

Group Formation: Fragmentation Transitions in Network Coevolution Dynamics

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Dynamics of Networks:

1. Dynamics **OF** network formation: Structure created by individual choices/actions
2. Dynamics **ON** the network: Actions of individuals constrained by the social network

Rightwing view



Leftwing view



3. Co-evolution of agents and network :

Circumstances make men as much as men make circumstances

..new research agenda in which the structure of the network is no longer a given but a variable.....explore how a social structure might evolve in tandem with the collective action it makes possible (Macy, Am. J. Soc. 97, 808 (1991))

Final Goal: Understanding dynamical processes of group formation and social differentiation: Emergence of social dynamical networks with

- Social structure
- Weak links (Granovetter)
- Community structure

Review paper: *T. Gross and B. Blasius, J. R. Soc. Interface 5, 259 (2008)*

Key ingredients.

- a) Going beyond dynamical models in which:
 - Network evolution is decoupled from the evolution of agents actions
 - Complete network redefined at each time step
- b) Social plasticity as ratio of time scales of evolution of network and action



Generic result: Network fragmentation transition

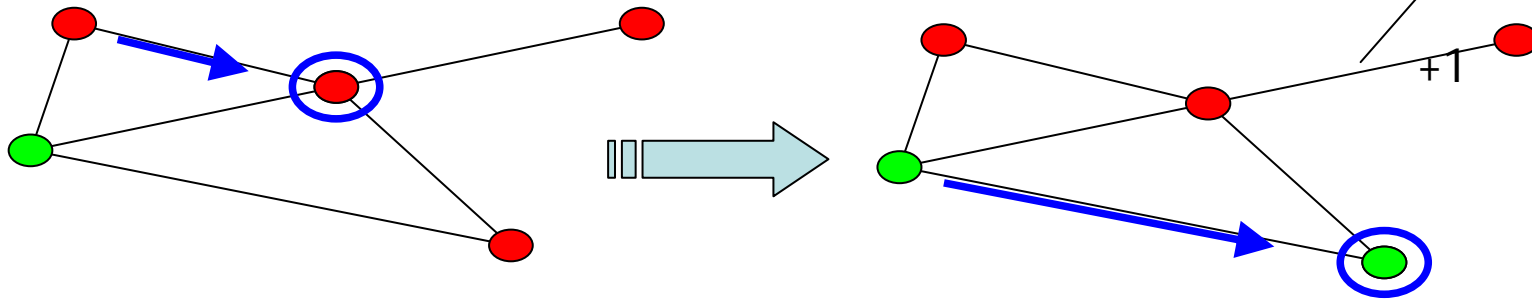
(Independent of link conservation, rewiring rule, interaction...)
Zachary's karate club

Two examples in model of consensus dynamics:

- * **Voter model:** Minimal model
F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. 100, 108702 (2008)
- * **Axelrod's cultural model:** Robustness of globalization-polarization transition
F. Vazquez et al. Physical Review E, 76, 046120 (1-5) (2007)
D. Centola et al. Journal of Conflict Resolution, 51, 905-929 (2007)

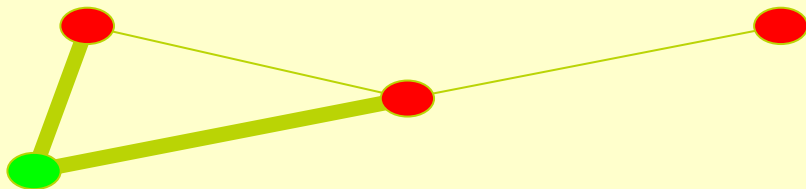
“Agents” located in the nodes of a network have a binary option $\sigma_i=1$ or $\sigma_i=-1$.

A randomly chosen voter takes the option of one of its neighbors: *Imitation process*



Qs?: When and how one of the two absorbing states (*consensus*) is reached? Effect of network of interactions?

Order Parameter: Average interface density (measure of *active links*)



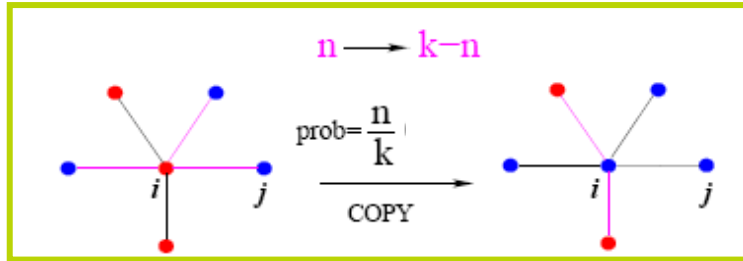
$$\rho = \frac{1}{2N\langle k \rangle} \left(1 - \sum_{i=1}^N \sum_{j \in \nu(i)} \sigma_i \sigma_j \right)$$

$\rho=0$ in absorbing state

Interface or active link: a link connecting nodes with different states.

* Mean Field Link Dynamics:

ρ = global density of active links
 n = active links
 $k-n$ = inert links
 k = degree of node i



$$\frac{d \langle \sigma \rangle}{dt} = 0$$

$$\Delta \rho = \frac{2(k - 2n)}{\langle k \rangle N}$$

Node i of degree k :

$$\left. \frac{d\rho}{dt} \right|_k = \frac{1}{1/N} \sum_{n=0}^k B(n, k) \frac{n}{k} \frac{2(k - 2n)}{\langle k \rangle N}$$

$B(n, k)$ = Prob. that node i has n active links

$$B(n, k) \approx \frac{k!}{n!(k-n)!} \rho^n (1-\rho)^{k-n} \quad \leftarrow \text{Mean Field: } \rho \sim \text{prob that a link from node } i \text{ is active}$$

$$\frac{d\rho}{dt} = \sum_k P_k \left. \frac{d\rho}{dt} \right|_k = \frac{2\rho}{\langle k \rangle} [(\langle k \rangle - 1)(1 - 2\rho) - 1]$$

$$\rho^s = \frac{\langle k \rangle - 2}{2(\langle k \rangle - 1)}$$

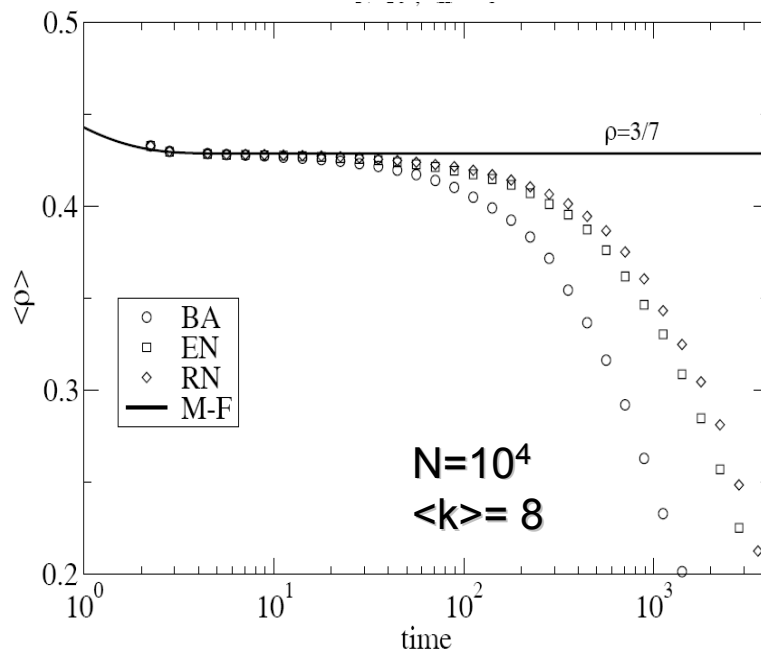
* Mean Field Node Dynamics: $\frac{d \langle \sigma \rangle}{dt} = 0$

Mean Field Link Dynamics:

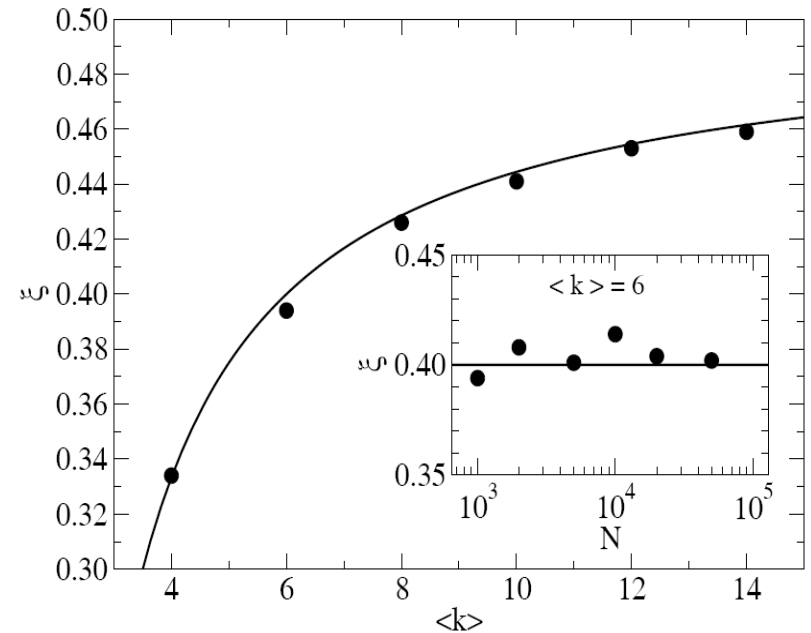
Single parameter theory

$$\rho^s = \xi = \frac{\langle k \rangle - 2}{2(\langle k \rangle - 1)}$$

Network topology independence



Barabasi-Albert Scale Free Networks



Initial: Degree-regular random graph with μ neighbors.

Nodes take state $S = -1$ or $S = +1$ with the same probability $1/2$.

1. Pick a node i and a neighbor j at random.

2. If $S_i = S_j$ nothing happens.

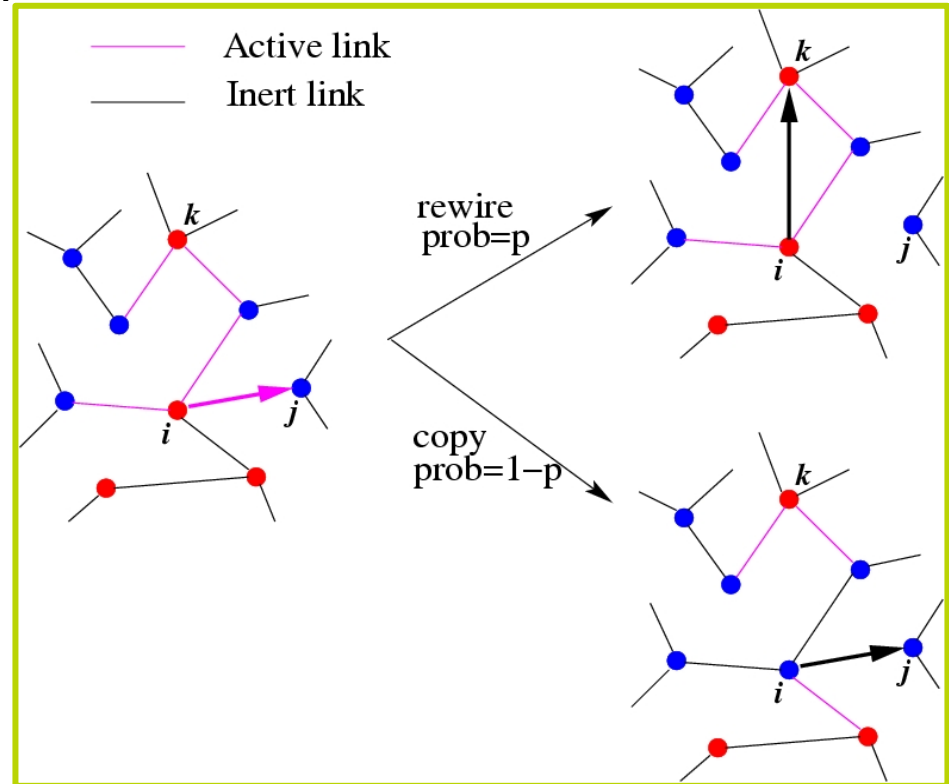
3. If $S_i \neq S_j$ then:

- Network dynamics: *rewire*
with probability p delete link $i-j$
and create link $i-k$ ($S_i = S_k$).
- State dynamics: *copy*
with probability $1-p$ set $S_i = S_j$.

4. Repeat ad infinitum.

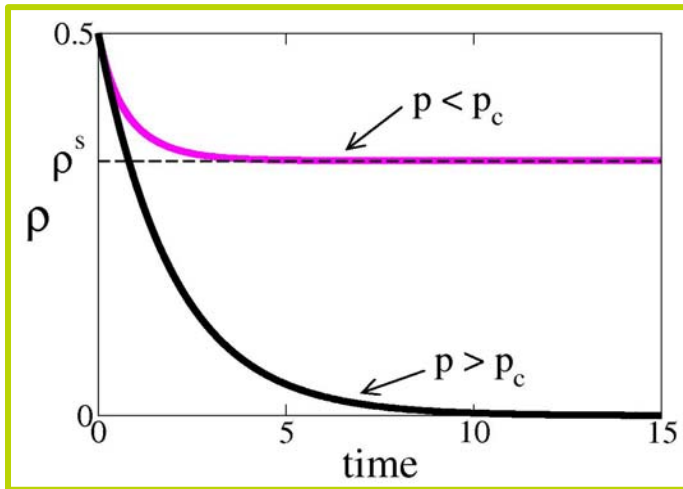
* Agents select interacting partner according to their state

* p gives a ratio of time scales of evolution of state of nodes and network



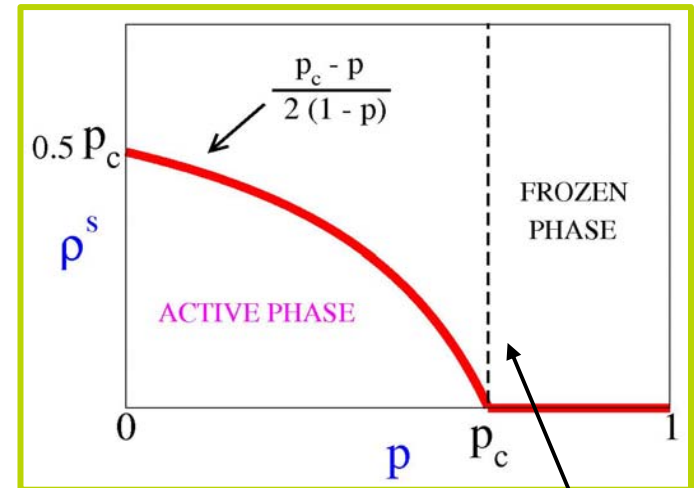
Master equation for the density of active links in the $N \rightarrow \infty$ limit:

$$\frac{d\rho}{dt} = \frac{2\rho}{\mu} [(1-p)(\mu-1)(1-2\rho) - 1]$$



Active - Frozen Transition at

$$p_c = \frac{\mu - 2}{\mu - 1}$$

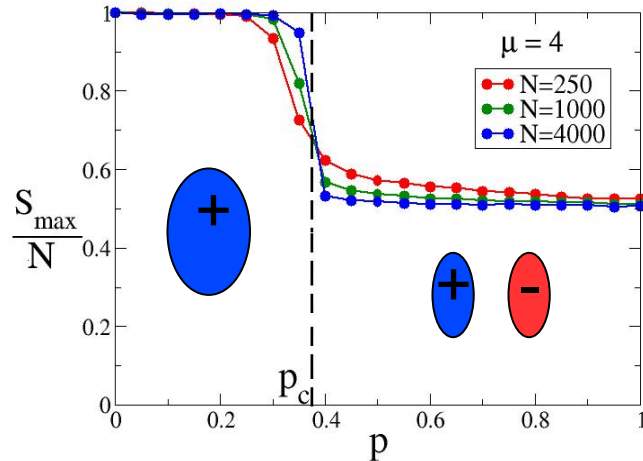


$$\rho_{++} = \rho_{--} = 1/2 \quad ?$$

- * **Active phase:** Links continuously being rewired and nodes flipping states
- * **Frozen phase:** Fixed network where connected nodes have the same state

Fragmentation Transition

Size of largest network component.



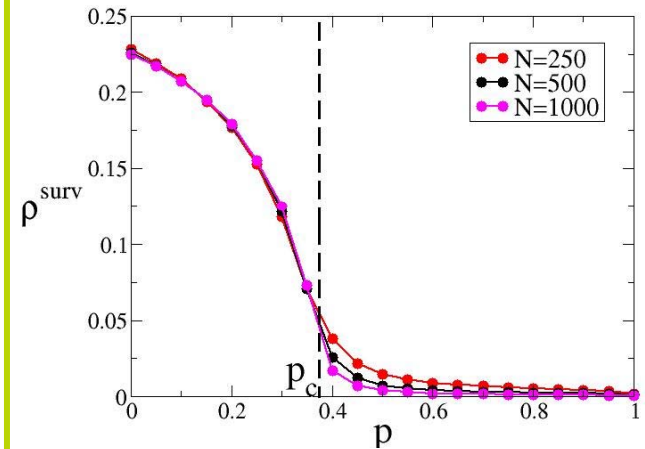
Active phase → Connected network ($S_{\max}/N = 1$)
($N = \infty$)

Frozen phase → Fragmented network ($S_{\max}/N \approx 0.5$)

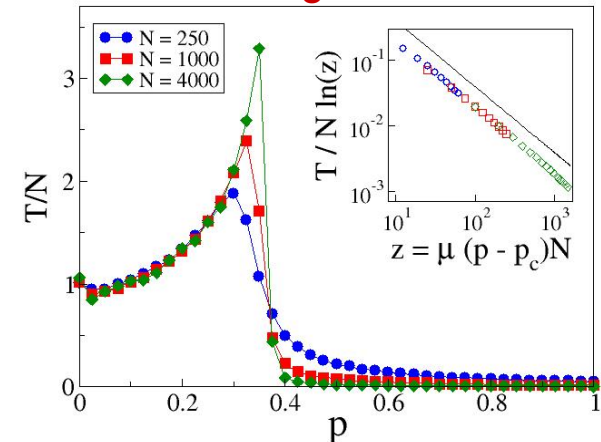
* $p < p_c$: slow rewiring keeps network connected until system fully orders and freezes in a single component.

* $p > p_c$: fast rewiring leads to fragmentation of network into two components before system reaches full order.

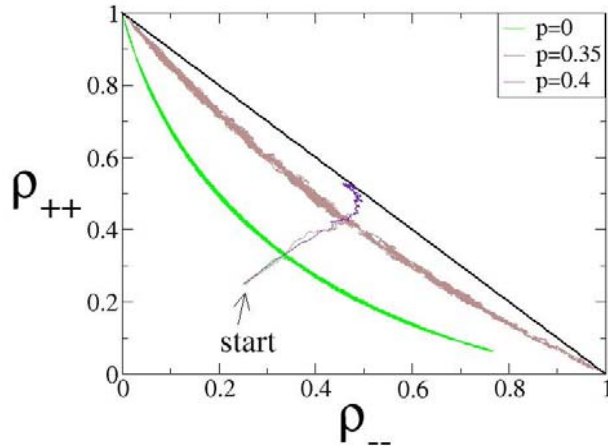
Active links in surviving runs.



Convergence times

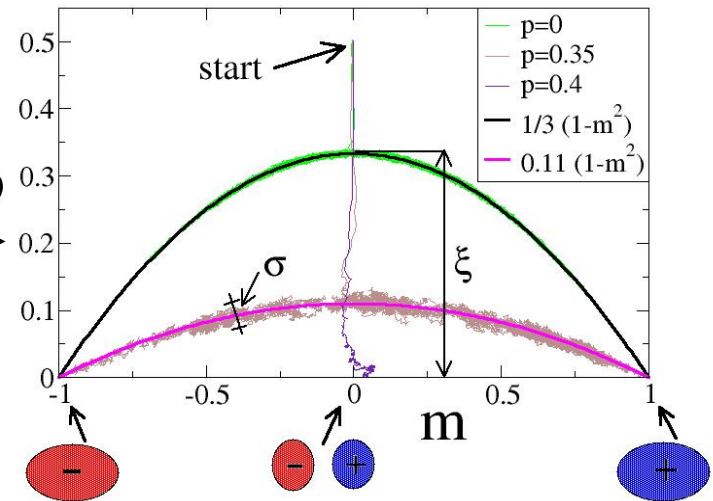


$$T \sim \begin{cases} N(p_c - p)^{-1} & \text{for } p \lesssim p_c \\ (p - p_c)^{-1} \ln[\mu(p - p_c)N] & \text{for } p \gtrsim p_c \end{cases}$$



$$m = \rho_{++} - \rho_{--} \quad \rho$$

$$\rho = 1 - \rho_{--} - \rho_{++}$$



absorbing points

$p < p_c : \rho \sim \xi(1 - m^2)$	\rightarrow	$m = -1, 1$	(one component)
$p > p_c : \rho \sim e^{-t/\tau}$	\rightarrow	$m = 0$	(two components)

$p < p_c$: **slow rewiring** keeps **network connected** until system fully orders and freezes in a single component.

$p > p_c$: **fast rewiring** leads to **fragmentation** of network into two components before system reaches full order.

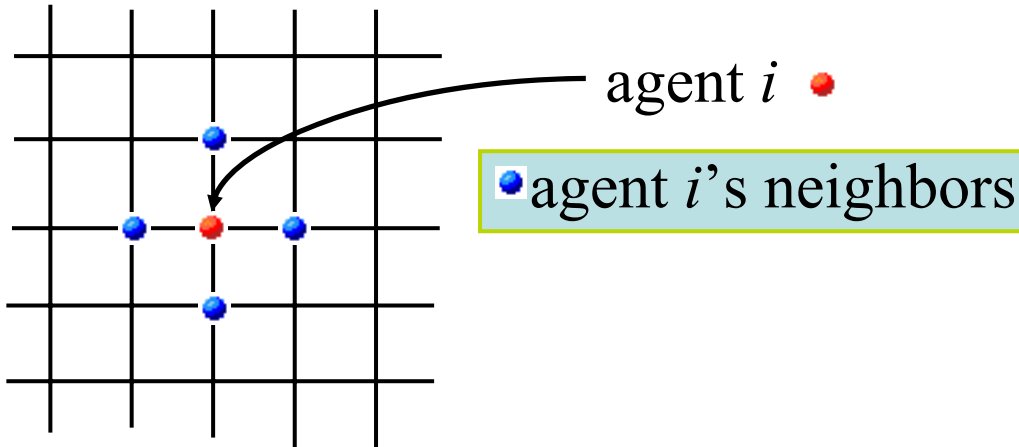
Proposal: Model to explore mechanisms of competition between *globalization* and persistence of *cultural diversity ("polarization")*

Definition of culture: Set of individual attributes subject to social influence

Principle of Homophily: Promotes interaction between similar.
"like attracts like"

Principle of Social Influence: Promotes cultural similarity. *The more two interact the more similar they become.*

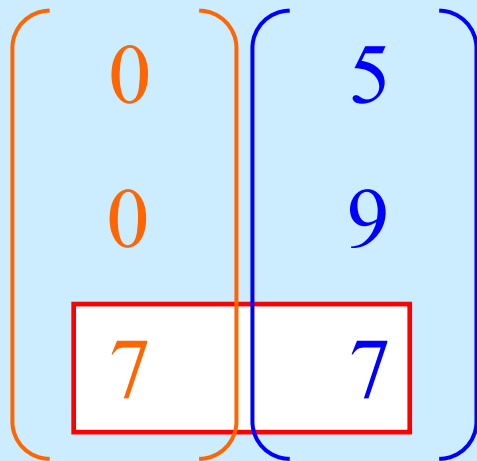
Axelrod's conclusion: Combination of homophily and social influence produces and sustains polarization (cultural diversity)



$$\begin{pmatrix} \sigma_{i1} \\ \sigma_{i2} \\ \vdots \\ \sigma_{iF} \end{pmatrix} \quad \begin{array}{l} F = \# \text{ Features} \\ q = \# \text{ Traits per} \\ \text{feature} \\ \sigma_{if} \in \{0, \dots, q-1\} \end{array}$$

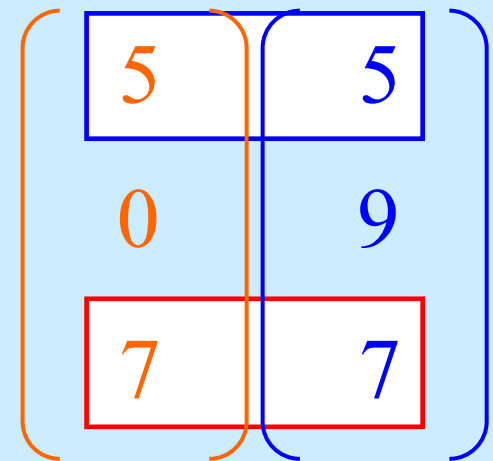
$F=3; q=10$

$q^F (10^3)$ equivalent cultural options.



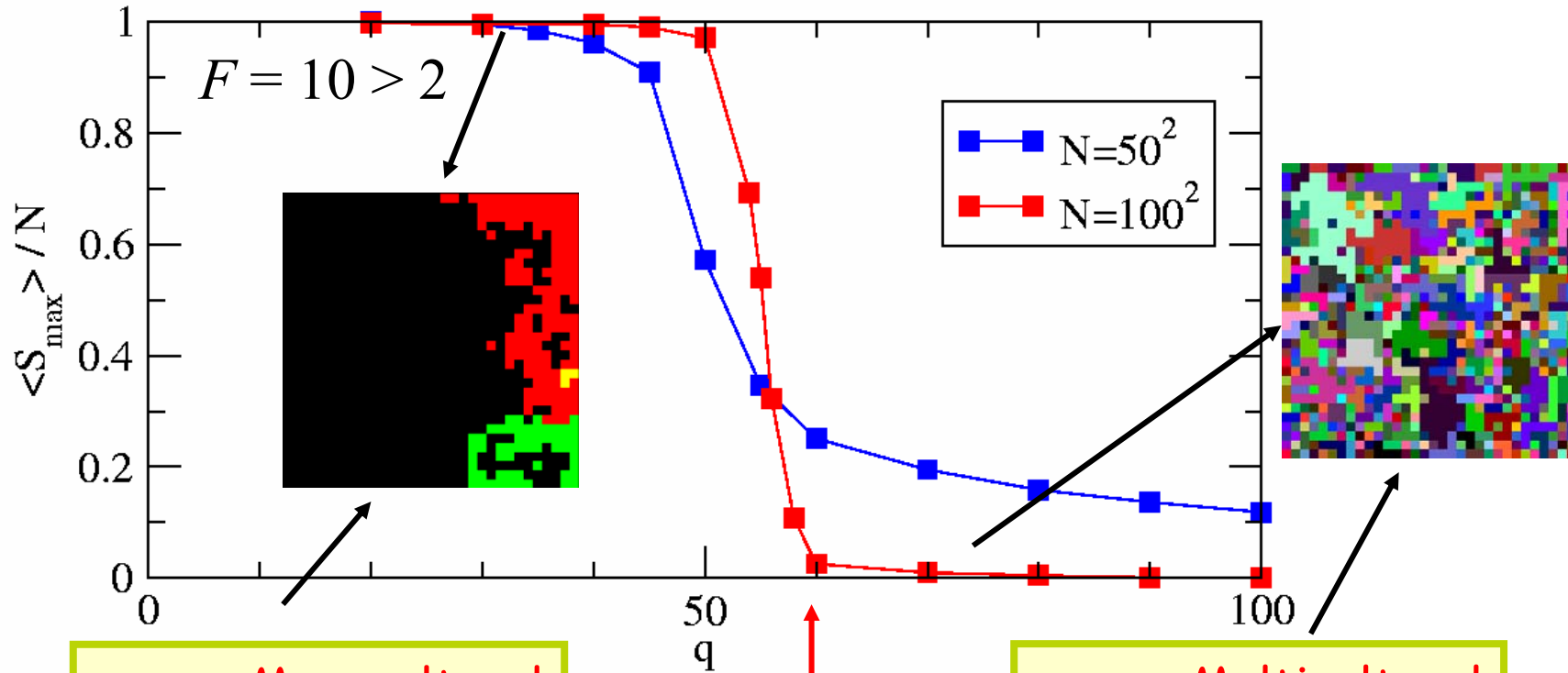
Mechanism of local convergence:

$$\begin{array}{l} \text{Prob to interact} = \\ \hline \frac{\text{Common features}}{F} = \frac{1}{3} \end{array}$$



- **Order parameter:** S_{\max} size of the largest homogeneous domain
- **Control parameter:** q measures initial degree of disorder.

Lewenstein et al (1992)

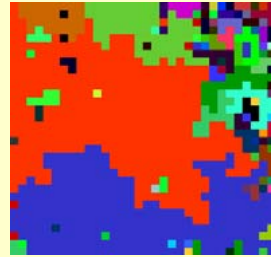
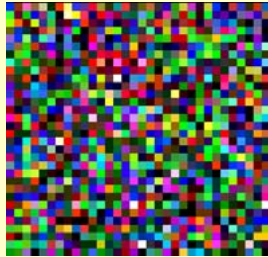


$q < q_c$: Monocultural
Global culture

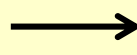
$q > q_c$: Multicultural
Cultural diversity
Global polarization

http://ifisc.uib.es/eng/lines/APPLET_Axelrod/Culture.html

Illustration of how **local convergence** can generate **global polarization**.



$t = 0$



System freezes in an absorbing multicultural state

➔ Frozen polarized states stable?

➔ Robustness of globalization-polarization transition?

* **Cultural drift:** “Perhaps the most interesting extension and at the same time, the most difficult one to analyze is cultural drift (modeled as spontaneous change in a trait).”

R. Axelrod, J. Conflict Res. (1997)

➔ Polarized states are not stable and cultural diversity is destroyed

Klemm et al., Phys Rev. E 67, 045101R (2003); J. Economic Dynamics and Control 29, 321 (2005)

* **Coevolution:**

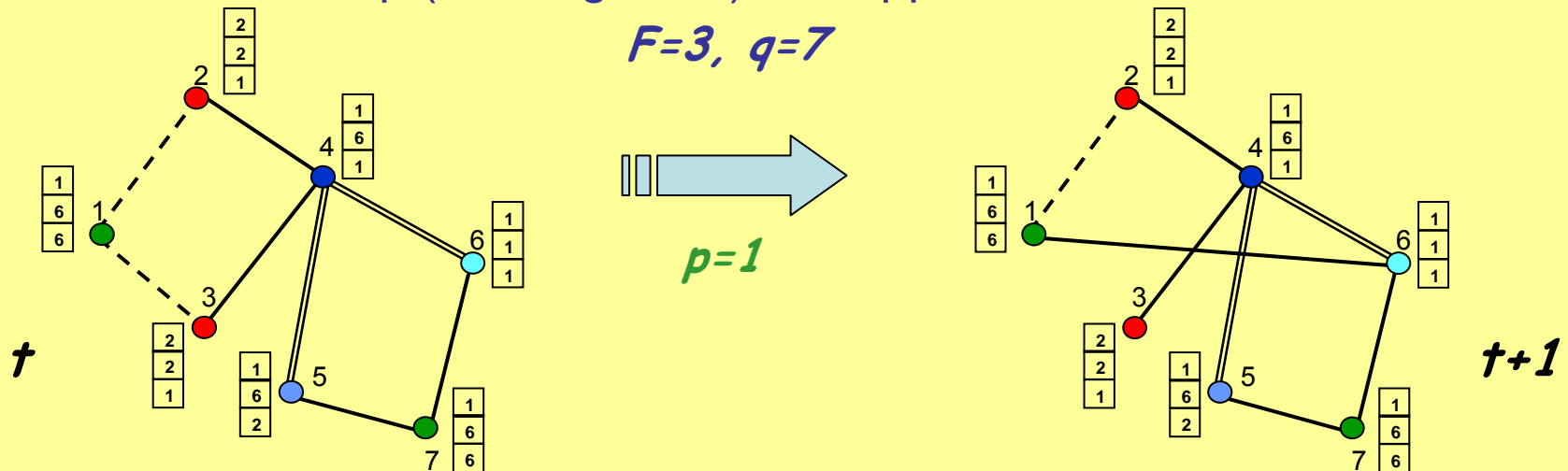
➔ New specification of homophily

➔ Transition robust. Culturally polarized states robust vs cultural drift

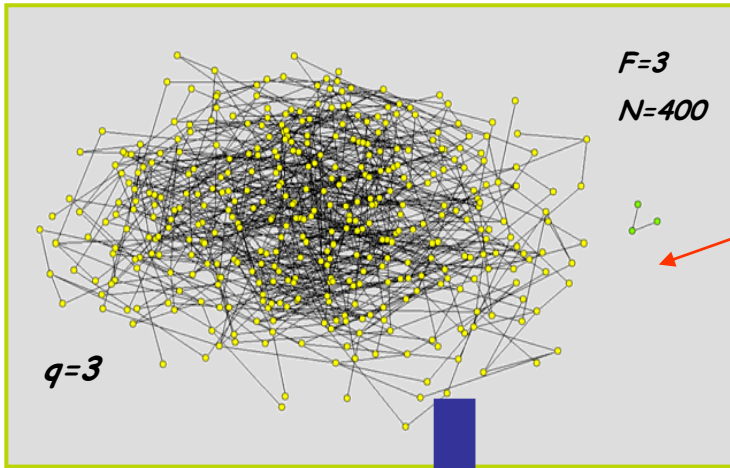
Step 1: Choose randomly a link connecting two agents and calculate the overlap (number of shared features). Probability of interaction is proportional to the overlap (if overlap is not maximum)

Step 2: Social influence dynamics: interaction results in one more common trait

Step 3: NETWORK DYNAMICS: New homophily specification
A link with zero overlap (cleavage-link) is dropped + new link established

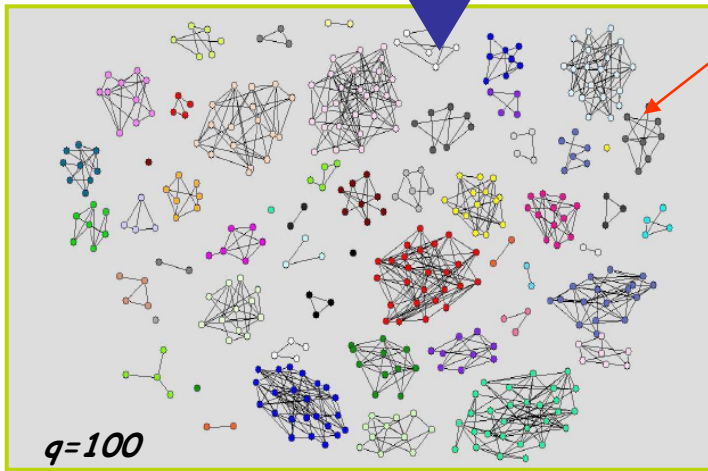


Region I (frozen configuration)



Fragmentation

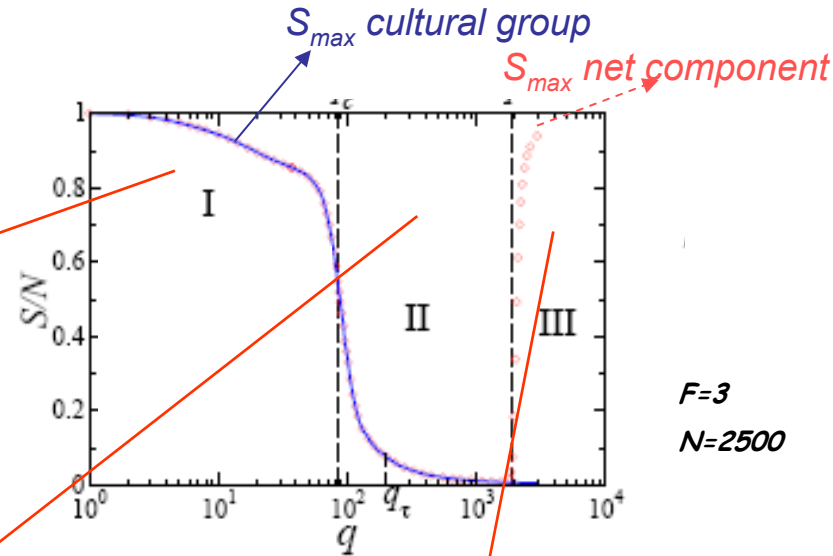
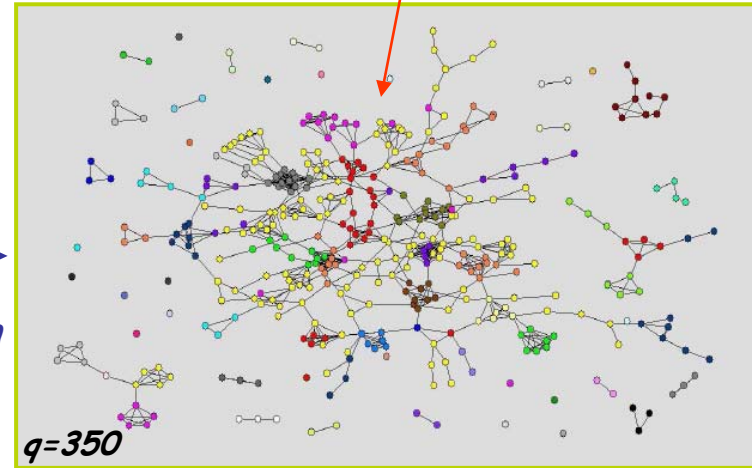
Region II (frozen)



Recombination

$$q^* \cong \frac{NF}{\langle k \rangle}$$

Region III (dynamic frustrated configuration)



Region I
giant network component

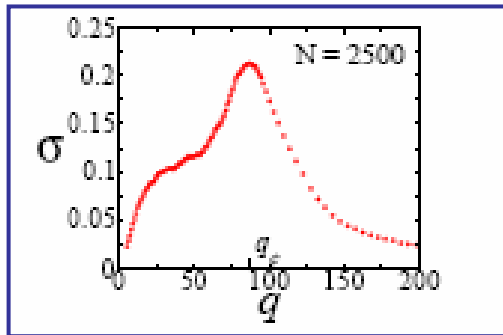


Region II
many small network components

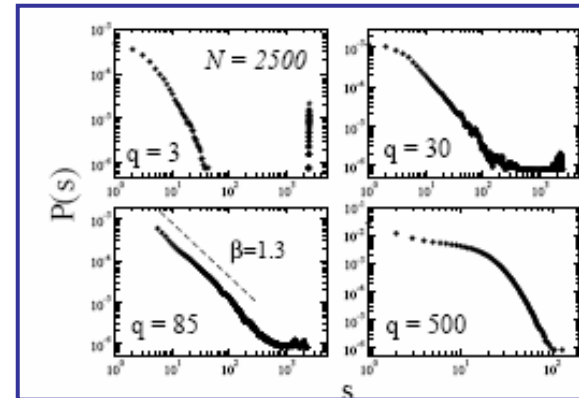
Maximum of fluctuation in S

Power law distribution for size components

$F=3$



$q_c' = 85$



$$P(s) \sim s^{-\beta}$$

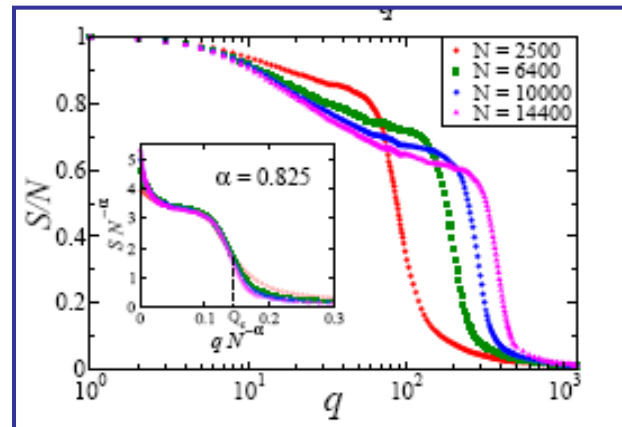
$$\beta = 1.3$$

Finite size scaling

$$S = N^\alpha f(N^{-\alpha} q) \text{ for } q < q_c.$$

$$q_c \sim N^\alpha \rightarrow \infty$$

$$S/N \sim N^{\alpha-1} \rightarrow 0 \text{ as } N \rightarrow \infty$$

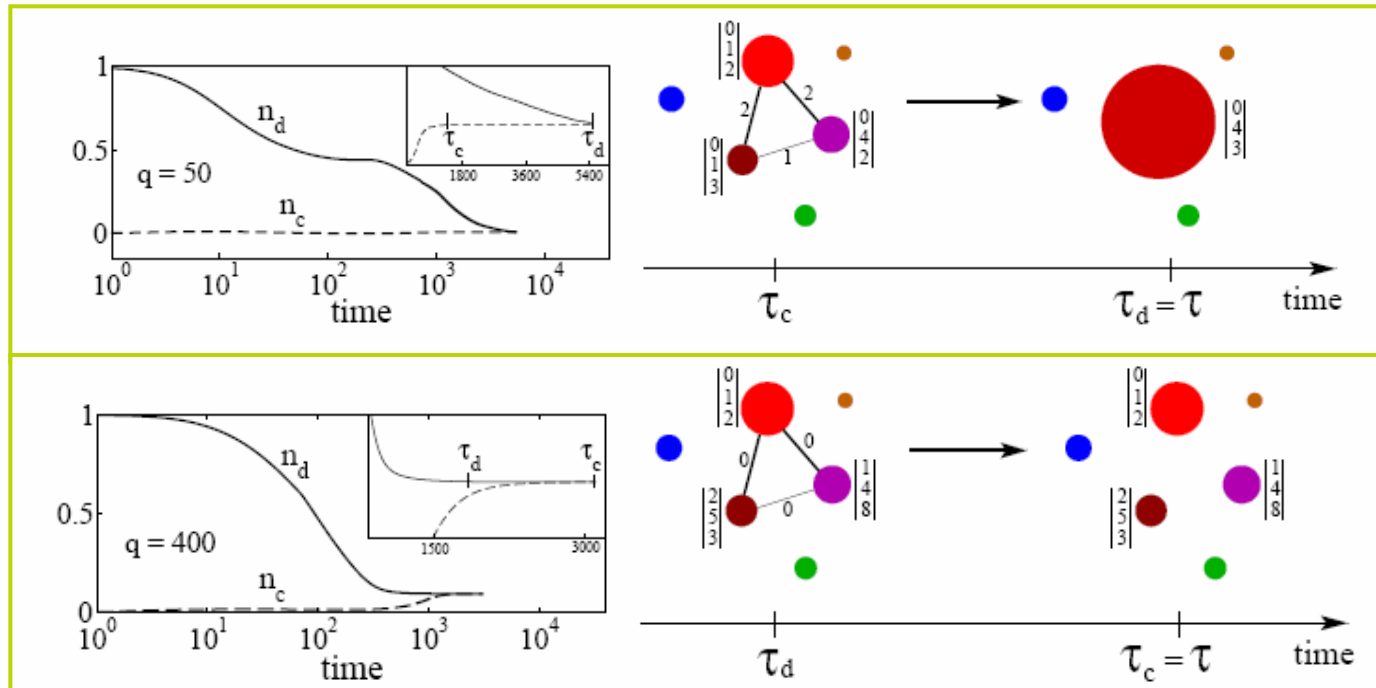


Transition becomes continuous and disappears in the limit $N \rightarrow \infty$

Two internal time scales (τ_c and τ_d) spontaneously emerge from a model in which states and network are updated at the same rate

$n_c(t)$ = # of network **components** /N

$n_d(t)$ = # of cultural **domains** /N



$\tau_d > \tau_c$

$\tau =$
freezing time
 $\text{Max}(\tau_d, \tau_c)$

$\tau_d < \tau_c$

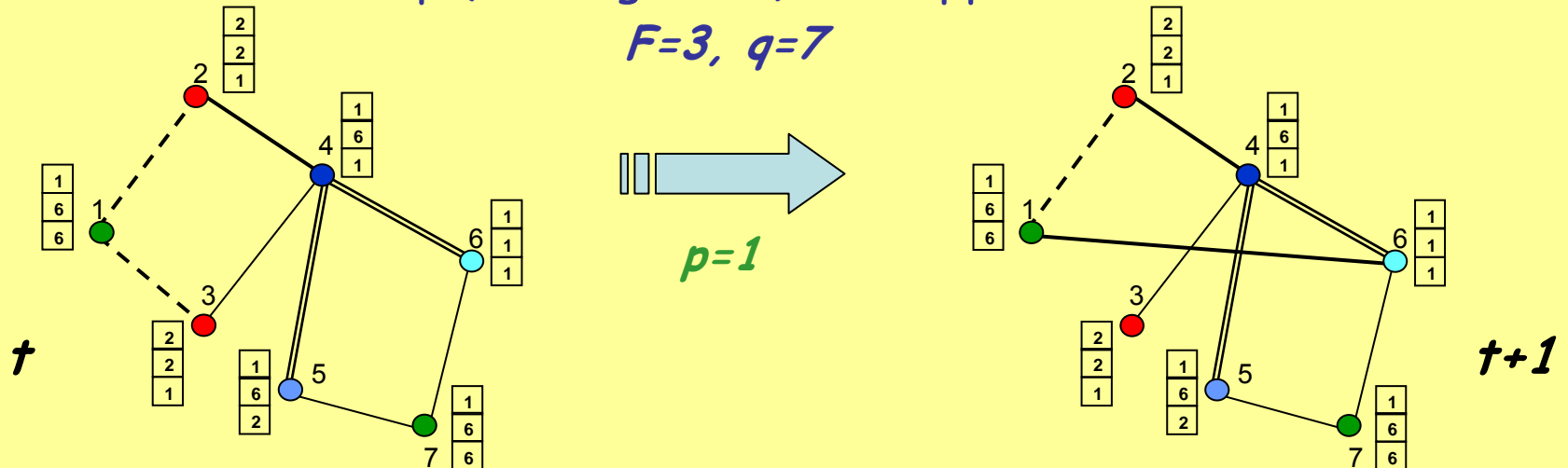
n_d reaches stationary value at τ_d \rightarrow Controls states (**domains**) formation
 n_c reaches stationary value at τ_c \rightarrow Controls network (**component**) formation

* Fragmentation transition occurs for $\tau_c \approx \tau_d$

Step 1: Choose randomly a link connecting two agents and calculate the overlap (number of shared features). Probability of interaction is proportional to the overlap (if overlap is not maximum)

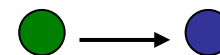
Step 2: **Social influence dynamics:** interaction results in one more common trait

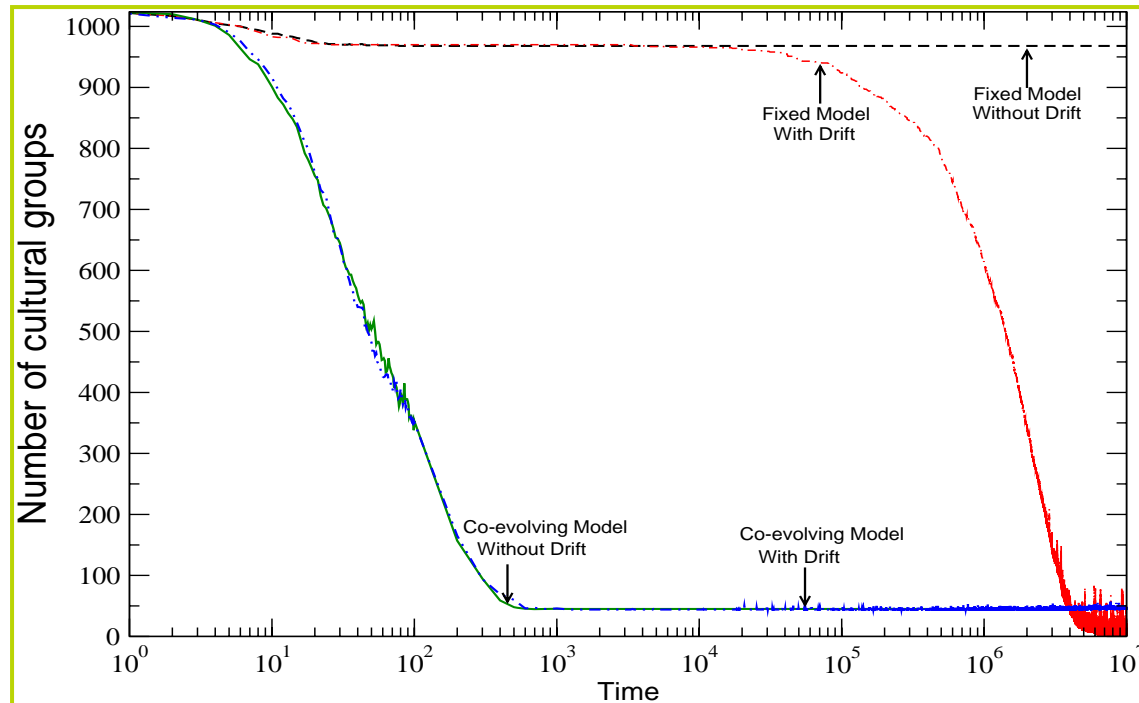
Step 3: NETWORK DYNAMICS: New homophily specification
A link with zero overlap (cleavage-link) is dropped + new link established



Step 4: Cultural drift:

Single feature perturbation with probability r





Region II
 $F=3, q=100$
 $N=1024$
 $r=10^{-5}$

- * Dynamical network maintains polarization in spite of cultural drift of slow rate: Insensitive to noise
- * Noise is not efficient to produce globalization in a co-evolving network during large time scales

• **Basics:** Interaction of several cultural features based on homophily and social influence produces a transition between global culture and polarization.

• **Fixed networks:** Long range links and degree heterogeneity favor globalization. High clustering restores polarization in scale free networks with large number of nodes.

Klemm et al., Phys. Rev. E 67, 026120 (2003)

• **Cultural drift in fixed networks:** Essential \longrightarrow Qualitative changes. q -independent, N -dependent noise induced transition between metastable global culture and noise dominated polarized state.

Klemm et al., Phys. Rev. E 67, 045101 (2003); J. Econ. Dyn. Control 29, 321(2005)

Co-evolution (Dynamic networks):

* Network Fragmentation and recombination transitions

F. Vázquez et al., Phys. Rev. E 76, 046120(2007)

* Stable cultural polarization: Cultural drift of slow rate becomes inefficient.

D. Centola et al. J. of Conflict Resolution 51, 905 (2007)

Thanks!



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