Coexistence, consensus and polarisation in a continuous opinion dynamics model with quenched disorder

Joseph W. Baron

IFISC (CSIC-UIB) Palma de Mallorca – Spain.

josephbaron@ifisc.uib-csic.es

UNIT OF EXCELLENCE MARÍA DE MAEZTU

AGENCIA ESTATAL DE INVESTIGACIÓN

Introduction

■ A model of continuously varying opinions with heterogeneous interactions between individuals is introduced

■ It is shown that consensus, polarisation and a spread of moderate opinions (coexistence) are all possible in this model

■ We deduce what kinds of interactions between individuals lead to each of the aforementioned states

Model

■ Consider a set of individuals $i = 1, 2, 3, \ldots, N$ with opinions $0 \leq x_i \leq 1$

■ Opinions evolve continuously according to (inspired by the Lotka-Volterra model in Ecology)

$$\dot{x}_i = g(x_i)\left[N^{-1}\sum_j x_j - x_i + \sum_j z_{ij}\left(x_j - \frac{1}{2}\right)\right]. \qquad (1)$$

■ The precise form of $g(x_i)$ does not affect the phase transitions (see below), but it is convenient to choose $g(x_i) = x_i(1 - x_i)$ for the numerical integration.

■ Quenched interaction coefficients z_{ij} are Gaussian random variables with statistics

$$\mathrm{E}[z_{ij}] = \frac{\mu}{N}, \qquad \mathrm{Var}[z_{ij}] = \frac{\sigma^2}{N}, \qquad \mathrm{Cov}[z_{ij}, z_{ji}] = \frac{\Gamma\sigma^2}{N}. \qquad (2)$$

■ μ is the agreeableness – how likely individuals are to copy each other

■ σ is the amount of heterogeneity in the interactions

■ $0 \leq \Gamma \leq 1$ is the reciprocity – how closely related are z_{ij} and z_{ji}

Dynamic mean-field theory

■ In the limit $N \to \infty$, the following single-opinion process has the same statistics as Eqs. (1)

$$\dot{x} = g(x)\left\{m(t) - x + \mu\left[m(t) - \frac{1}{2}\right] + \Gamma\sigma^2\int_0^t dt' G(t,t')\left[x(t') - \frac{1}{2}\right] + \sigma\eta(t)\right\}. \qquad (3)$$

■ The coloured Gaussian noise $\eta(t)$ has statistics

$$\langle\eta(t)\rangle = 0, \qquad \langle\eta(t)\eta(t')\rangle = \left\langle\left[x(t) - \tfrac{1}{2}\right]\left[x(t') - \tfrac{1}{2}\right]\right\rangle. \qquad (4)$$

■ The process is determined self-consistently so that

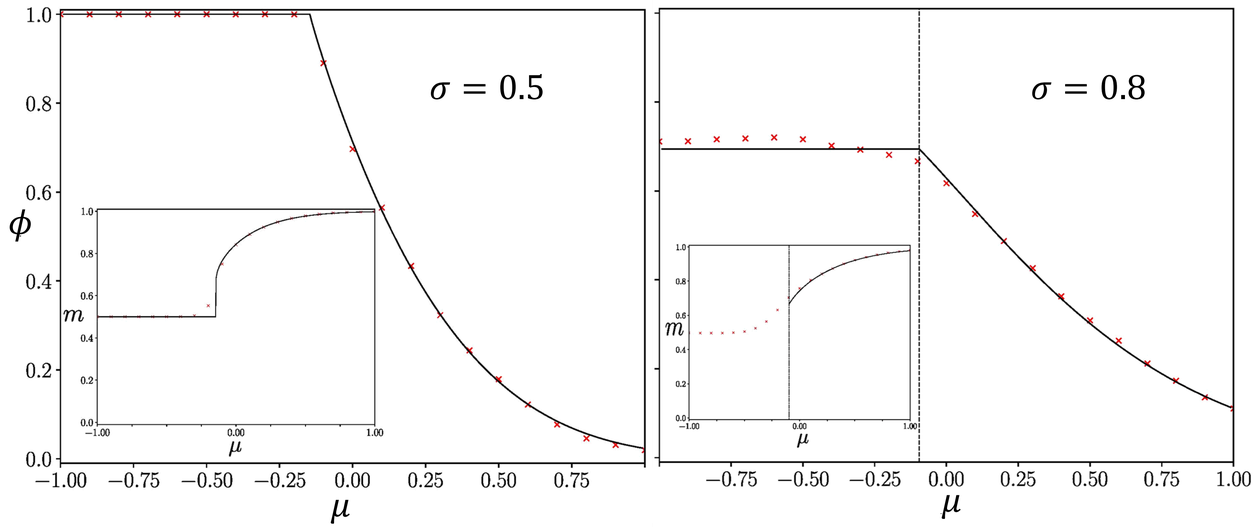
$$m(t) = \langle x(t)\rangle, \qquad G(t,t') = \left\langle\frac{\delta x(t)}{\delta\eta(t')}\right\rangle. \qquad (5)$$

■ This single-opinion process has a fixed-point solution. One can find the distribution of opinions at the fixed point and analyse the stability.

Phase transitions

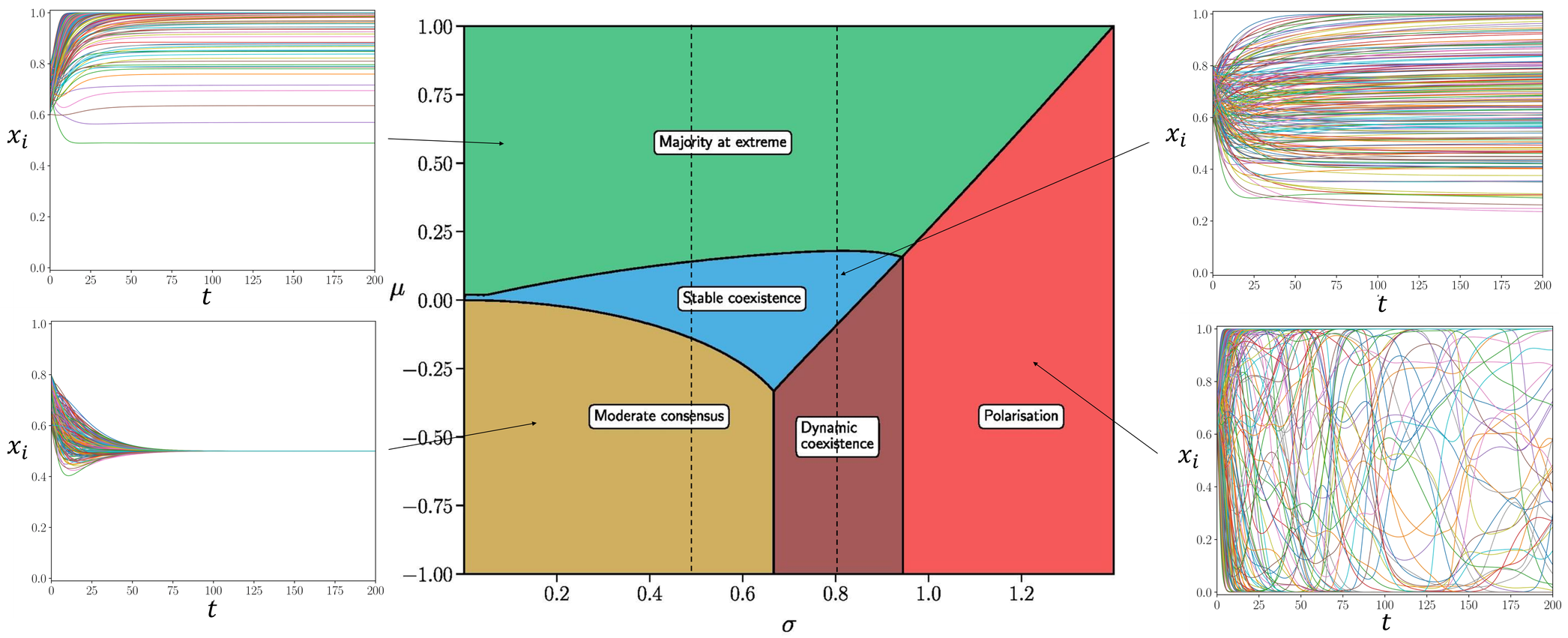
Moderate consensus

Dynamic coexistence



ϕ – Fraction of opinions with $x_i \neq 0$ and $x_i \neq 1$ at the fixed point

Phase diagram



Conclusions

■ Agreeableness (a tendency to adopt the same opinion as others – large positive μ) leads to a majority at the extreme opinion

■ Disagreeableness (a tendency to contradict others – large negative μ) leads to consensus at the moderate opinion

■ Highly heterogeneous interactions between individuals (large σ) leads to unstable opinions and polarization

■ Reciprocity of interactions ($\Gamma \approx 1$) accentuates the effect of heterogeneity and promotes extreme opinions (not shown here)