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Wealth distribution in modern and medieval societies

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Abstract. The power-law form of the upper part of the distribution of individual wealth/income (Pareto's law) is very well established for many countries and years. The Pareto index is however non-universal, varying typically from 1.5 to around 3. A recently introduced model for wealth exchange on an evolving family-network [Physica A **353**, 515 (2005)] is reviewed and compared with empirical data. While the model mimics very well recent individual wealth data in a modern society (U.K.), it fails to explain results for a feudal society, based on the number of serf families owned by nobles (Hungary, mid XVI century). The unusually low (around 1) Pareto index found in this case is not compatible with the previous model. It is suggested that this fact may be interpreted as a result of the absence of active trading among agents.

1 Introduction

Wealth distribution has been a topic of active research in recent years (two books, [1] and [2], have just come out on this subject!). Early empirical studies of the distribution of wealth/income in society were undertaken by the Italian economist Vilfredo Pareto at the end of the XIX century [3]. He analyzed 22 datasets from several (mainly European) countries and cities and concluded that the cumulative distribution of wealth $P_>(\omega)$ (probability to have wealth greater than a certain value) had always a power-law form, in the higher argument region,

$$P_>(\omega) = \int_\omega^\infty P(\omega')\, d\omega' \sim \omega^{-\alpha} \tag{1.1}$$

with an exponent α around 1.5. This behavior is known as Pareto law and α as the Pareto index. Recent economic data, for a variety of capitalistic countries from four continents and spanning a period of more than thirty years (see references in [1] and [2]), largely confirm Pareto's functional form for the upper part of the distribution, with a non-universal exponent $1 < \alpha < 3$, while the low and medium income ranges are found to follow another law – exponential or lognormal. Most of the available data about personal richness comes from individual income tax declarations, so it actually refers to *income* rather than *wealth*. In contrast, Forbes Magazine rankings of the richest people are based on their total wealth (fortune). Inheritance tax also provides information on the wealth (of a deceased person).

Detailed economic data for earlier times in history is however scarce. Finding an appropriate measure of wealth for a specific historic/geographical context is the first task to be considered in such studies. The area of houses, for instance, was used in ref. [4] to study a community in ancient Egypt. In the present work, we present and discuss some recent results of wealth distribution in a medieval society – the Hungarian aristocratic society around the year 1550 [5]. In medieval times rich people (nobles) were land owners who had serfs (villeins) at their service. The wealth of the Hungarian nobles is expressed here in the number of owned serf families, which is a good measure of the size and potential of the owned lands.

A variety of statistical models of wealth exchange between agents have been introduced in order to explain the empirical studies [2]. In most of those works, one assumes the agents live on a predefined interaction network [6], exchanging wealth with their neighbors, according to some stochastic rule. Recently, a model was proposed [7] where the network structure is not fixed a-priori but rather dynamically coupled to the wealth-exchange rule. This social/economical network evolves as a consequence of the wealth-transfer mechanism and one may study its statistical and topological properties. This model successfully reproduces both the characteristics of the modern family relation networks and the measured shape of the wealth distribution curves. It is however unable to account for the unusually low value of α found for the medieval society under study [5]. Possible reasons for this discrepancy are examined.

2 Family network model

This model was designed to simultaneously reproduce the observed wealth distribution in a society and the main characteristics of the social network of first-degree family relations. The main mechanisms for wealth exchange are *inheritance* and *social costs* associated with raising a new family. Families are represented by nodes and a first-degree (parent/child) family relation is a link. Node variables are wealth $\omega_i \geq 0$ and age a_i. The number of nodes N and the total wealth W are fixed. Initially, the nodes are randomly linked and uniformly distributed wealth values are assigned to them. The model has two parameters: q, the cost of raising a new family and p, the fraction of parents' wealth given to a new family/child as start-up money. The Monte Carlo dynamics has the following steps:

i) the oldest node *dies* – its links are removed and its wealth is distributed among its neighbors (*children* nodes)
ii) a new node *is born* and linked to two randomly chosen *parents* nodes, whose wealth is above a threshold q.
iii) wealth q is taken from each parent node and distributed among other nodes **preferentially according to respective wealth**; then a fraction p of the parents' remaining wealth is given to the new node.
iv) the age of all nodes is increased by 1.

One Monte Carlo step (MCS), or *generation*, corresponds to repeating i) to iv) N times. The redistribution of the amount $2q$ of wealth in step iii) models expenses (housing, food, clothes, etc) related to raising a new family. The preferential distribution rule was introduced to account for the fact that the richest members of society are more likely to benefit from these social costs, since they hold more business.

The model was simulated for several values of the parameters p and q. A stationary state was reached, independent of initial conditions and number of agents ($N > 5000$). For reasonable parameter values ($p \simeq 0.3$, $q \simeq 0.7-0.9$) the distribution $P_>(\omega)$ exhibits an exponential form for low/medium ω and a power-law dependence for the upper 5%–10%. The Pareto index depends on p and q and takes values in the 1.7–2.5 range. In fig. 1 the best results of the simulation are shown and compared with recent data obtained from inheritance tax in the U.K. (2001) [8]. The agreement is most satisfactory.

The statistical properties of the family network were also analyzed. The degree distribution decays exponentially for large values and the average connectivity varies between 1.8–1.9 (see

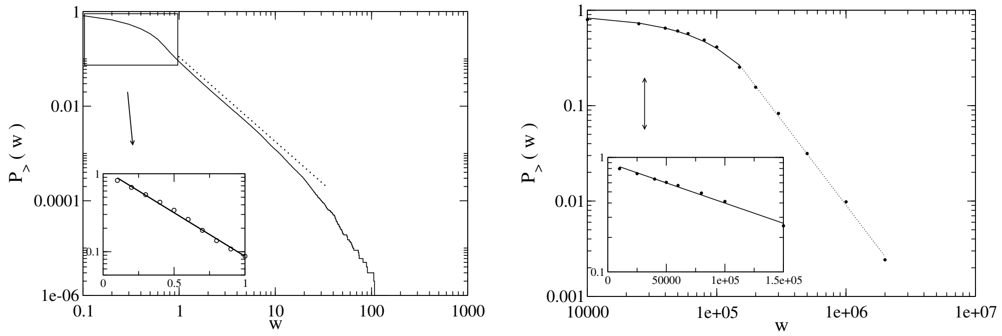

Fig. 1. Cumulative wealth distribution function. Left: model simulations for $p = 0.3$, $q = 0.7$, $N = 10000$. Right: wealth data for the population of U.K. in 2001 [8]. The tails yield exponents $\alpha = 1.80$ for the simulations and $\alpha = 1.78$ for U.K. data. The initial part of both curves is exponential, as shown in the insets.

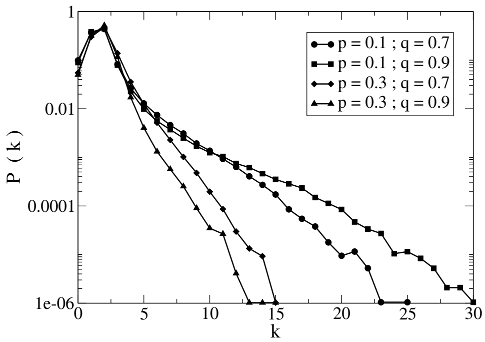

Fig. 2. Stationary degree distribution for various parameter values and $N = 10000$.

fig. 2). Those are reasonable characteristics for a first-degree (modern) family network. Other network properties, as well as interesting correlations between wealth and degree, were also revealed (see [7] for details).

The preferential mechanism of wealth distribution was found to be the key to get a Pareto law (assuming a uniform redistribution rule did not yield a power law tail). One limitation of the model is the unrealistic requirement of a minimum wealth (q) for reproduction, also unavoidable in order to obtain the observed form of the wealth distribution!

3 Wealth distribution in Hungarian medieval society

Economic life, barter and wealth exchange developed very slowly in Central-Eastern European aristocratic medieval societies. The aristocrat families were more or less self-supporting and no relevant barter existed. Hence, in model terms, the underlying social network and the wealth exchange on it are expected to have no major influence on the agents' wealth in such case and the no-trade limit of the standard wealth-exchange model of Bouchaud-Mezard [9] may be realized.

Taking the number of serf families living on a nobleman's land as a measure of his wealth, and disregarding those who had less than 10 serfs, a dataset was organized containing the top 1283 aristocratic families and 116 religious institutions in Hungary 1550 (see [5] for details). Of course, only the tail of the distribution of the wealth of the Hungarian population is obtained from this set. A convenient way to look for a Pareto law is to make a Zipf plot [10]: data is ranked according to decreasing wealth and log rank is plotted versus log wealth. This is shown in fig. 3, where the linear (Pareto) region is seen to extend for two decades, with a Pareto exponent $\alpha = 0.92$. This value is lower than the vast majority of currently reported values referring to personal wealth (a similar result was however found for India [11]). The family-network

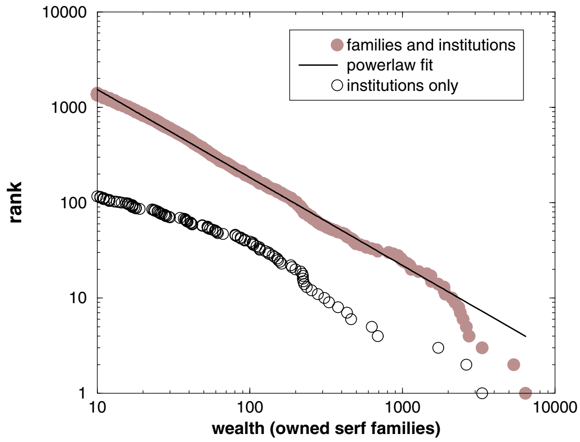

Fig. 3. The rank of the top hungarian aristocrat families and institutions as a function of their estimated total wealth on a log-log scale. Pareto index $\alpha = 0.92$.

model presented above does not predict such a low value, therefore it is not adequate to model the dynamics of wealth in the medieval society. This is consistent with the social/economic characteristics of the studied society, namely the absence of active (land/serfs) trading. Though a bit lower, $\alpha = 0.92$ is rather close to the prediction ($\alpha = 1$) of Bouchaud-Mezard model [9] in the case of independent agents. The random multiplicative model, developed to explain cities population distribution [12] may also be given a formulation, appropriate for the present case, in terms of the fluctuations of the wealth of independent individuals (see [5] for details).

There are other alternative ways to produce an exponent $\alpha = 1$ (or even lower) [13] but they seem less adequate to the present situation.

4 Summary and conclusion

A recently introduced wealth-exchange model, in which the wealth dynamics is coupled to the structure of the underlying social network, was reviewed. Though very simplistic, the model is able to generate a stationary wealth distribution in qualitative, and even quantitative, agreement with contemporary data, aside with realistic characteristics of the modern family-relations network. In the framework of this model, the preferential redistribution of wealth (a *riches get richer process*) is a necessary condition to obtain a Pareto-type distribution.

A study of wealth distribution in a feudal society was also presented. The tail of this distribution is a power-law with a rather small (around 1) Pareto index, which the previous model cannot explain. Such value is however consistent with the prediction of a random multiplicative model of wealth evolution for independent agents [9]. This no-trade condition is in agreement with the known absence of active economic life in the society under study. Alternative ways to produce an exponent $\alpha \leq 1$ [13] should also be further examined.

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