Dynamical phase transitions in quantum reservoir computing

Rodrigo Martínez-Peña, Gian Luca Giorgi, Johannes Nokkala, Miguel C. Soriano and Roberta Zambrini

IFISC (CSIC-UIB), Palma de Mallorca – Spain

rmartinez@ifisc.uib-csic.es

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Introduction

Closed quantum systems may exhibit different dynamical regimes, such as Many-Body Localization or thermalization [1], that can affect their ability to process information. Specifically, we establish the **role of dynamical phases** of Ising spin networks in the field of quantum **reservoir computing**. Reservoir computing is an unconventional computing paradigm that consists in exploiting classical or quantum dynamical systems **to solve nonlinear and temporal tasks** [2]. We observe that the thermal phase of the spin model is naturally adapted to the requirements of reservoir computing while the localized phase is detrimental for the purposes of this computational approach, with **improved performance** for linear and mildly nonlinear tasks identified **in the transition regime**. We uncover the physical mechanisms behind optimal information processing capabilities of the spin networks, essential for future experimental implementations [4].

Dynamical Phases

Closed Quantum Systems may exhibit different dynamical regimes. Examples:

Thermalization:

- Local observables forget the initial condition.
- Their stationary values are predicted by Statistical Mechanics.
- Intuition: subsystems use the rest of the system as a bath.

Many-Body Localization:

- Produced by the presence of strong disorder in the system.
- Local observables retain their initial conditions at the stationary state.
- Emergence of quasi-local conserved quantities.

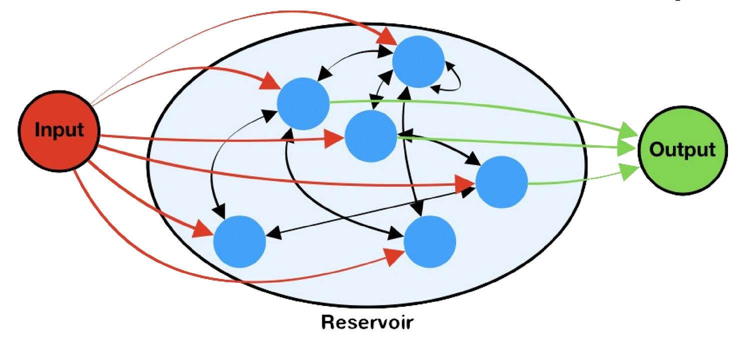
Reservoir Computing

Description:

- Machine learning technique used for non-linear and **temporal** tasks.
- A reservoir computer is composed by 3 layers: input layer, reservoir layer and output layer.
- The reservoir layer is a **dynamical system**. For example, a Recurrent Neural Network (RNN).
- The **only trained** connections are those from **output layer**.

Characteristics:

- Fast training (a **linear regression** in the output layer)
- **Multitasking** (one output layer per task).
- **Hardware implementations** (facilitated by fixed connections in the reservoir).



Quantum Dynamical System

Dynamical system: Transverse-field Ising model

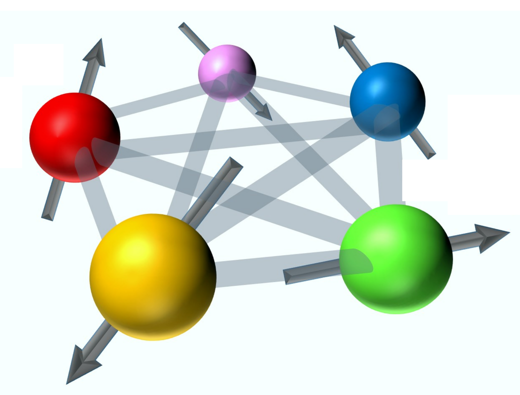
$$H=\sum_{i>j=1}^{N} J_{ij}\sigma_i^x\sigma_j^x+\frac{1}{2}\sum_{i=1}^{N}(h+D_i)\sigma_i^z$$

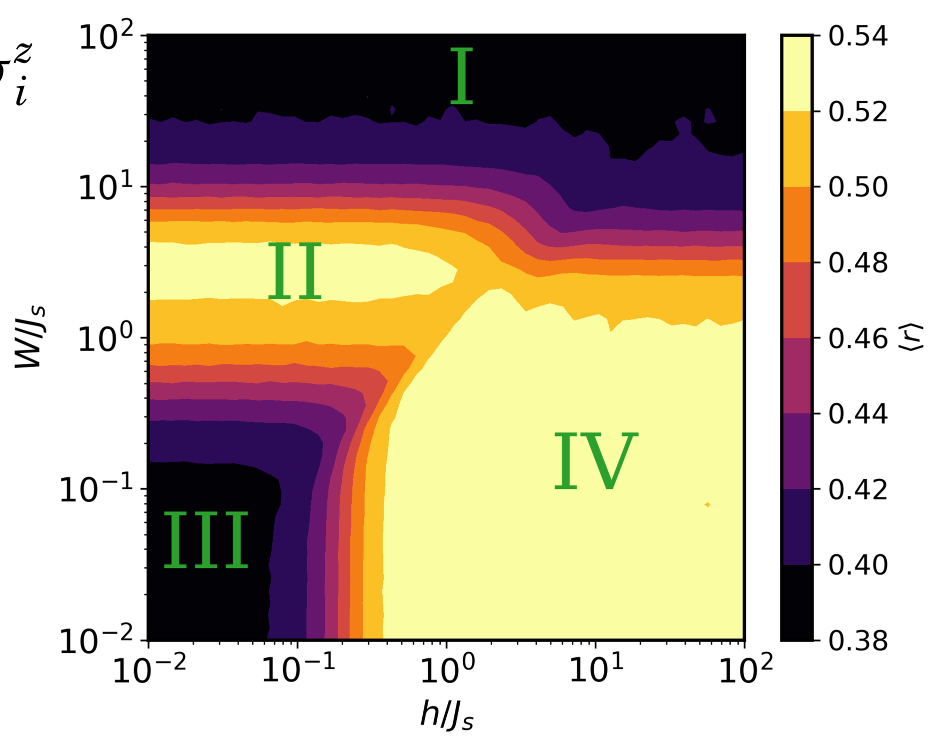
Random uniform distributions:

$D_i\in[-W,W]$

$J_{ij}\in[-J_s/2,J_s/2]$

$N=10$

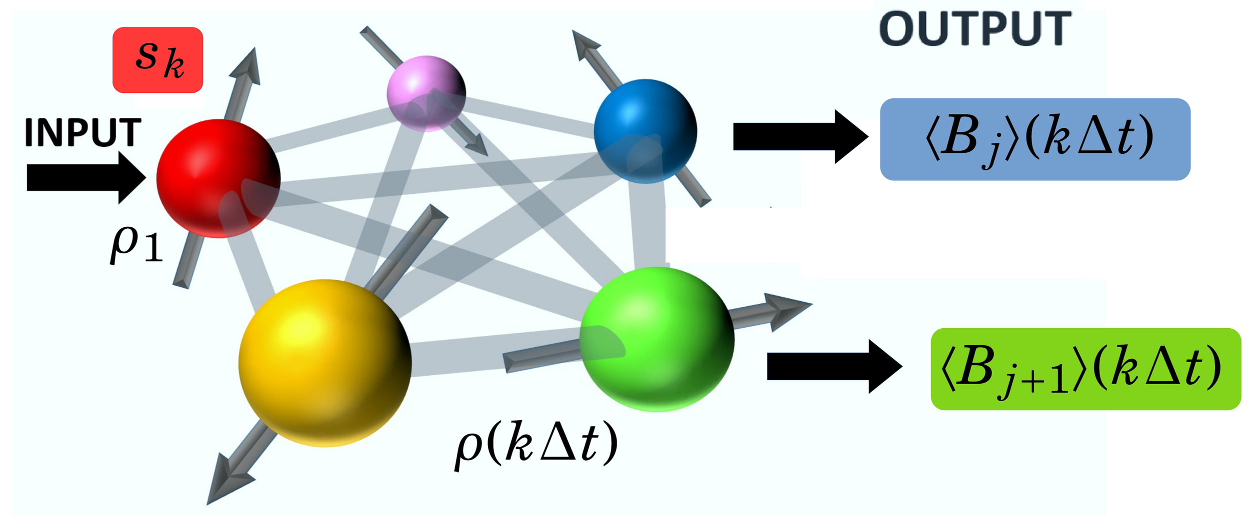




$$r=\min(\delta_n,\delta_{n+1})/\max(\delta_n,\delta_{n+1})$$
$$\delta_n=E_n-E_{n-1}$$

Quantum Reservoir Computing (QRC)

[3] Fuji and Nakajima, 2017



Input: $\rho_1=|\psi_k\rangle\langle\psi_k|$ $\quad |\psi_k\rangle=\sqrt{1-s_k}|0\rangle+\sqrt{s_k}|1\rangle$

Dynamics: $\rho(k\Delta t)=e^{-iH\Delta t}\rho_1\otimes\mathrm{Tr}_1\left[\rho[(k-1)\Delta t]\right]e^{iH\Delta t}$

Output: $x_j(k\Delta t)=\mathrm{Tr}\left[B_j\rho(k\Delta t)\right]=\langle B_j\rangle(k\Delta t)$

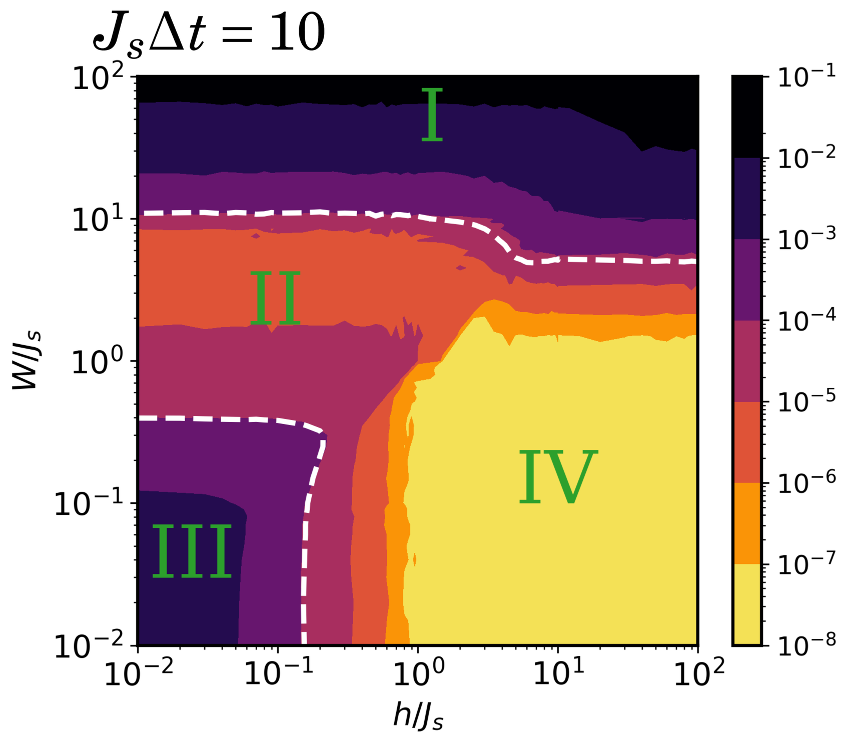
$$\{B_j\}_{j=1}^{4^N}=\{I,\sigma^x,\sigma^y,\sigma^z\}^{\otimes N}\qquad \bar{y}_k=\sum_j^{O} w_j x_j(k\Delta t)$$

Convergence Property

- Fundamental property of Reservoir Computing systems: **convergence property.**
- Convergence property: **independence of initial conditions** after several input injections (we used 200 in this plot).

Quantum case:
$t\to\infty$
$||\rho_A-\rho_B||\to 0$

$J_s\Delta t=10$



The convergence property is **enhanced in the ergodic phase**, while the influence of different **initial conditions persists in the localized phases.**

QRC Performance

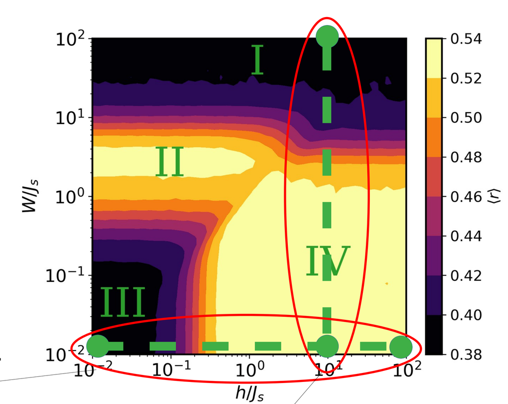
Tasks:

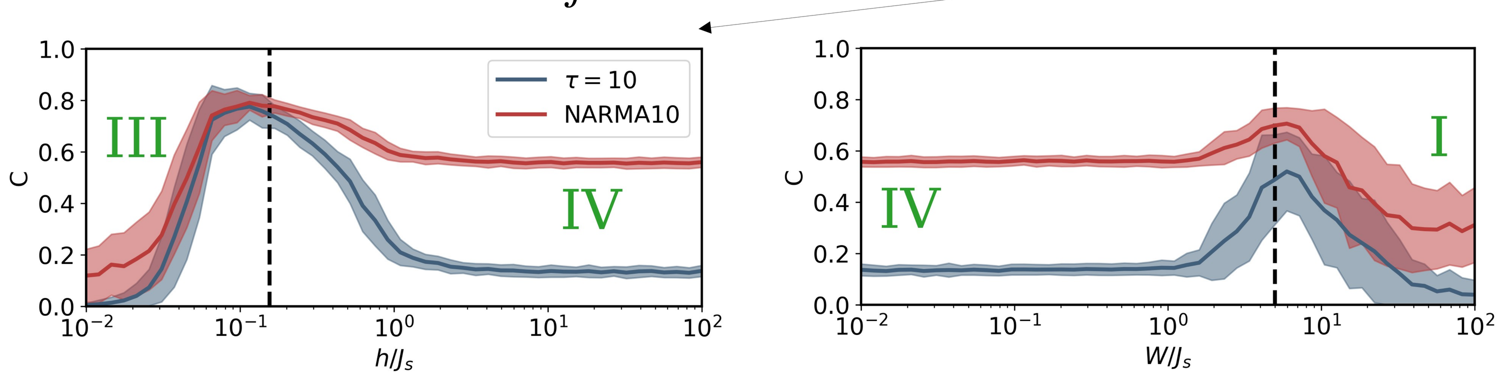
τ=10:
$y_k=s_{k-10}$

$$C=\frac{\mathrm{cov}^2(y,\bar{y})}{\sigma^2(y)\sigma^2(\bar{y})}\quad\begin{cases}C=0\\C=1\end{cases}$$

NARMA10:

$$y_k=0.3y_{k-1}+0.05y_{k-1}\left(\sum_{j=1}^{10}y_{k-j}\right)+1.5s_{k-10}s_{k-1}+0.1$$





$O=75$
$J_s\Delta t=10$

The ergodic phase **performs better** than the localized one. The linear memory and NARMA10 task are **improved at the transition.**

Conclusions

- Thermal phase enhances the convergence property and performs better than localized phases. It is naturally adapted to Reservoir Computing.
- Localized phases are detrimental for Reservoir Computing.
- Different tasks can be solved by exploiting the trade-off between linear and non-linear memory at the transition between phases.

References

[1] D.A. Abanin, E. Altman, I. Bloch, and M. Serbyn. Reviews of Modern Physics, 91(2), 2019.

[2] M. Lukoševičius and H. Jaeger. *Computer Science Review*, 3(3):127–149, 2009.

[3] K. Fujii and K. Nakajima. *Physical Review Applied*, 8(2):024030, 2017.

[4] R. Martínez-Peña, G.L. Giorgi, J. Nokkala, M.C. Soriano and R. Zambrini. ArXiv: 2103.05348