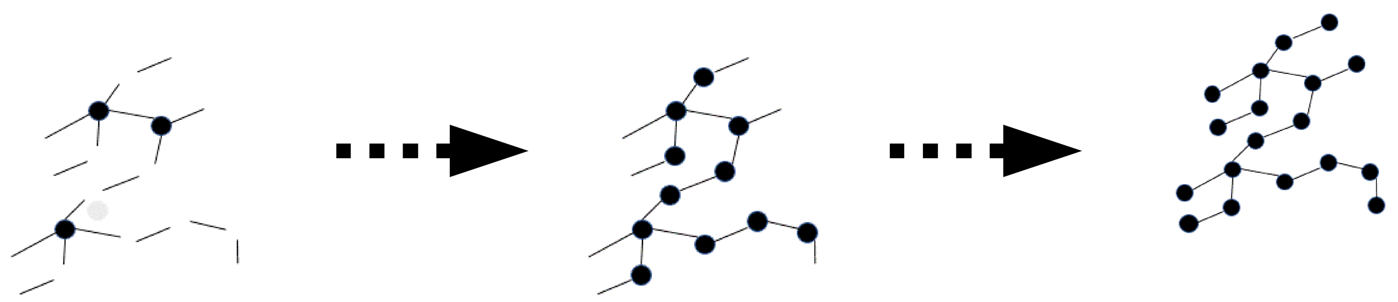
What is it ?

The Degree-Ordered-Percolation (DOP) is a process in which **nodes are added in degree-descending order**.



Critical exponents DOP VS Random Percolation (RP)

By **using the generating function formalism** we have **computed the following critical exponents** of the DOP:

ν: $p_c(N) - p_c \propto N^{-\frac{1}{\nu}}$;

β: $|G(p)| \sim \Delta^{\beta}$ where $\Delta = p - p_c$;

τ: $n_s(p) \sim s^{-\tau}$ where n_s is the probability that a finite cluster has size s at the percolation critical point.

$$\frac{1}{\nu}$$

	$2<\gamma\leq 3$	$3<\gamma<4$	$4<\gamma$
DOP	1	$\frac{\gamma-3}{\gamma-1}$	$\frac{1}{3}$
RP	$\frac{3-\gamma}{\gamma-1}$	$\frac{\gamma-3}{\gamma-1}$	$\frac{1}{3}$

$$\beta$$

	$2<\gamma\leq 3$	$3<\gamma<4$	$4<\gamma$
DOP	1	$\frac{1}{\gamma-3}$	1
RP	$\frac{1}{3-\gamma}$	$\frac{1}{\gamma-3}$	1

$$\tau$$

	$2<\gamma\leq 3$	$3<\gamma<4$	$4<\gamma$
DOP	not a power law	$\frac{2\gamma-3}{\gamma-2}$	$\frac{5}{2}$
RP	3	$\frac{2\gamma-3}{\gamma-2}$	$\frac{5}{2}$

Why are we interested in this process?

We analyse the birth of the giant component G on uncorrelated networks of size N with a power law degree distribution $P(k) = Ak^{-\gamma}$.

We determine the percolation threshold p_c

To verify what has been argued by Lee *et al.* [1]:

if p_c *vanishes, for* $N \to \infty$ *and* $\gamma > 3$*, then it is at the origin of the vanishing of the SIS threshold*

It is natural to wonder how p_c depends on the network properties and

whether the critical exponents are different from those of standard random percolation.

Conclusions

1- For networks with $3 < \gamma < 4$ p_c **is finite** but extremely small.

2- **DOP and RP are in different universality classes** for scale-free networks.

What's next ?

What is **the effect of degree correlations** on p_c?

References

1 Gallos L K, Cohen R, Argyrakis P, Bunde A and Havlin S 2005 Phys. Rev. Lett. 94 188701

2 Lee H K, Shim P-S and Noh J D 2013 Phys. Rev. E 87 062812

3 Pastor-Satorras R, Castellano C, Van Mieghem P and Vespignani A 2015 Rev. Mod. Phys. 87 925

4 C. Castellano and R. Pastor-Satorras, Phys. Rev. X 10, 011070 (2020).

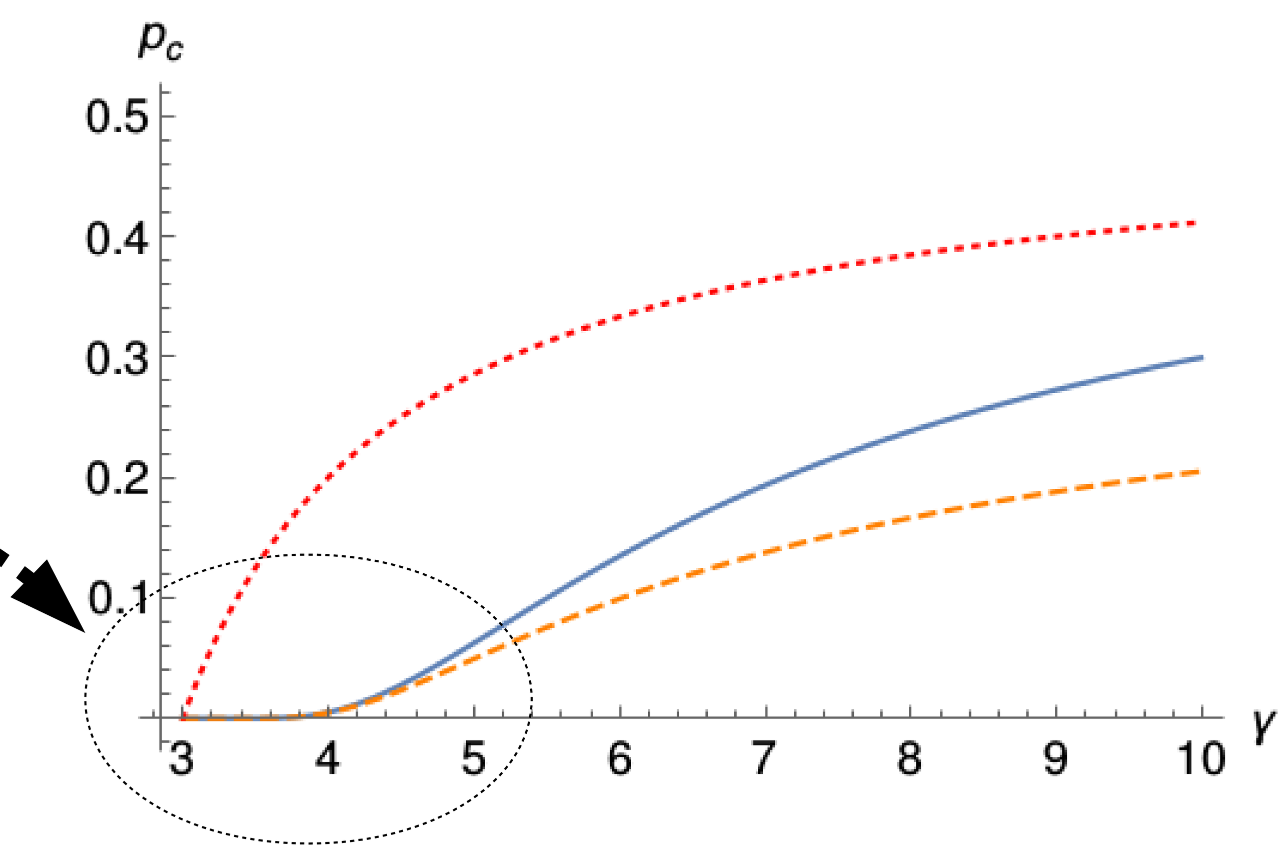
The percolation threshold p_c

By **applying the Molloy-Reed criterion** we obtain

$$\sum_k \frac{k^2 - k}{\langle k \rangle} \Theta(k - k_c) P(k) = 1$$

from this we determine the value of k_c that is the minimum degree left in the network at the critical point. Once k_c is known **we find the percolation threshold by the condition**:

$$p_c = \sum_k \Theta(k - k_c) P(k). \qquad (1)$$



The blue solid line represents the exact solution of Eq. (1), the orange dashed line represents the approximate one. The dotted red line is the threshold for standard percolation.