Quantum thermodynamics for a model of an expanding universe

David Edward Bruschi

Formerly: Racah Institute of Physics and Quantum Information Science Centre
the Hebrew University of Jerusalem
the Holy Land
Currently: York Centre for Quantum Technologies, Department of Physics
University of York
the United (for the moment) Kingdom

April 24, 2015

arXiv:1409.5283
In collaboration with: N. Liu, J. Goold, I. Fuentes, V. Vedral, K. Modi
Relativistic and Quantum Physics

Relativity + Quantum Information = Relativistic Quantum Information
(Classical) Thermodynamics and general relativity

Important achievements in general relativistic scenarios

- Three laws of black holes;
- Cosmology and big bang;
- Firewall issues.
Outlook, aims and motivations

Outlook

- (Classical) Thermodynamics useful in the study of the Universe.
- Quantum processes in the universe do not necessary involve large numbers of constituents.
- **Important**: Von N. Entropy of the Universe cannot change.
Introduction

Outlook, aims and motivations

Outlook

- (Classical) Thermodynamics useful in the study of the Universe.
- Quantum processes in the universe do not necessary involve large numbers of constituents.
- Important: Von N. Entropy of the Universe cannot change.

Aim

Use quantum thermodynamics to understand work, entropy and energy flows in relativistic and cosmological setups.
Outlook, aims and motivations

Outlook

- (Classical) Thermodynamics useful in the study of the Universe.
- Quantum processes in the universe do not necessarily involve large numbers of constituents.
- Important: Von N. Entropy of the Universe cannot change.

Aim

*Use quantum thermodynamics to understand work, entropy and energy flows in relativistic and cosmological setups.*

Motivations

- The Universe is relativistic and quantum system and processes can involve small numbers of constituents.
- Mainly: We cannot compute energy, entropy and work flows in relativistic quantum systems.
Simple cosmology model

Expanding universe line/metric element

\[ ds^2 = \Omega^2(\tau) \left[ -d\tau^2 + dx^2 + \ldots \right], \quad g_{\mu\nu} = \Omega^2(\tau) (-1, 1, \ldots) \]
# Simple cosmology model

## Expanding universe line/metric element

\[
 ds^2 = \Omega^2(\tau) \left[ -d\tau^2 + dx^2 + \ldots \right], \quad g_{\mu\nu} = \Omega^2(\tau) (-1, 1, \ldots) 
\]

## Scalar quantum field

\[
 \phi(t, x) = \sum_k \left[ u_k \ a_k + u_k^* \ a_k^{\dagger} \right], \quad [a_k, a_{k'}^{\dagger}] = \delta^d(k - k') 
\]
## Simple cosmology model

### Expanding universe line/metric element

$$\text{Expanding universe line/metric element}$$

$$ds^2 = \Omega^2(\tau) \left[ -d\tau^2 + dx^2 + \ldots \right], \quad g_{\mu\nu} = \Omega^2(\tau) (-1, 1, \ldots)$$

### Scalar quantum field

$$\text{Scalar quantum field}$$

$$\phi(t, x) = \int \sum_k \left[ u_k a_k + u_k^* a_k^\dagger \right], \quad [a_k, a_k^\dagger] = \delta^d(k - k')$$

### Frequency shift and choice of conformal factor

$$\text{Frequency shift and choice of conformal factor}$$

$$\omega_{\text{in/out}} = \sqrt{k^2 + m \Omega^2(\tau_{\mp \infty})}, \quad \Omega(\tau) = \sqrt{1 + \epsilon (1 + \tanh(\sigma \tau))}$$
Simple cosmology model

Expanding universe line/metric element

\[ ds^2 = \Omega^2(\tau) \left[ -d\tau^2 + dx^2 + \ldots \right], \quad g_{\mu\nu} = \Omega^2(\tau) (-1, 1, \ldots) \]

Scalar quantum field

\[ \phi(t, x) = \int \frac{dk}{\pi} \left[ u_k a_k + u_k^* a_k^\dagger \right], \quad \left[ a_k, a_{k'}^\dagger \right] = \delta^d(k - k') \]

Frequency shift and choice of conformal factor

\[ \omega_{\text{in/out}} = \sqrt{k^2 + m \Omega^2(\tau_{\mp\infty})}, \quad \Omega(\tau) = \sqrt{1 + \epsilon (1 + \tanh(\sigma \tau))} \]

Bogoliubov transformations and squeezing

\[ a_{\text{out}, k} = \cosh r_k \ a_{\text{in}, k} + e^{i\theta_k} \sinh r_k \ a_{\text{in}, -k}^\dagger, \quad \tanh r_k = \frac{\sinh \left( \frac{\pi (\omega_{\text{out}} - \omega_{\text{in}})}{2\sigma} \right)}{\sinh \left( \frac{\pi (\omega_{\text{out}} + \omega_{\text{in}})}{2\sigma} \right)} \]
Work and energy of two mode squeezing

All couple of modes \((k, -k)\) decouple. We can focus on a single couple and redefine \(a_{in,k} \equiv a_{in}\) and \(a_{in,-k} \equiv b_{in}\).

Initial Hamiltonian

\[
H_{in} = \omega_{in} \left[ a_{in}^{\dagger} a_{in} + b_{in}^{\dagger} b_{in} + \frac{1}{2} \right]
\]

Final Hamiltonian

\[
H_{out} = \omega_{out} \left[ a_{out}^{\dagger} a_{out} + b_{out}^{\dagger} b_{out} + \frac{1}{2} \right]
\]
Work and energy of two mode squeezing

All couple of modes \((k, -k)\) decouple. We can focus on a single couple and redefine \(a_{in,k} \equiv a_{in}\) and \(a_{in,-k} \equiv b_{in}\).

**Initial Hamiltonian**

\[
H_{in} = \omega_{in} \left[ a_{in}^\dagger a_{in} + b_{in}^\dagger b_{in} + \frac{1}{2} \right]
\]

**Final Hamiltonian**

\[
H_{out} = \omega_{out} \left[ a_{out}^\dagger a_{out} + b_{out}^\dagger b_{out} + \frac{1}{2} \right]
\]

Start with an initial thermal state \(\rho\) with \(n_i\) particles. The work \(W\) done by spacetime is

**Work performed**

\[
W = \text{Tr} \left( (H_{out} - H_{in}) \rho \right) = \omega_{out} n_c + (\omega_{out} - \omega_{in})(n_i + 1)
\]

with \(n_c\) particles that are created.
The “inner friction”

An adiabatic process would lead to a work cost $W_{ad}$ of the form

Adiabatic work

$$W_{ad} = (\omega_{out} - \omega_{in})(n_i + 1)$$
The “inner friction”

An adiabatic process would lead to a work cost $W_{\text{ad}}$ of the form

\[
W_{\text{ad}} = (\omega_{\text{out}} - \omega_{\text{in}})(n_i + 1)
\]

which allows us to find the inner friction $W_{\text{fric}}$ for our case as

\[
W_{\text{fric}} = W - W_{\text{ad}} = \omega_{\text{out}} n_c
\]
The “inner friction”

An adiabatic process would lead to a work cost $W_{ad}$ of the form

\[
W_{ad} = (\omega_{out} - \omega_{in})(n_i + 1)
\]

which allows us to find the inner friction $W_{fric}$ for our case as

\[
W_{fric} = W - W_{ad} = \omega_{out} n_c
\]

Our last step

We proceed to show that $W_{fric}$ can be interpreted as an entropic quantity.
Entropy and inner friction

Forward process

\[ p_{\text{in,out}} = |\langle n_{\text{out}}|n_{\text{in}} \rangle|^2 \langle n_{\text{in}}|\rho|n_{\text{in}} \rangle \]

Backward process

\[ p_{\text{out,in}} = |\langle n_{\text{in}}|n_{\text{out}} \rangle|^2 \langle n_{\text{out}}|\rho|n_{\text{out}} \rangle \]
# Entropy and inner friction

## Forward process
\[
p_{\text{in,out}} = \langle n_{\text{out}} | n_{\text{in}} \rangle^2 \langle n_{\text{in}} | \rho | n_{\text{in}} \rangle
\]

## Backward process
\[
p_{\text{out,in}} = \langle n_{\text{in}} | n_{\text{out}} \rangle^2 \langle n_{\text{out}} | \rho | n_{\text{out}} \rangle
\]

## Fluctuation relation:
\[
s_{\text{in,out}} := - \log \langle n_{\text{out}} | \rho | n_{\text{out}} \rangle + \log \langle n_{\text{in}} | \rho | n_{\text{in}} \rangle
\]
\[
p_{\text{in,out}} = p_{\text{out,in}} e^{s_{\text{in,out}}}
\]
## Entropy and inner friction

### Forward process

\[ p_{\text{in},\text{out}} = \langle n_{\text{out}}|n_{\text{in}} \rangle^2 \langle n_{\text{in}}|\rho|n_{\text{in}} \rangle \]

### Backward process

\[ p_{\text{out},\text{in}} = \langle n_{\text{in}}|n_{\text{out}} \rangle^2 \langle n_{\text{out}}|\rho|n_{\text{out}} \rangle \]

### Fluctuation relation: \( s_{\text{in},\text{out}} := -\log \langle n_{\text{out}}|\rho|n_{\text{out}} \rangle + \log \langle n_{\text{in}}|\rho|n_{\text{in}} \rangle \)

\[ p_{\text{in},\text{out}} = p_{\text{out},\text{in}} e^{s_{\text{in},\text{out}}} \]

### Forward process

\[ P_{\text{in},\text{out}}(s) = \sum_{n_{\text{in}},n_{\text{out}}} p_{\text{in},\text{out}} \delta(s - s_{\text{in},\text{out}}) \]

### Backward process

\[ P_{\text{out},\text{in}}(s) = \sum_{n_{\text{in}},n_{\text{out}}} p_{\text{out},\text{in}} \delta(s - s_{\text{out},\text{in}}) \]
## Entropy and inner friction

### Forward process

$$p_{\text{in,out}} = |\langle n_{\text{out}}|n_{\text{in}}\rangle|^2 \langle n_{\text{in}}|\rho|n_{\text{in}}\rangle$$

### Backward process

$$p_{\text{out,in}} = |\langle n_{\text{in}}|n_{\text{out}}\rangle|^2 \langle n_{\text{out}}|\rho|n_{\text{out}}\rangle$$

### Fluctuation relation: $s_{\text{in,out}} := -\log \langle n_{\text{out}}|\rho|n_{\text{out}}\rangle + \log \langle n_{\text{in}}|\rho|n_{\text{in}}\rangle$

$$p_{\text{in,out}} = p_{\text{out,in}} e^{s_{\text{in,out}}}$$

### Forward process

$$P_{\text{in,out}}(s) = \sum_{n_{\text{in}},n_{\text{out}}} p_{\text{in,out}} \delta(s - s_{\text{in,out}})$$

### Backward process

$$P_{\text{out,in}}(s) = \sum_{n_{\text{in}},n_{\text{out}}} p_{\text{out,in}} \delta(s - s_{\text{out,in}})$$

### Average entropy: $K(X||Y) := -\sum_n p_x(n) \left[ \log p_y(n) - \log p_x(n) \right] \geq 0$

$$s = K(P_{\text{in,out}}||P_{\text{out,in}})$$
### Results

#### Initial state

\[ \rho = \frac{e^{-\beta_{in} H_{in}}}{\mathcal{Z}} \]

#### Entropy

\[ s_{in, out} = \frac{\omega_{in}}{T} n_c \]
## Results

### Initial state

\[
\rho = \frac{e^{-\beta_{in} H_{in}}}{\mathcal{Z}}
\]

### Entropy

\[
s_{\text{in,out}} = \frac{\omega_{in}}{T} n_c
\]

### Final result

\[
s = \frac{\omega_{in}}{T} n_c
\]

This is our main result.

### Extendable result

Number of created particles \( n_c = \sinh^2 r \). This result can be extended to:

- Unruh effect: \( \tanh r = \exp\left[ \frac{\hbar \omega}{k_B T_U} \right] \);
- Schwarschild black hole: \( \tanh r = \exp\left[ \frac{\hbar \omega}{k_B T_H} \right] \);
- Analogue gravity models.
Conclusions and outlook

Conclusions

* We have studied applications of quantum thermodynamics to setups that appear in quantum field theory;
* Have found some entropic quantity that increases in (simple) cosmological processes;
* The results apply to different cosmological and quantum field theoretical scenarios.

Outlook

* Can teach us more about the physics at the overlap of relativity and quantum mechanics;
* Drive future theoretical efforts to uncover novel physics;
* **Hopefully**: provide energy balance relations in quantum field theoretical scenarios.
Thank You.