Electronics Letters, **37**, 1062 (2001)

Coherence Resonance in Chaotic Electronic Circuits

Oscar Calvo[1], Claudio R. Mirasso[1], Raúl Toral[1,2]
[1]*Departament de Física, Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain*
[2]*Instituto Mediterráneo de Estudios Avanzados (IMEDEA), CSIC-UIB, E-07071 Palma de Mallorca, Spain*

We experimentally demonstrate that a chaotic electronic circuit, in this case the Chua circuit, exhibits the main features of coherence resonance when subjected to an external white-noise input.

Random fluctuations (or noise) usually constitute a source of disorder and are considered, in general, as detrimental in, e. g., linear information theory, electrical engineering or neurobiology. However, it has been shown in the recent past that the amount of order on the response of a system can be improved if a right amount of noise is added. One of the most well known examples is that of stochastic resonance [1], by which a system driven by an external signal optimizes its response for an intermediate value of the noise level. This behaviour has been observed in the information transfer in crayfish mechanoreceptors [2], tunnel diodes [3], electronic circuits [4], amongst other systems. More surprisingly, a quasi-periodic signal can be made to appear just by applying a noise source, without the need of an external forcing. The periodicity of the signal becomes optimal for some intermediate values of the noise intensity. This effect is called *coherence resonance* [5] and has been observed in excitable [5,6], bistable and oscillatory [7] and optical systems [8], and a similar phenomenon appears in systems close to a limit cycle [9], .

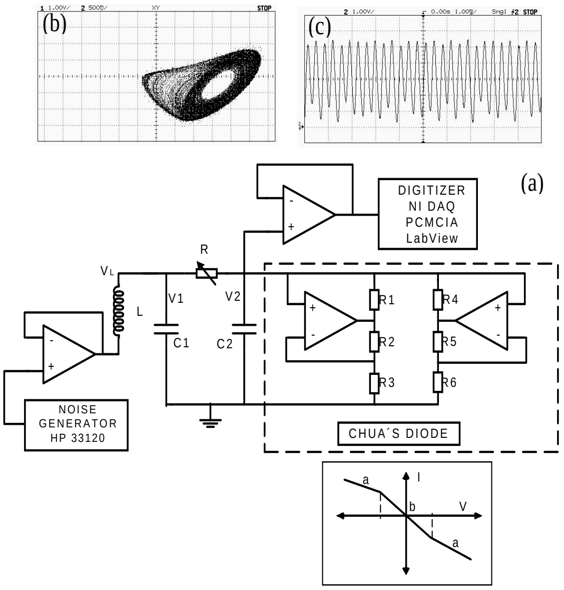

FIG. 1. (a) Block of the Chua Circuit. The inset plots the characteristics of the non-linear; (b) Phase space representation V_1 vs. V_2; (c) Typical time trace of $V_2(t)$. A digital acquisition board from National Instruments NI-DAQ, sampling at 10KHz, plugged into the PCMC/A slot of a laptop was used to digitize the signal. A Labview program controlled the board in a continuous acquisition mode. An HP 3312 function generator was used to provide the noise signals from 0 to 10 V[pp].

In this letter we experimentally demonstrate that these ideas can be extended to chaotic systems, for which we find the main features of coherence resonance. Although we choose as an example a Chua circuit, we believe that the same behaviour should be observed in other chaotic systems with similar characteristics. A Chua circuit [10] is shown in figure 1. The output voltages V_1, V_2 and V_L chaotically oscillate in time for a large region of parameter values, yielding different chaotic attractors. Mathematically, a Chua circuit is described in terms of three non-linear first order differential equations [10]. These equations predict that the system has, for some range of parameters, three unstable fixed points. We choose the set of parameters $C_1 = 10$ nF, $C_2 = 100$ nF, $R = 1673$ Ω, $a = -1/7$, $b = 2/7$, where a and b characterize the slope of the non-linear Chua resistance (see inset fig. 1). For this set of parameters, there is a *single scroll* attractor (as shown in the fig. 1 (b)), and its mirror image (not shown in the figure). In the absence of any external perturbation, V_1 and V_2 chaotically oscillate around one of the unstable fixed points with a mean oscillation frequency of ~ 2.3 KHz and without any possibility to jump to its mirror image attractor. However, when a small amount of noise is added to the circuit, V_1 and V_2 start to switch quasi-periodically between the two single scroll attractors. Three typical time traces are shown in the figure 2 for the voltage V_2 at low, intermediate and large noise levels. As can be seen in panel (b) for the optimum noise level, the regularity of the jumping process becomes evident. This is the first indication that noise is inducing a more regular behaviour in this chaotic system.

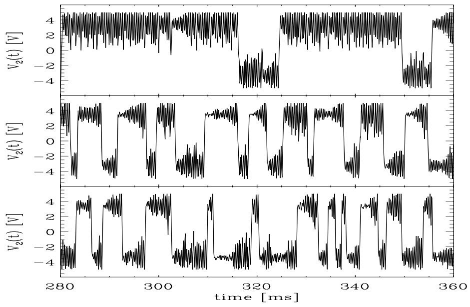

FIG. 2. Voltage time series for three different noise levels (maximum values): a) 0.5 V[rms], b) 1.5 V[rms], optimum noise level, and c) 2.5 V[rms].

To better quantify the situation we calculate the variance σ of the residence time in the attractors normalized to the mean value $\langle t \rangle$. In figure 3 (a) it can be clearly seen that this quantity has a minimum at an intermediate noise level, in this case for a maximum noise voltage of $\sim$ 1.5 V[rms]. Another indicator has been also calculated: the normalized autocorrelation function of the time series, whose minimum value C_{min} measures the strength of the (anti)correlation between the residence time in the two states. As can be seen in figure 3 (b) this value of C_{min} reaches a minimum around the noise level of 1.5 V[rms], in agreement with the previous indicator. These results reveal the existence of a constructive effect of the noise that is capable to yield a maximum regularity in the process of transition from one chaotic attractor to the other.

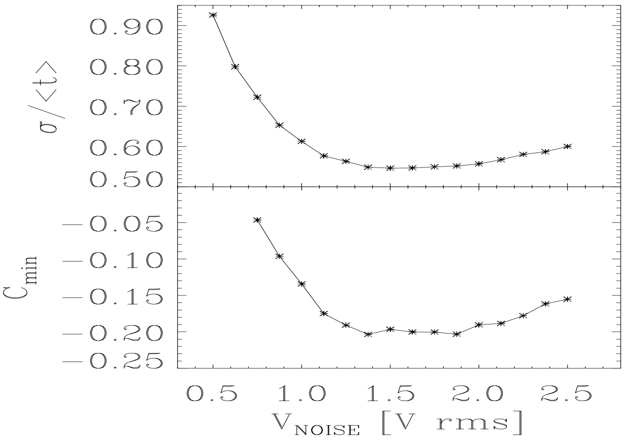

FIG. 3. (a) Variance of the residence time normalized by the mean time $\sigma/\langle T \rangle$ and (b) minimum of the correlation function C_{min} as a function of the noise level V_{NOISE}.

In conclusion, we have given the first evidence of coherence resonance in a chaotic electronic circuit. It has been demonstrates experimentally that there is an improvement in the response of the circuit, as measured by the variance of the residence time and the minimum of the time correlation functios, when an optimal amount of noise is added. These results show that noise, instead of being a source of disorder, can actually induce a quasi-periodic movement in an electronic, chaotic, circuit.

Acknowledgements This work has been supported by DGES (Spain) project PB97-0141-C02-01 and MCyT (Spain) project BMF2000-1108.

---

[1] L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, “Stochastic Resonance”, Rev. Mod. Phys. **70**, (1), pp. 223-287 (1998).

[2] J. K. Douglass, L. Wilkens, E. Pantazelou and F. Moss, “Noise Enhancement of Information Transfer in Crayfish Mechanoreceptors by Stochastic Resonance”, Nature **365**, pp. 337-340 (1993).

[3] R. Mantegna and B. Spagnolo, “Stochastic Resonance in a Tunnel Diode” Phys. Rev. E **49**, (3), pp. R1792-R1795 (1994).

[4] X. Godivier, J. Rojas-Varela and F. Chapeau-Blondeau, “Noise-assisted signal transmission via stochastic resonance in a diode nonlinearity”, Electron. Lett. **33**, (20), p. 1667-1667 (1997).

[5] A.S. Pikovsky and J. Kurths, “Coherence Resonance in a Noise-Driven Excitable System”, Phys. Rev. Lett. **78**, (5), pp. 775-778 (1997).

[6] D.E. Postnov, S. K. Han, T. Yim, and O.V. Sosnovtseva “Experimental Observation of Coherence Resonance in Cascaded Excitable Systems”, Phys. Rev. E **59**, (4), 3791-3794 (1999).

[7] B. Lindner and L. Schimansky-Geier, “Coherence and stochastic resonance in a two-state system” Phys. Rev. E **61**, (6), pp. 6103-6110 (2000).

[8] G. Giacomelli, M. Giudici, S. Balle and J.R. Tredicce, “Experimental Evidence of Coherence Resonance in an Optical System”, Phys. Rev. Lett. **84**, (15), pp. 3298-3301 (2000); J. Buldú, J. García-Ojalvo, C. Mirasso, M. C. Torrent and J. M. Sancho “Effect of External Noise Correlation in Optical Coherence Resonance”, paper nlin.AO/0104047, http://arXiv.org/abs/nlin/0104047.

[9] H. Gang, T. Ditzinger, C. Z. Ning, and H. Haken, “Stochastic resonance without external periodic force”, Phys. Rev. Lett. **71**, (6), pp. 807810 (1993); W. J. Rappel and S. H. Strogatz, “Stochastic resonance in an autonomous system with a nonuniform limit cycle”, Phys. Rev. E **50**, (4), pp. 3249-3250, (1994).

[10] *Chua’s Circuit: A Paradigm for Chaos*, R.N. Madan, ed. World Scientific Publishing (1993).