Discriminating Chaotic and Stochastic Dynamics in an Optoelectronic Oscillator with Delayed Feedback

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Delay dynamics accounts for a wide variety of phenomena in life sciences, physics and technology, chemistry and economics. Optoelectronic oscillators have become a model system to study delay dynamics and to utilize them for novel applications [1].

Our aim is to identify the time-scales at which either stochastic or chaotic dynamics dominate in an experimental optoelectronic feedback system. We address this critical issue by estimating two information-theory quantifiers, namely the permutation entropy and the permutation statistical complexity, of experimentally obtained time series as functions of the embedding delay τ of a certain symbolic reconstruction.

Deterministic chaotic time series share several properties with those generated by stochastic processes, e.g. a wide-band power spectrum, and/or a delta-like autocorrelation function. Therefore, they can be hard to distinguish in practical situations where they co-exist. Several works aimed at elucidating the deterministic or random nature of a time series justify the use of the permutation entropy and statistical complexity to study the properties of chaotic time series [2]. Furthermore, the permutation entropy and statistical complexity are able to successfully identify the internal structures of time series originating from delay systems [3].

We present results on an experimental optoelectronic set-up, which performs a nonlinear delay dynamics of the Ikeda type [4]. The main dynamical features of this test-bed system have already been characterized, both from the theoretical and the experimental point of view [1]. Our experimental set-up can be seen in Figure 1(a).

Figure 1(b) illustrates the results of the permutation entropy (empty circles) and the statistical complexity (crosses) as a function of the embedding delay when the system is in the chaotic regime. Two extrema can be seen in this figure, one for each of the quantifiers. The boundary between non-deterministic and deterministic dynamics is given by the position of these extrema. For large values of the permutation entropy and small ones of the statistical complexity the dynamics can be considered to be non-deterministic ($\tau = 0.10$ and 70-200 samples). Deterministic (chaos) dynamics dominate for intermediate values of the permutation entropy and the statistical complexity ($\tau = 10.70$ samples).



Fig. 1 (a) The experimental implementation includes a semiconductor laser diode, an integrated optics Mach–Zehnder modulator (performing a sine squared nonlinear transformation), a fiber delay line, and an optoelectronic feedback for the intensity detection, the linear filtering, and the amplification. This feedback serves as the drive of the MZ modulator, closing thus the delayed oscillation loop. (b) Permutation entropy (H_s empty circles) and statistical complexity (C_{js} crosses) analysis of an experimentally recorded chaotic time-trace as a function of the embedding delay of the symbolic reconstruction. Sampling rate of the temporal acquisition: 500MSamples/s. Embedding dimension D=6.

We discuss that characteristic time scales present in the system dynamics can be detected through the presence of clear extrema of the quantifiers when they are calculated as a function of the embedding delay. **References**

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