Strongly coupled quantum heat machines

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Carnot machines

Components
1. Working fluid, the system (Ex. Gas)
2. Hot and cold bath
3. External cyclic driving of the system. It extracts or invests work (Piston)

The second law limits the machine efficiency

Engine

$$\eta \equiv \frac{-P}{J_H} \leq 1 - \frac{T_C}{T_H} \equiv \eta_{Carnot}$$

Absorbed heat (Invested energy)

$$P = \dot{W}$$
Continuous quantum machine

K Szczygielski, D. G. –K., R Alicki Physical Review E 87 (1), 012120

Components
1. Working fluid: (qubit, TLS).
2. Hot and cold bath (normal modes), permanently coupled to the system (weak coupling).
3. A piston (External driving) periodically drives the system and gets or gives work.

\[ H_{Tot} = H_S(t) + \sum_i (H_{B_i} + \xi_i S \otimes F_i) \quad i \in H, C \]

Coupling spectrum

\[ G_i(\omega) = \int_{-\infty}^{\infty} e^{it\omega} \xi_i^2 \langle F_i^\dagger(t) F_i(0) \rangle, \quad H_{SB} = \sum_{i \in H, C} \xi_i S \otimes F_i \]

\[ G_i(-\omega) = e^{-\beta_i \omega} G_i(\omega) \quad \text{KMS Condition} \]

\( \omega_0 \)
(resonant baths) \[ H_{Tot} = H_S(t) + \sum_i H_{B_i} + H_{SB_i} \quad i \in H, C \]

Lindblad master equation (weak coupling, “small” $\xi_i$)

$\rho_S(t)$

$J_i(G_i(\omega))$  \[ \text{Thermodynamic quantities (analytic expressions)} \]

Baths at equilibrium  \[ \xrightarrow{\text{KMS condition}} \]

$\eta \leq 1 - \frac{T_C}{T_H}$
Weak coupling limitations

Weak coupling QHM \( P \propto \gamma \propto \xi_i^2 \) \((\xi_i \text{ coupling strength})\)

\[ t \propto 1/\gamma \]

Also strokes QHMs are limited

The coupling strength limits the output

? \[ \xi_i \]
Strong coupling

Lindblad equations are no longer valid!

Weak Coupling

Strong Coupling
Redefining the system and the bath

\[ \tilde{\mathcal{H}} \xrightarrow{\mathcal{H} = U\tilde{\mathcal{H}}U^\dagger} \]

\( U \) Polaron transformation

D. P. McCutcheon and A. Nazir, NJP 12, 113042 (2010).
Particular example

Cold bath  Driving  Hot bath

Working fluid

$\sigma_z \otimes \xi_C \sum_K g_{C,k} (a_k^\dagger + a_k)$  $\sigma_x \otimes \xi_H \sum_K g_{H,k} (b_k^\dagger + b_k)$

First coupling  Second coupling
Transformed Hamiltonian

\[ U = e^S \quad S = \sigma_Z \otimes \xi_C \sum_K \frac{g_{C,k}}{\omega_{C,k}} (a^\dagger_k - a_k) \]

First coupling

\[ \sigma_Z \otimes \xi_C \sum_K g_{C,k} (a^\dagger_k + a_k) \]

\[ \sigma_+ \otimes (A_+ - A) + h.c \]

\[ A_+ = \Pi_K e^{2\xi_C \frac{g_{C,k}}{\omega_{C,k}} (a^\dagger_k - a_k)} \]

\[ A = \langle A_+ \rangle \]
Transformed Hamiltonian

\[ U = e^S \quad S = \sigma_Z \otimes \xi_C \sum_K \frac{g_{C,k}}{\omega_{C,k}} (a_k^\dagger - a_k) \]

Second coupling

\[ \sigma_x \otimes \xi_H \sum_K g_{H,k} (b_k^\dagger + b_k) \]

\[ (\sigma_+ \otimes A_+ + h.c.) \otimes \xi_H \sum_K g_{H,k} (b_k^\dagger + b_k) \]
Effective weak coupling

First coupling

Original spectrum

Weak coupling
Validity limit

Second coupling

\[
\left(\sigma_+ \otimes A_+ + h.c\right) \otimes \xi_H \sum_K g_{H,k} (b_k^\dagger + b_k)
\]

Effectively weak if \(\xi_H \sim \xi_C\)
Coupling Spectrum

WEAK COUPLING:

\[ a_{k'}^{\dagger}(t) = a_{k'}^{\dagger} e^{i\omega_k t} \]

Modes should be resonant!

STRONG COUPLING:

\[ A_+(t) = \prod_k \sum_n C_{n,k} e^{i n \omega_k} \]

Harmonic Modes also contribute!
First coupling

\[ F_1^\dagger = A_+ - A \]

\[ G_1(-\omega) = e^{-\beta \omega} G_1(\omega) \]
Second coupling

\[ F_2^\dagger = A_+ \otimes \xi_H \sum_K g_{H,k} (b_k^\dagger + b_k) \]

\[ \omega = \omega_H + \omega_C \]

\[ G_2(-\omega) = e^{-\beta(\omega)\omega} G_2(\omega) \quad \beta(\omega) = \lambda(\omega)\beta_C + (1 - \lambda(\omega))\beta_H \]

Each \( \omega \) may be a different process
\[ \beta(\omega) = \lambda(\omega) \beta_C + (1 - \lambda(\omega)) \beta_H \]

- \( \lambda(\omega) = 1 \) \( \Rightarrow \beta(\omega) = \beta_C \)
- \( \lambda(\omega) = 0 \) \( \Rightarrow \beta(\omega) = \beta_H \)
- \( 0 \leq \lambda(\omega) \leq 1 \) \( \Rightarrow \beta_H \leq \beta(\omega) \leq \beta_C \)
- \( \lambda(\omega) \leq 0 \) \( \Rightarrow \beta(\omega) \leq \beta_H \)

\( \beta(\omega) \) is not restricted to \((\beta_H, \beta_C)\)
Power

Maximum power

Ultra-strong coupling

Weak coupling

\[ \xi_C / \omega_0 \quad (\xi_H \sim \xi_C) \]

Similar for cooling power!
Efficiency

\[ \eta = \frac{-P}{J_H} \]

\( (J_1, J_2) \)

Naive guess: \( J_H = J_2 \)

- May be larger than Carnot

Correct heat flow: \( J_H = J_2(1 - \lambda(\omega_0)) \)

\[ \eta = \frac{-P}{J_2(1 - \lambda(\omega_0))} \leq \eta_{car} \]

Similar for cooling
Conclusions

1. \[ \eta = \frac{-P}{J_2(1-\lambda(\omega_0))} \leq \eta_{car} \]

2. \[ \omega \]

3. \[ P \text{[a.u.]} \]

\[ \xi_c/\omega_0 \]