Large-scale transport in oceans

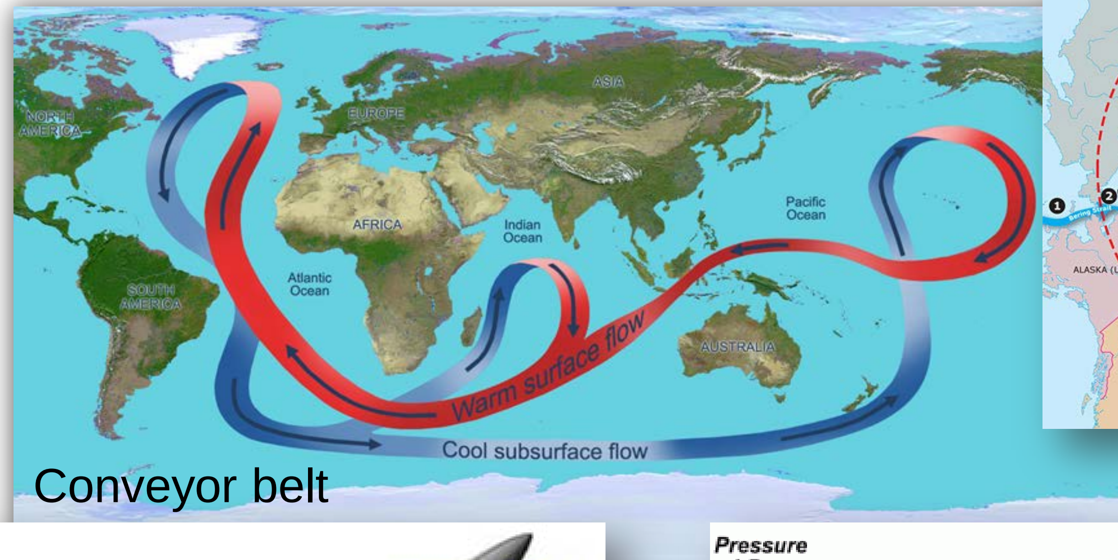
Emilio Hernández-García
IFISC (CSIC-UIB), Palma de Mallorca, Spain

Statistical Physics and Dynamical Systems approaches in Lagrangian Fluid Dynamics

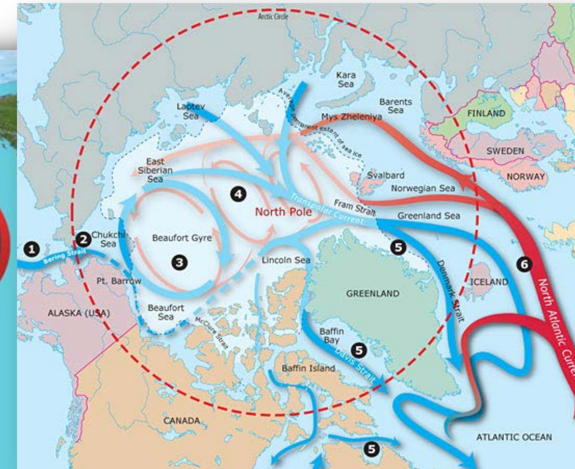
STATISTICAL PHYSICS AND DYNAMICAL SYSTEMS APPROACHES IN LAGRANGIAN FLUID DYNAMICS

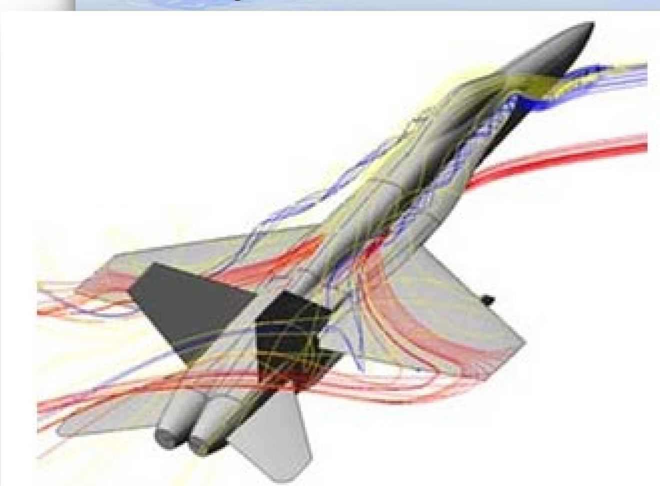
OUTLINE

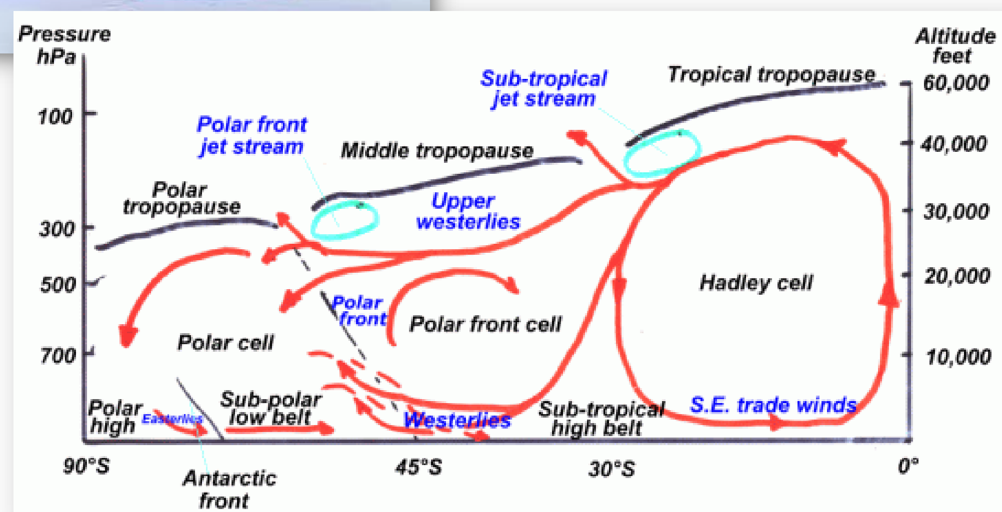
1. Lagrangian fluid dynamics and introduction to chaotic advection. Hamiltonian dynamics, KAM tori, Lyapunov exponents, open flows
2. Dispersion, diffusion and coherent structures in flows. Turbulent, pair and chaotic dispersion, gradient production, FTLE, FSLE, Lagrangian Coherent Structures
3. **Chemical and biological processes in flows.** Fisher and excitable plankton waves, filamental transitions, lamellar approaches, burning manifolds
4. Complex networks of fluid transport. Directed and weighted flow networks. Community detection



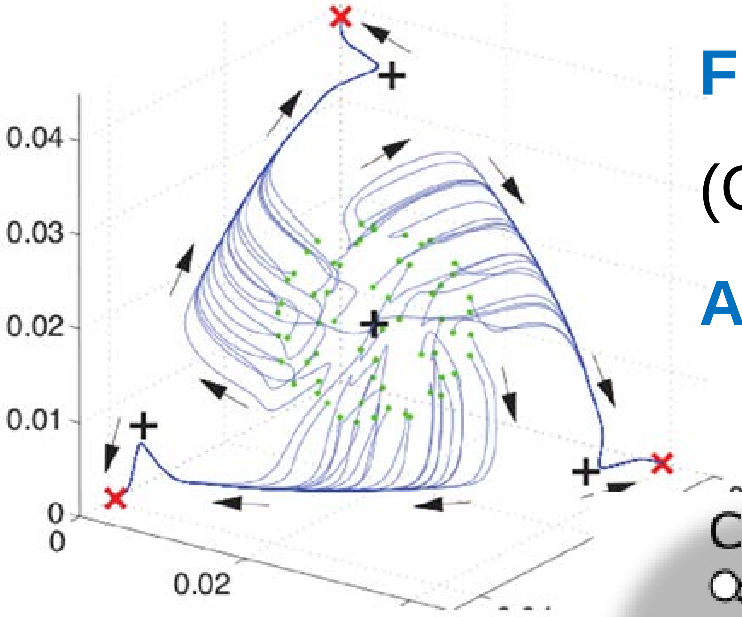
Conveyor belt







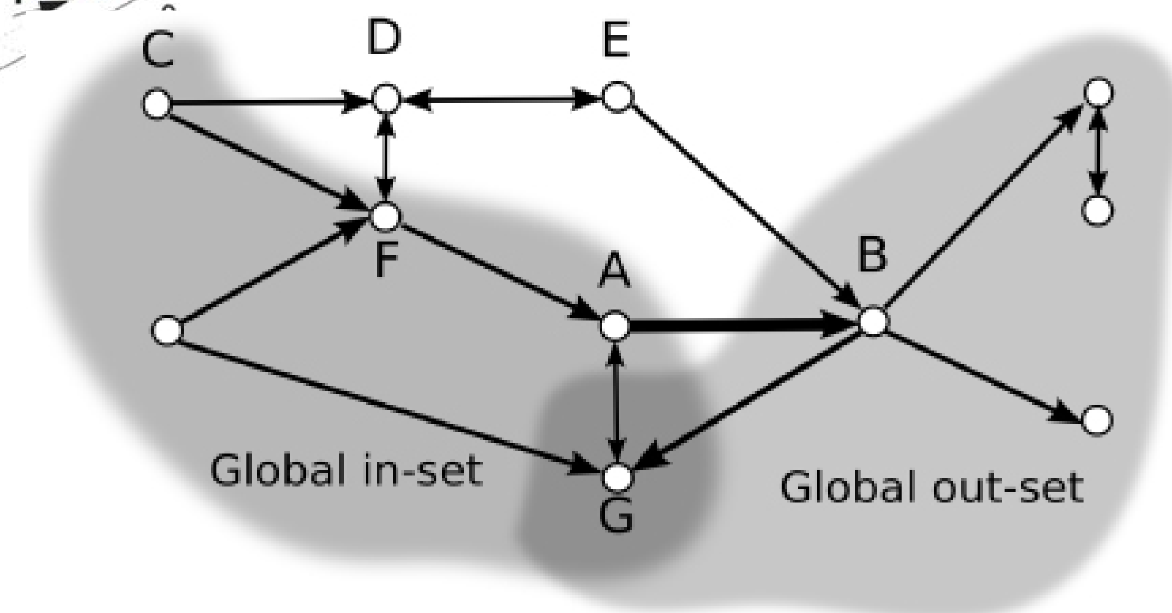
FLUID FLOWS: TRANSPORT OF WATER, AIR, MOMENTUM,HEAT, SUBSTANCES, …

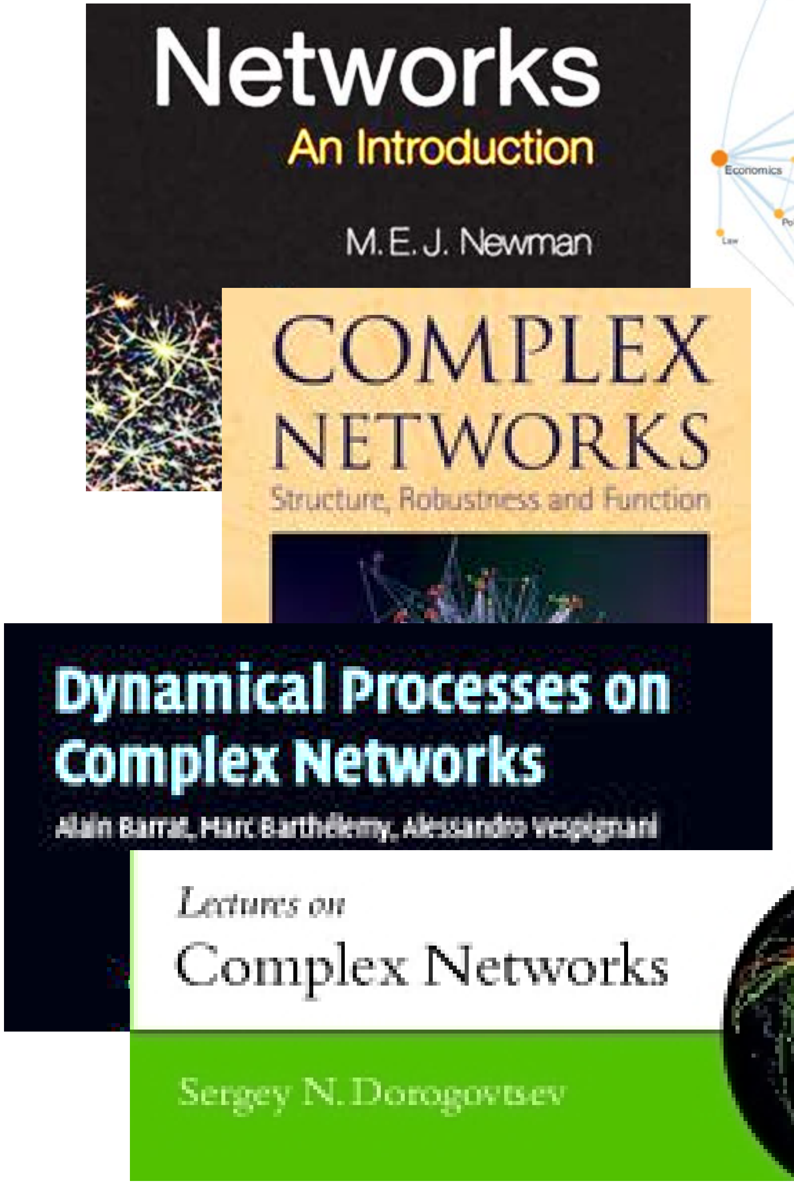


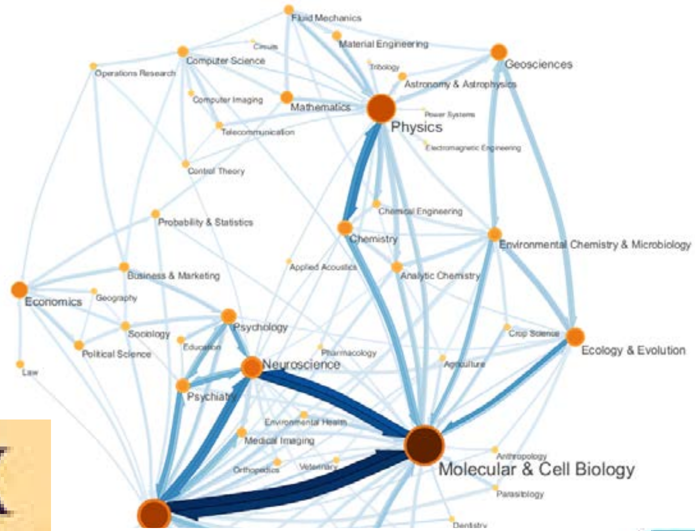
FLUID FLOW

(OR DYNAMICAL SYSTEMS FLOW)

AS TRANSPORT NETWORKS

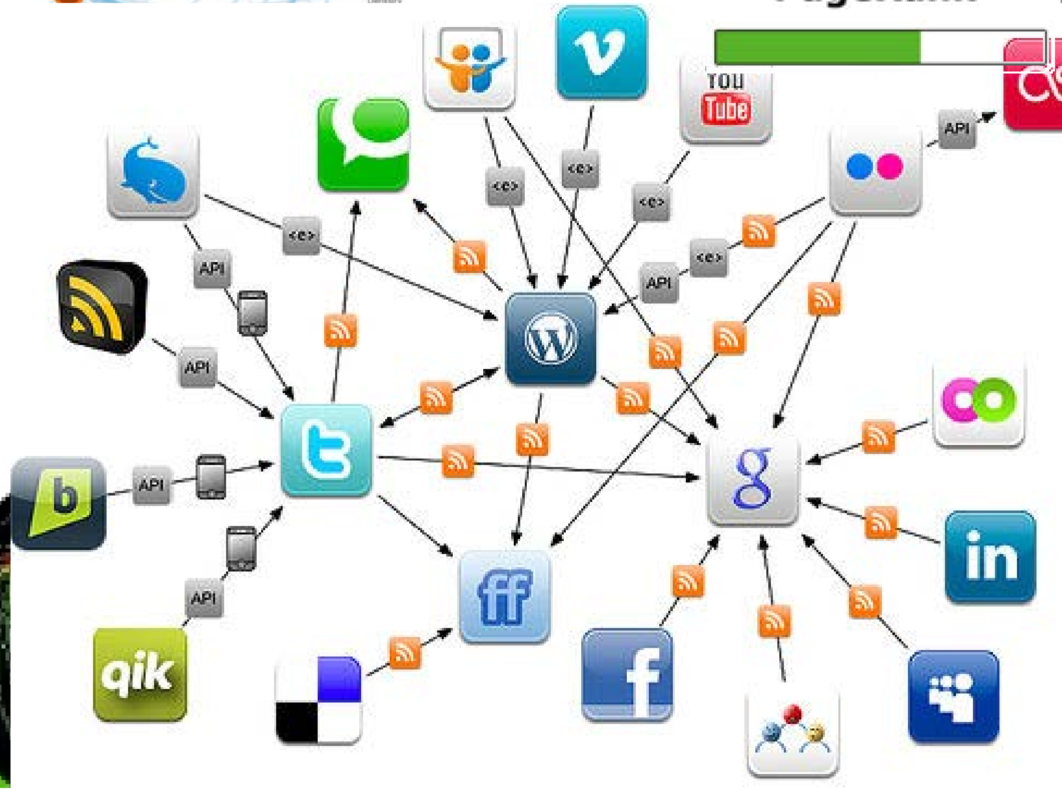



Networks
An Introduction
M. E. J. Newman
COMPLEX
NETWORKS
Structure, Robustness and Function
Dynamical Processes on
Complex Networks
Alain Barrat, Marc Barthélemy, Alessandro Vespignani
Lectures on
Complex Networks
Sergey N. Dorogovtsev


Physics
Geosciences
Mathematics
Computer Science
Chemistry
Environmental Chemistry & Microbiology
Economics
Psychology
Neuroscience
Ecology & Evolution
Molecular & Cell Biology


Big Data
Straight Ahead


Google
PageRank

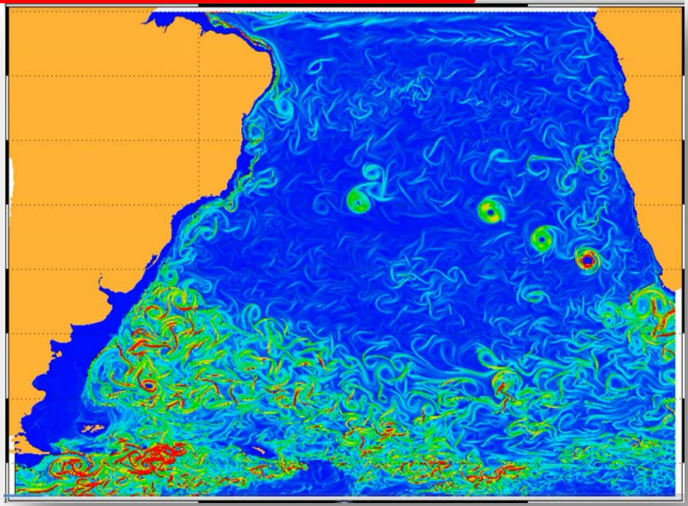


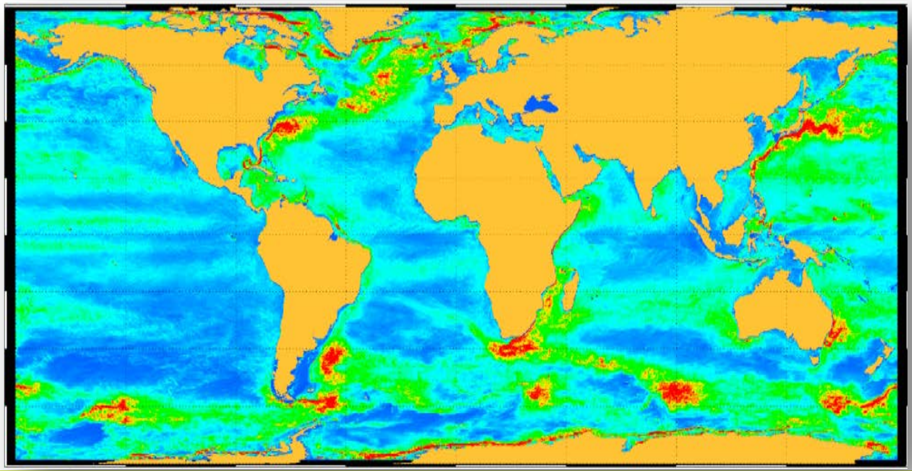
Lagrangian approaches to transport and mixing

- Geometric, local, … : FTLE, FSLE, geodesics, variational theory, M function, …
- Set-oriented, probabilistic ,…: Transfer operator, coherent sets, eigenvectors and singular vectors, …

- Detailed view of single events
- Statistical (climatological) descriptions

BIBLIOGRAPHY at `‘Resources’ for the School:` `www.gefenol.es/school2014/resources/`



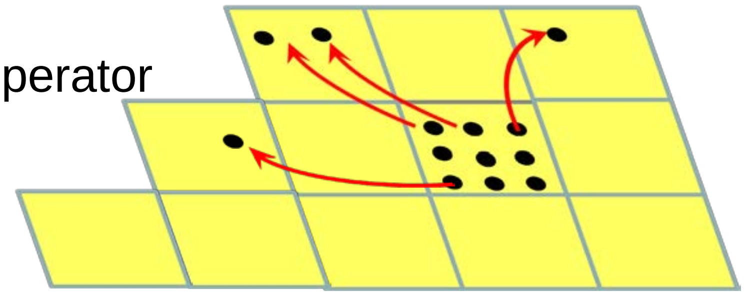


Single-time vs average FSLEs

Hernandez-Carrasco, Lopez, EHG, Turiel, (2012) JGR 117, C10007

Transfer or connectivity matrix

Discrete approximation to the Perron-Frobenius operator



P_{ij} = Prob(ending in j /starting in i)

Characterizes fluid flow from **INITIAL TIME** $\mathbf{t_0}$

to **FINAL TIME** $\mathbf{t_0+\tau}$

as acting on a discrete grid

Froyland et al. 2003, 2005

$$P(t_0,\tau)_{ij} = \frac{\#\text{ of tracers in box } i \text{ at time } t_0 \text{ going to box } j \text{ at time } t_0+\tau}{\#\text{ of tracers initially in box } i}$$

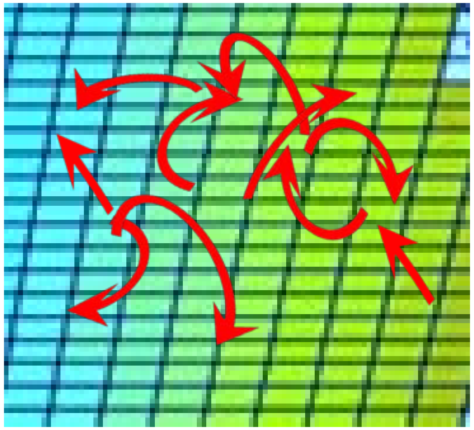
Network perspective

$P(t_0,\tau)$ → Adjacency Matrix

Box → Node

P_{ij} → Weight of link *i-j*

Weighted Directed Network



Example: How well-connected is the surface of the global ocean?

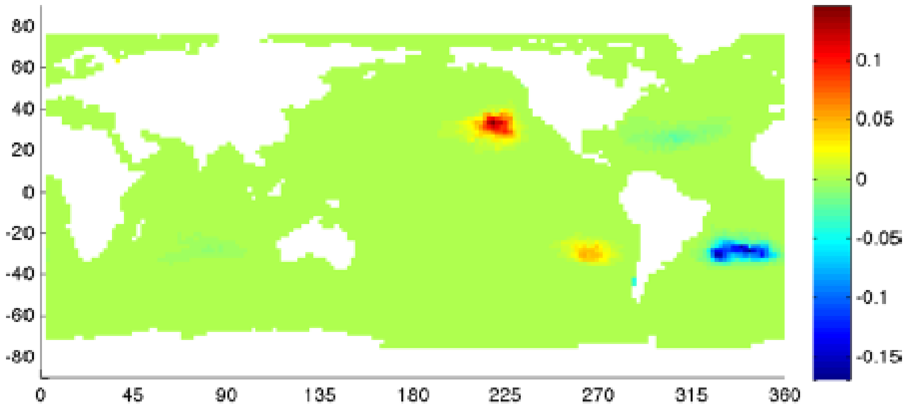
Froyland et al. Chaos 24, 033126 (2014)

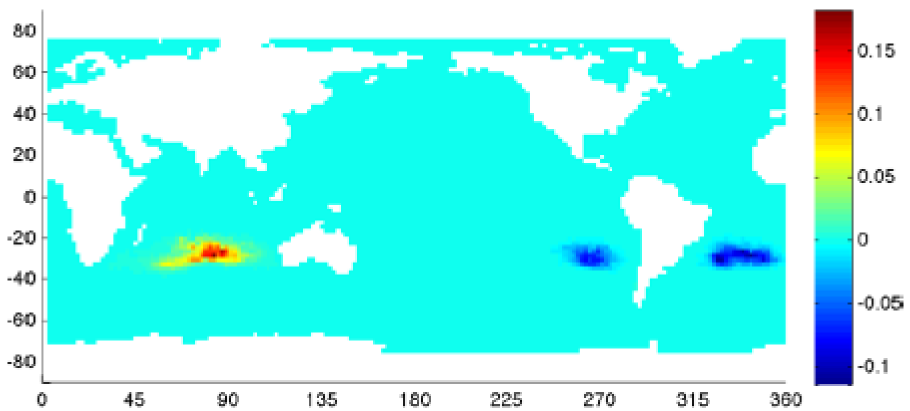
surface currents from OFES

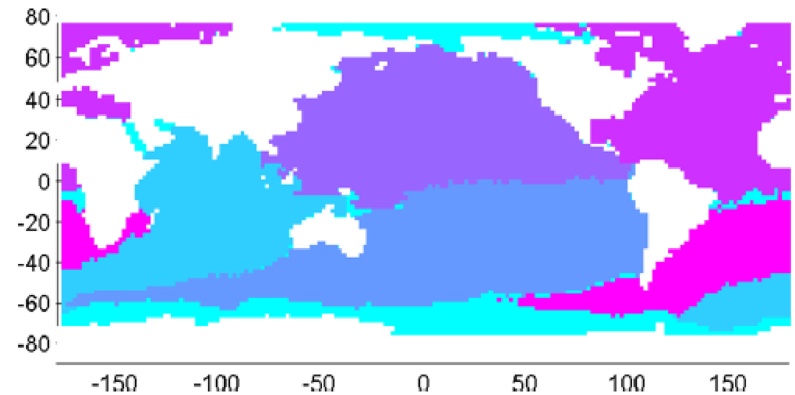
P_{ij}

Right eigenvectors: attractors

Left eigenvectors: basins of attraction







Relevant to garbage patches

Transport network in the Mediterrenean

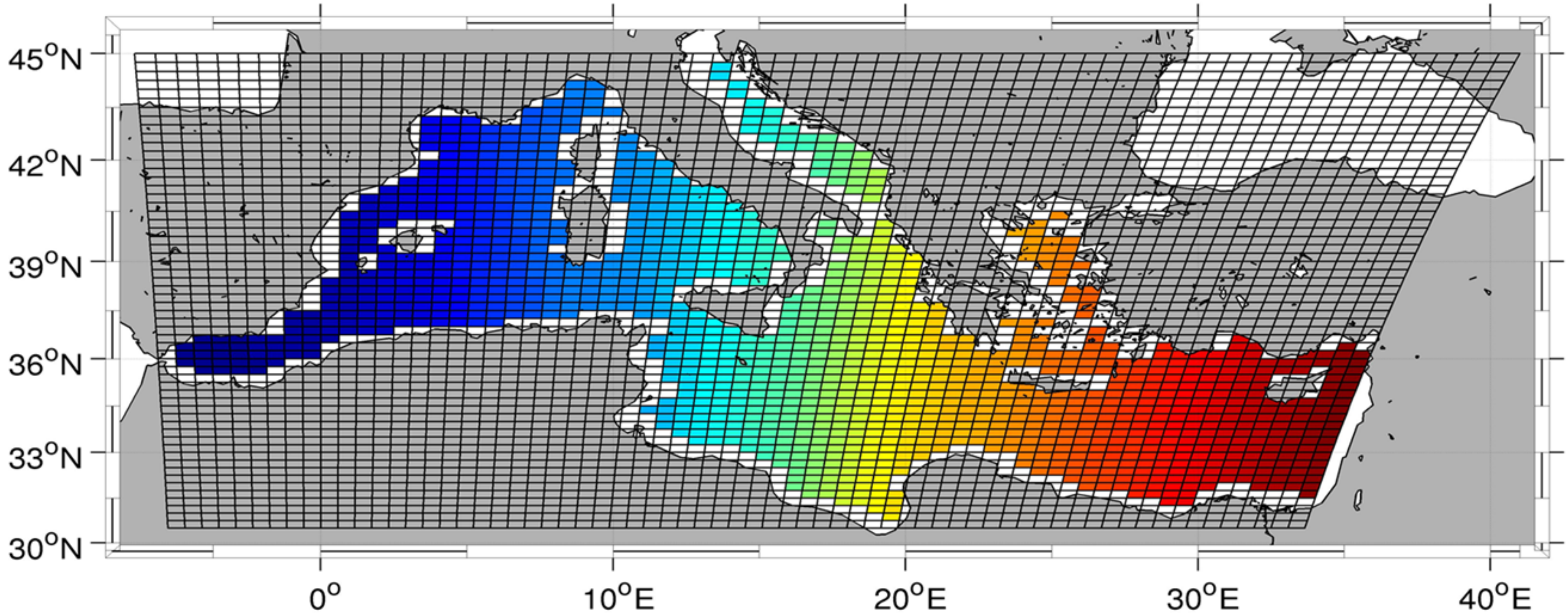
Rossi, Ser-Giacomi, Lopez, Hernandez-Garcia (2014), Geophys. Res. Lett. 41, 2883-2891
Ser-Giacomi, Rossi, Lopez, Hernandez-Garcia (2014)
http://ifisc.uib-csic.es/publications/publication-detail.php?indice=2556

Data

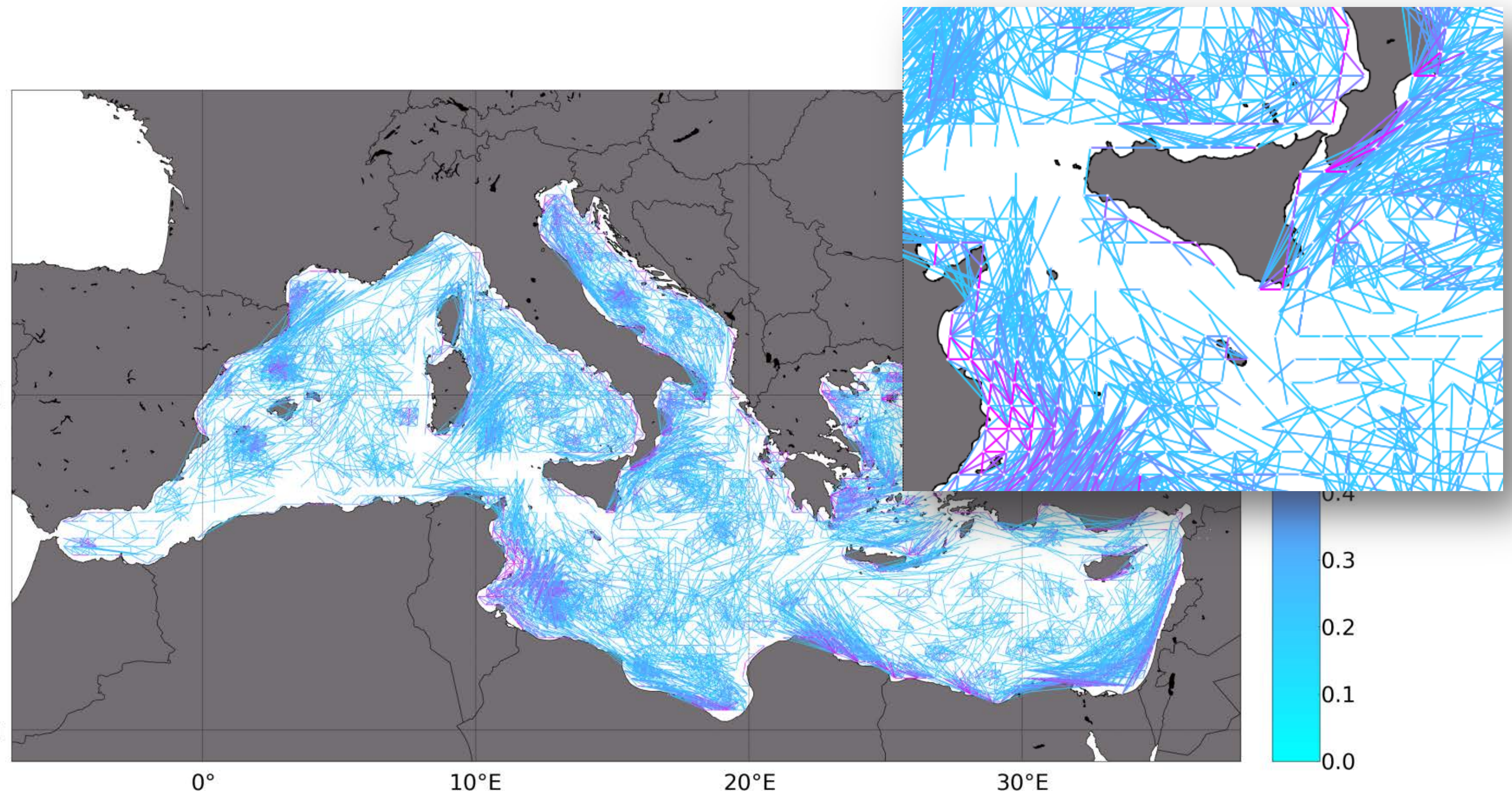
(1/16)˚ res, 8 m depth

Horizontal velocities from Eddy-resolving model NEMO (2002-2011).
Oddo et al 2009

- ECMWF wind fields
- assimilates satellite + ARGO data

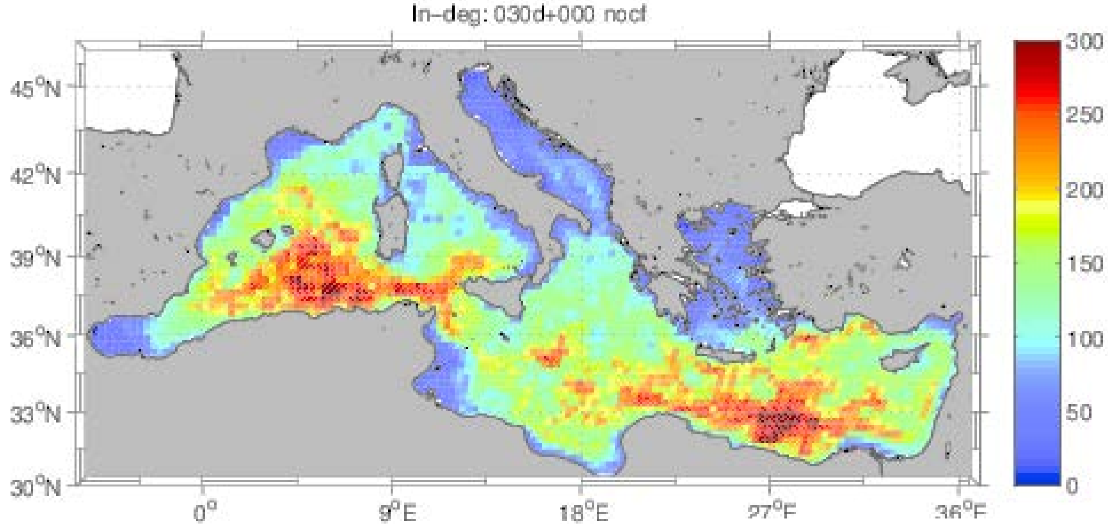


Network nodes: 3272 quasi-squares of (1/4)º (≈ 28 km) in an equal-area projection
500 particles released per node



start at time t_0 =July 1st . Integration time τ=30 days.

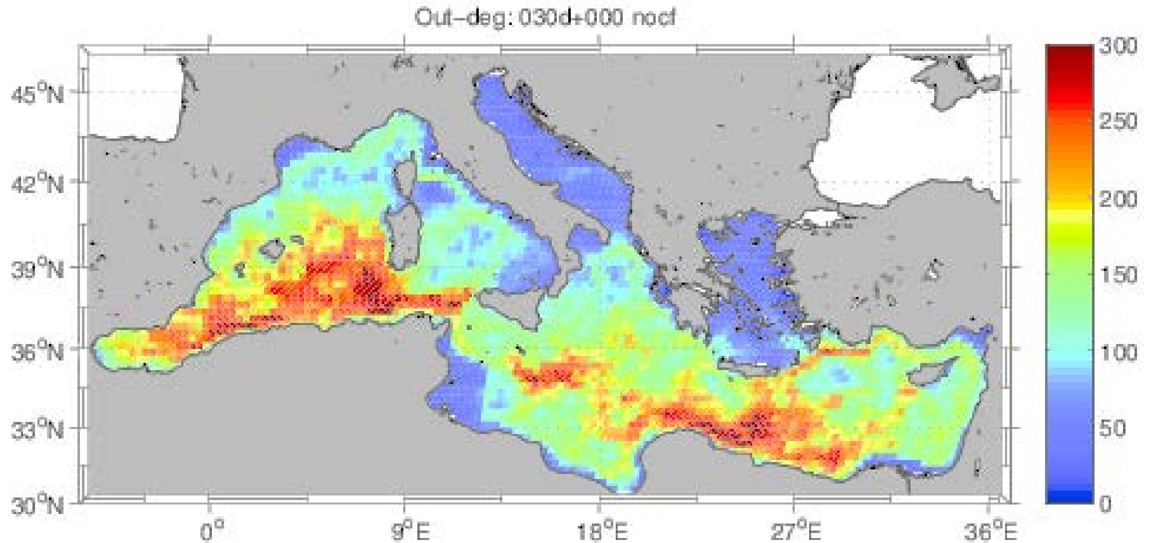
Standard local network descriptors can be given a fluid interpretation:



IN-DEGREE: number of nodes that connect towards a given one. A measure of **mixing**

OUT-DEGREE: number of nodes towards a given one sends tracers. A measure of **dispersion**

t_0=January 1st .
τ=30 days.



Perhaps too much weight to weakly receiving nodes?

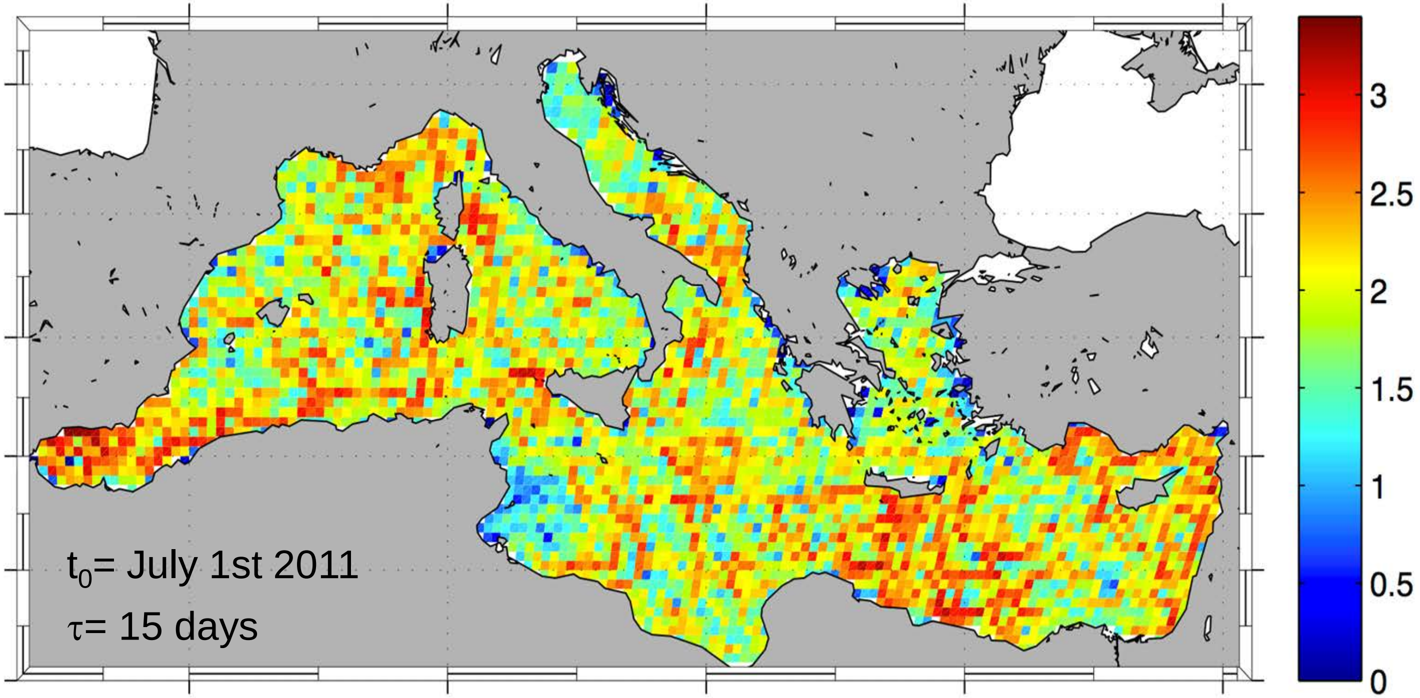
Weighted degree?
not informative because of normalization

Measure of dispersion: Finite time entropy
(in backwards time it would be a measure of mixing)

cf. Froyland and Padberg-Gehle (2012), Physica D 241, 1612

$$FTE_i(t_0, \tau) = -\sum_j P(t_0, \tau)_{ij} \log P(t_0, \tau)_{ij}$$

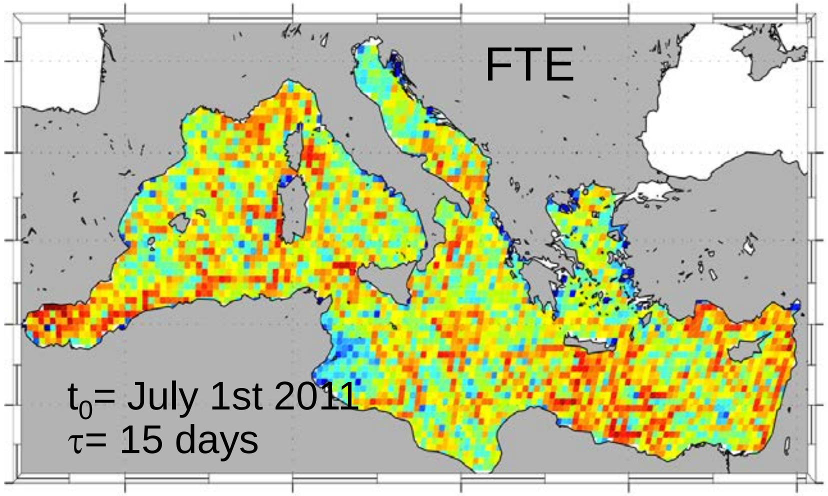
In fact it is an ε–τ entropy rate (Boffeta et al, Phys Rep 356 (2002) 367–474)

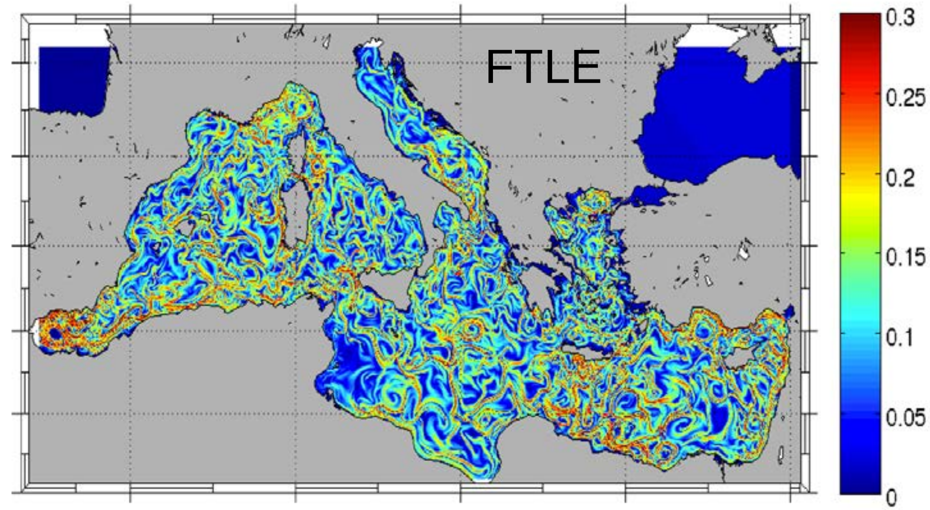


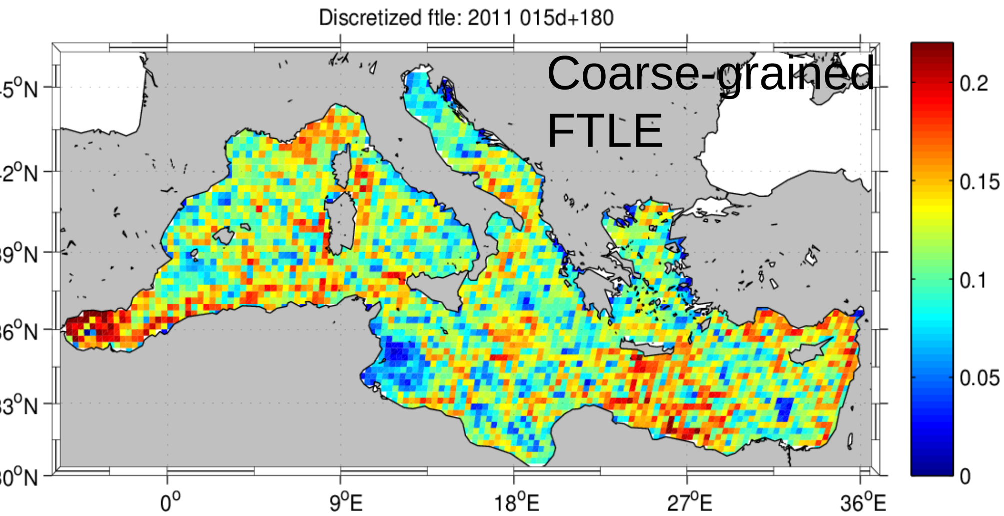
t_0= July 1st 2011

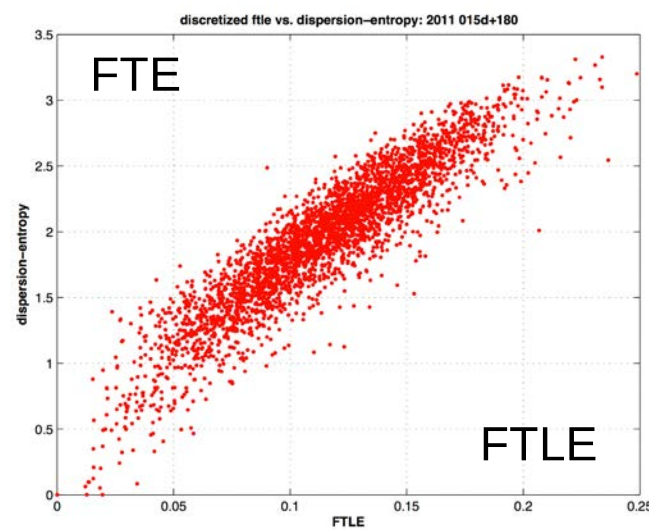
τ= 15 days

For small boxes, FTE should is related to the Kolmogorov-Sinai entropy, and then to the positive (finite-time) Lyapunov exponent (Boffeta et al 2002, Froyland et al 2012)









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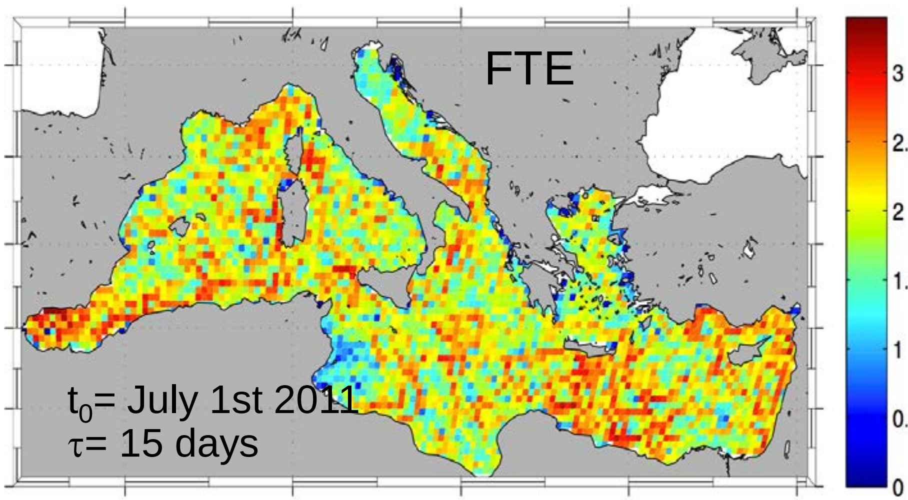
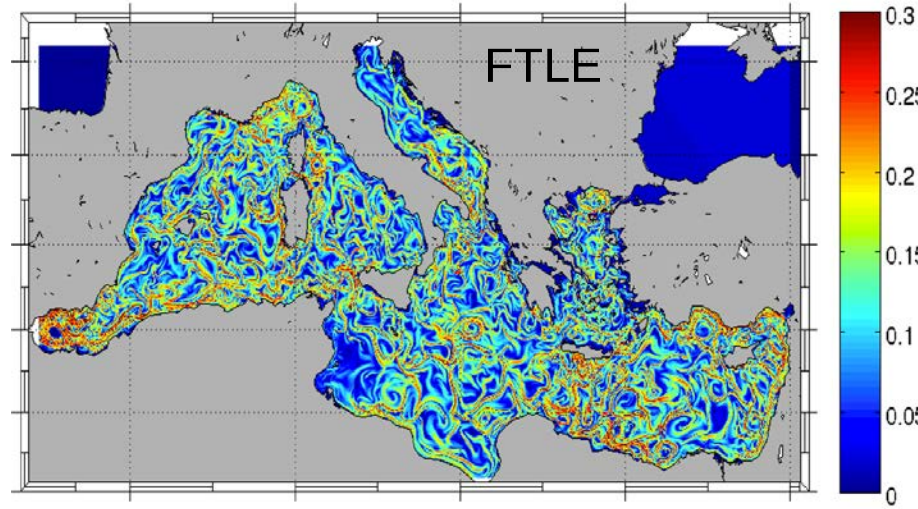
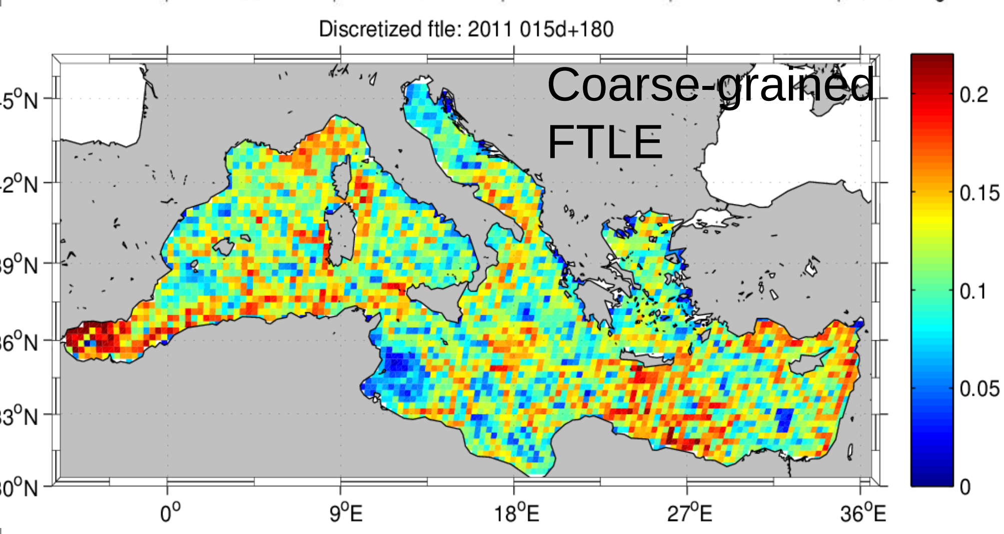
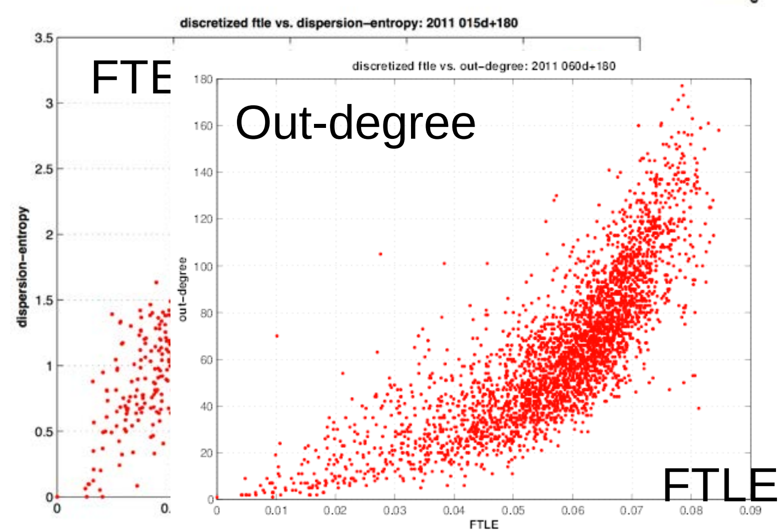








A family of network entropies

Ser-Giacomi et al. (2014)

$$H_i^q(t_0,\tau) \equiv \frac{1}{(1-q)|\tau|} \log \sum_{j=1}^{N} (\mathbf{P}(t_0,\tau)_{ij})^q$$

Can be defined for any weighted network. They quantify dispersion and mixing of walkers moving with these weights as transition probabilities

$$H_i^0(t_0,\tau) \equiv \frac{1}{\tau} \log K_O(i)$$

out degree

FTE

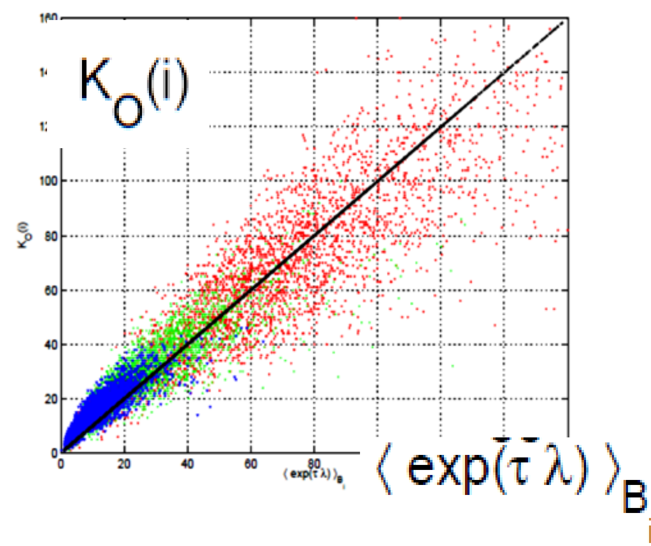
$$H_i^1(t_0,\tau) = -\frac{1}{\tau} \sum_{j=1}^{N} \mathbf{P}(t_0,\tau)_{ij} \log \mathbf{P}(t_0,\tau)_{ij}$$

$$e^{(1-q)\tau H_i^q(t_0,\tau)} \approx \left\langle e^{(1-q)\tau\lambda(\mathbf{x}_0,t_0,\tau)} \right\rangle_{B_i}$$

Related to the statistics of FTLE inside each box

$$\left\langle e^{\tau\lambda(\mathbf{x}_0,t_0,\tau)} \right\rangle_{B_i} = e^{\tau H_i^0(t_0,\tau)}$$

$$H^0{}_i \geq < \lambda >_i$$

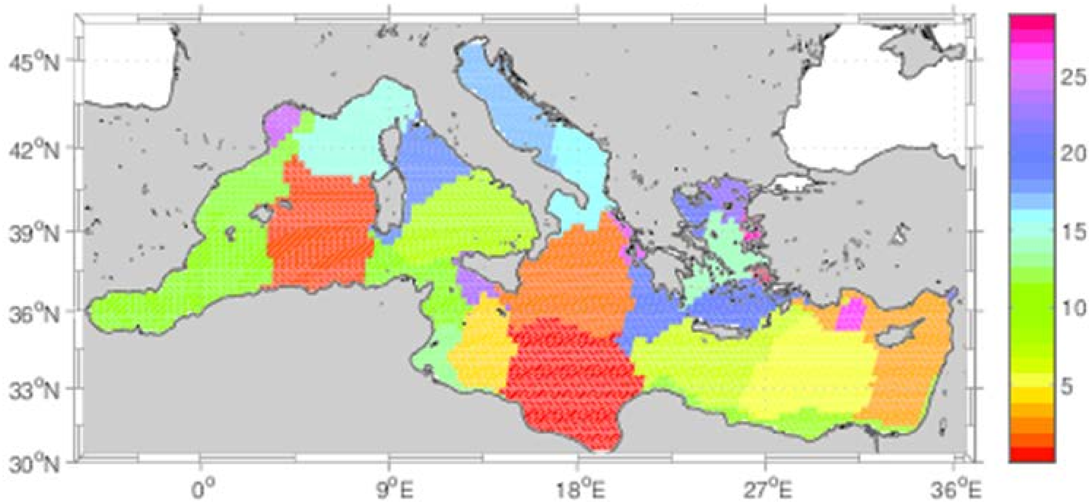


Partitioning the sea

- Assume larvae of fishes and crustaceans are passive tracers during some early-life stage, and that their arrival or not to some ocean areas is important to determine ecological and genetic community structure

- If Marine Protected Areas (MPAs) are established to preserve biodiversity, it has sense to create them in as many unconnected ocean regions as possible. But inside a well-mixed area, it would be redundant to establish many MPAs

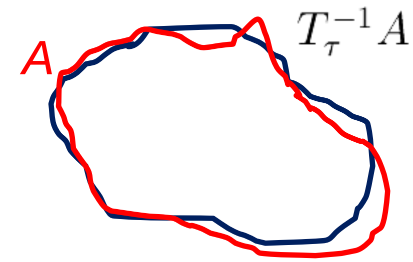
- Thus, it is of interest to find a partition of the sea into well-mixed areas (during a finite Pelagic Larval Duration, PLD) with a weak transport among them (provinces). PLDs of interest in the Mediterranean are 30-60 days. During this time the flow remains approximately bidimensional.



Detection of almost-invariant or (self)coherent sets

Froyland & Dellnitz, SIAM J Sci Comp (2003)
Froyland, Physica D (2005)
Dellnitz et al., Nonlin Proc Geophys (2009)
Froyland et al., Phys Rev Lett (2007)

Theory later extended to pairs of coherent sets



Coherence ratio of a measurable set *A* (during a time τ, flow map T_τ):

$$\rho(A) = \frac{m(A \cap T_\tau^{-1} A)}{m(A)}$$

A almost-invariant if

$$\rho(A) \approx 1$$

Optimal almost invariant decomposition in *q* sets $A_1, A_2, \ldots A_q$: the one maximizing the mean coherence

$$\rho = \frac{1}{q} \sum_{k=1}^{q} \rho(A_k)$$

subject to a constraint limiting the difference in size of the sets

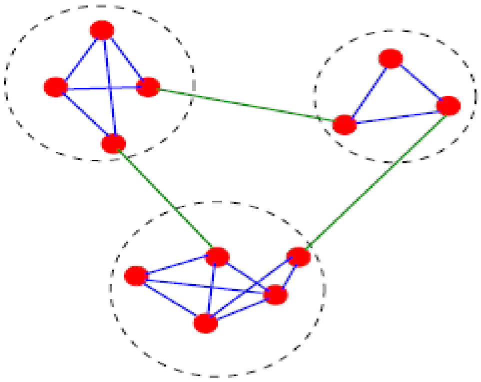
Maximization is done by considering eigenvectors and eigenvalues of a symmetrized version of $P(t_0, \tau)$, with further heuristic clustering (fuzzy c-means)

Is this what we need?

- The condition of good internal mixing is not imposed
- Sets are constrained to be of similar size
- The number of sets *q* has to be decided a priori
- Except for *q=2*, heuristics is needed to select the number of eigenvectors used and their clustering

Graph partitioning is a classical problem in network theory. Once the flow is interpreted as a network many alternative techniques are available. Community detection techniques try to find sets of nodes strongly connected among them and weakly connected with the rest

Author	Ref.	Label	Order
Girvan & Newman	[12,54]	GN	$O(nm^2)$
Clauset et al.	[174]	Clauset et al.	$O(n \log^2 n)$
Blondel et al.	[179]	Blondel et al.	$O(m)$
Guimerà et al.	[185,27]	Sim. Ann.	Parameter dependent
Radicchi et al.	[78]	Radicchi et al.	$O(m^4/n^2)$
Palla et al.	[28]	Cfinder	$O(\exp(n))$
Van Dongen	[271]	MCL	$O(nk^2)$, $k < n$ parameter
Rosvall & Bergstrom	[315]	Infomod	Parameter dependent
Rosvall & Bergstrom	[58]	Infomap	$O(m)$
Donetti & Muñoz	[48,423]	DM	$O(n^3)$
Newman & Leicht	[290]	EM	Parameter dependent
Ronhovde & Nussinov	[367]	RN	$O(m^\beta \log n)$, $\beta \sim 1.3$

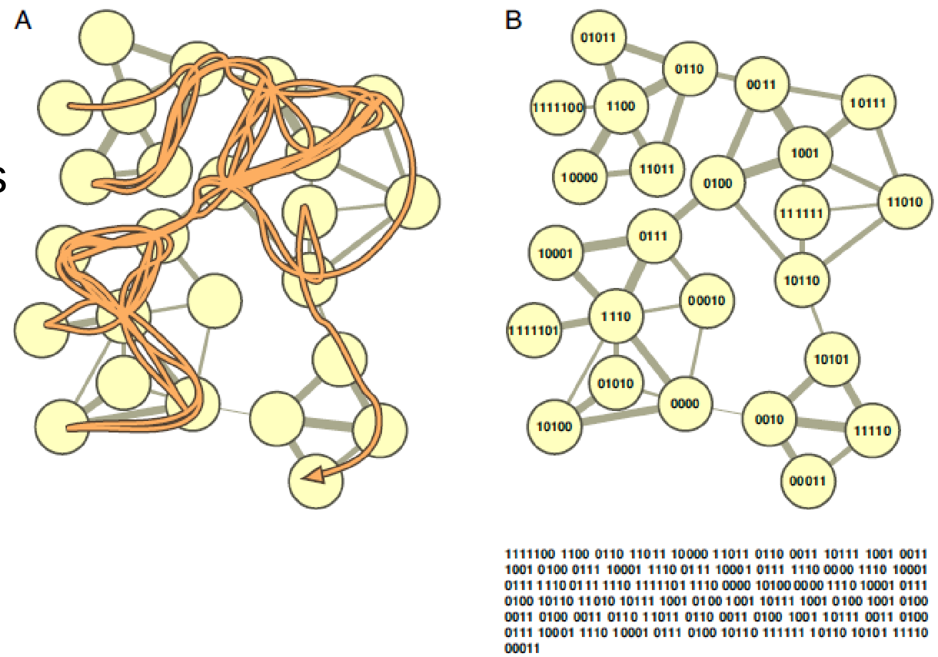


S. Fortunato, Phys Rep 486 (2010) 75-174

Among them: Infomap

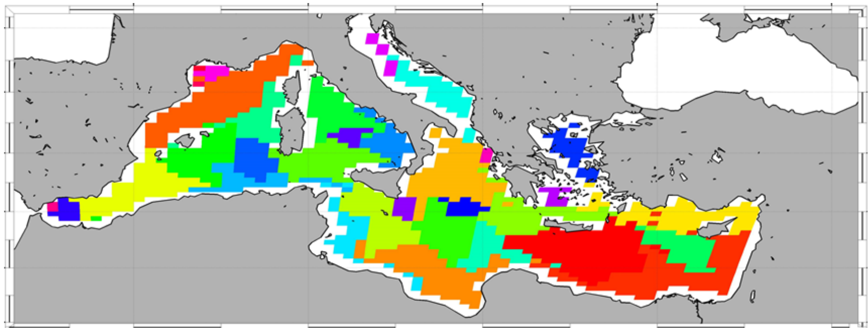
(Rosvall & Bergstrom (2008), PNAS 105, 1118–1123)

- The motion of random walkers in the flow network with transition probabilities P_{ij} is considered
- A partition in communities is assumed, and the minimum length of a codeword to describe the ensemble of random walks is estimated from information theory
- Then, the partition that minimizes the length of such code is found by a fast algorithm

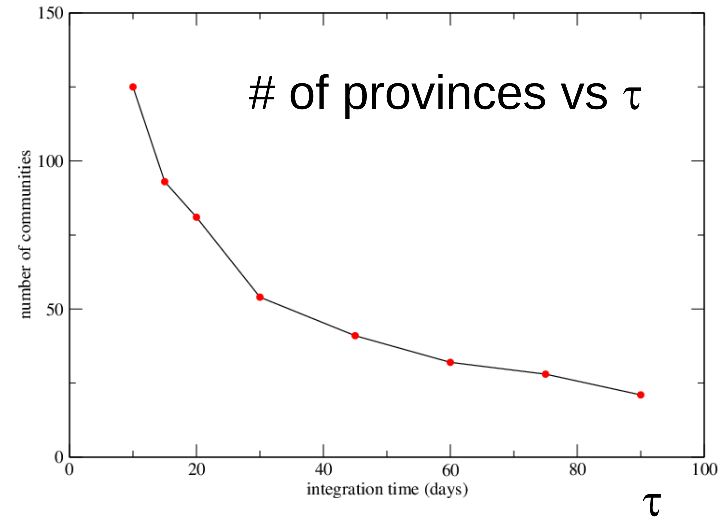


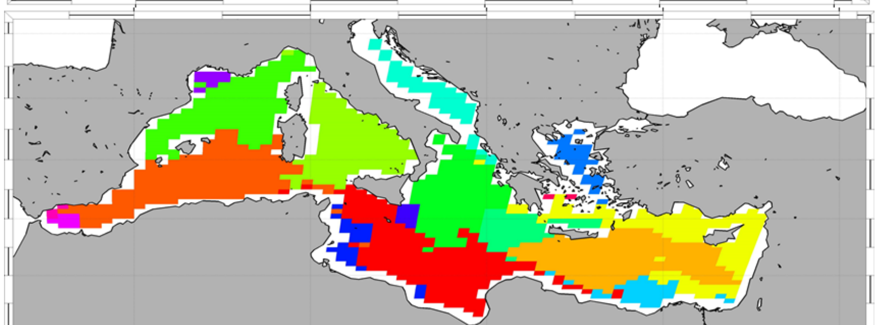
- It is based explicitly in the flow via the Markov walks defined by P_{ij}
- It determines modules internally well connected, with weak leaking
- The optimal number of communities is found
- There is no constrain in the size of communities

Provinces (≈ communities, almost-invariant sets)

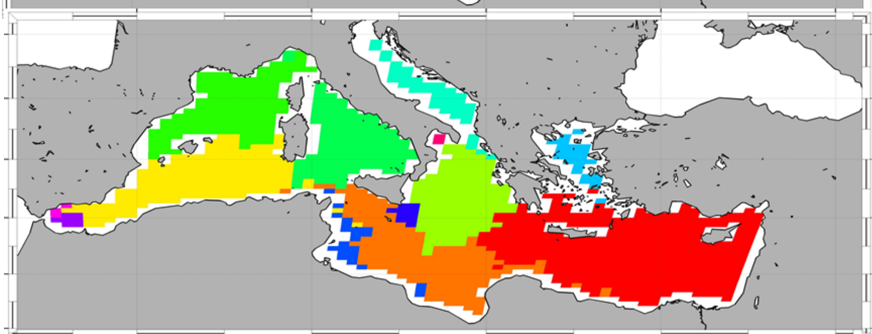


τ=30 days
33 provinces
ρ=0.77

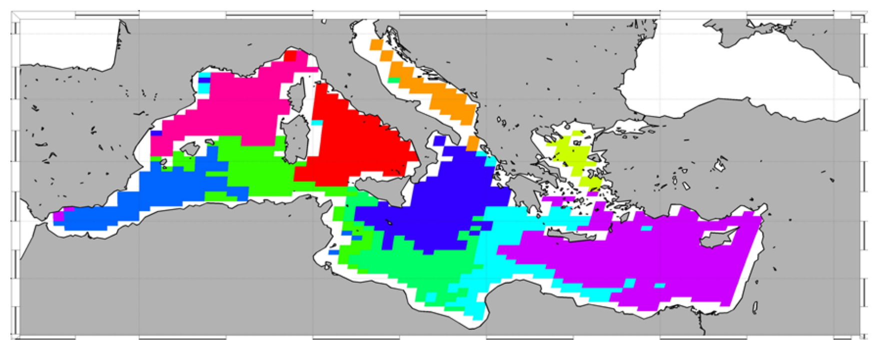




τ=60 days
17 provinces
ρ=0.78



τ=90 days, 13 provinces, ρ=0.80



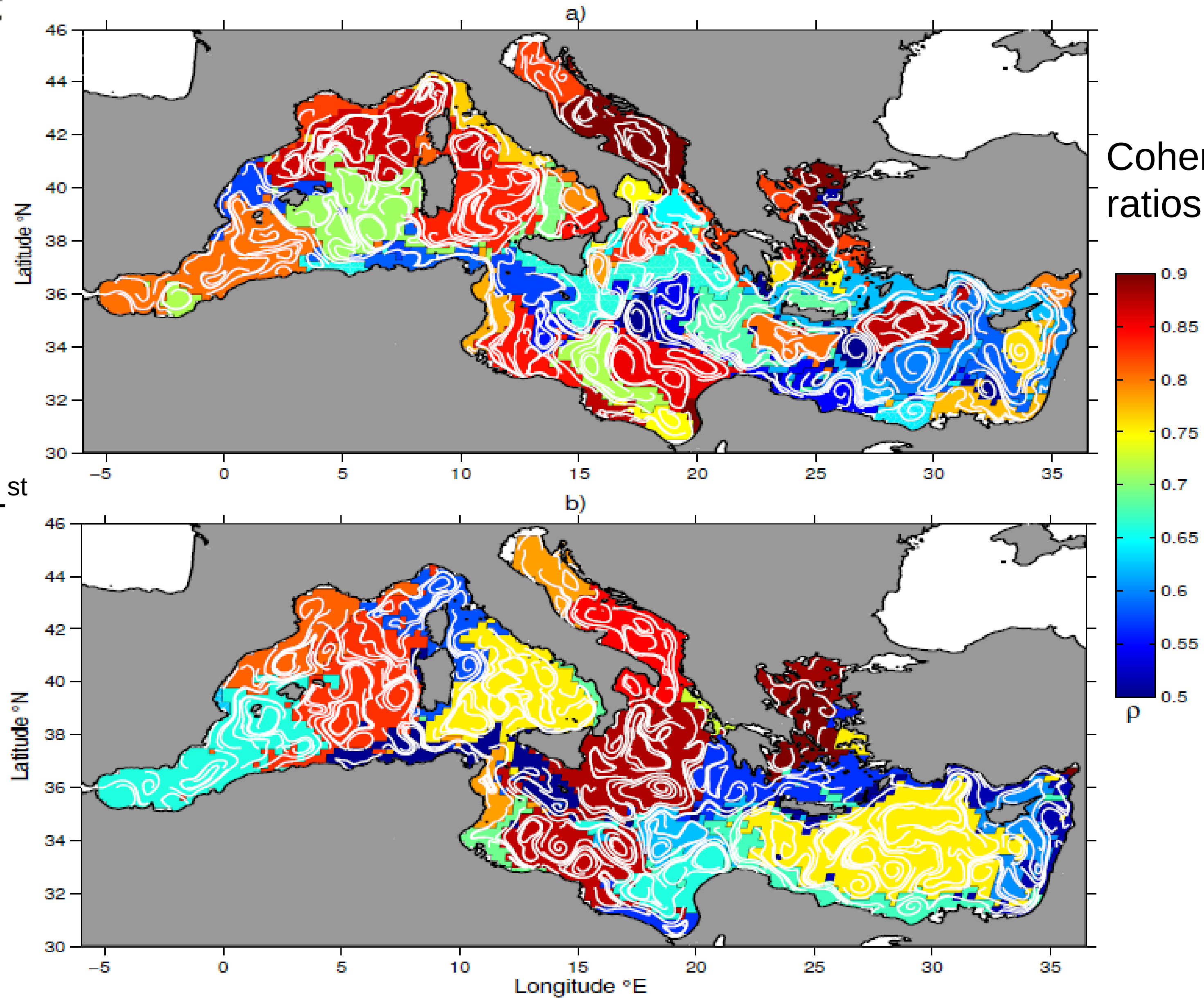
Spectral method, τ=90 days, 10 provinces, ρ=0.78

τ=30

t_0=January 1st 2011

τ=60

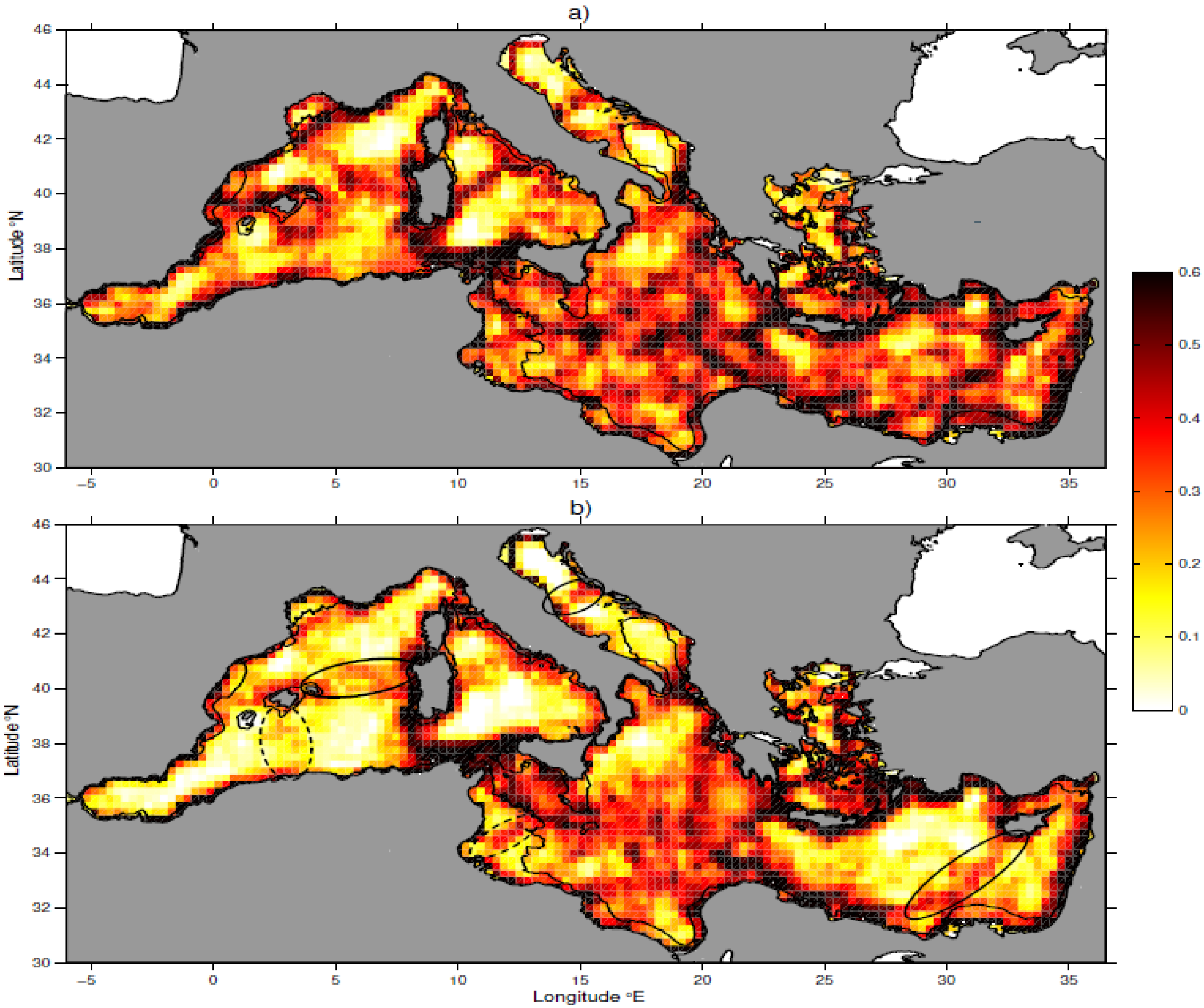
Coherence ratios $\rho(A_k)$



Persistence of the community borders during 20 values of t_0
(10 years, summer and winter)

τ=30

τ=60



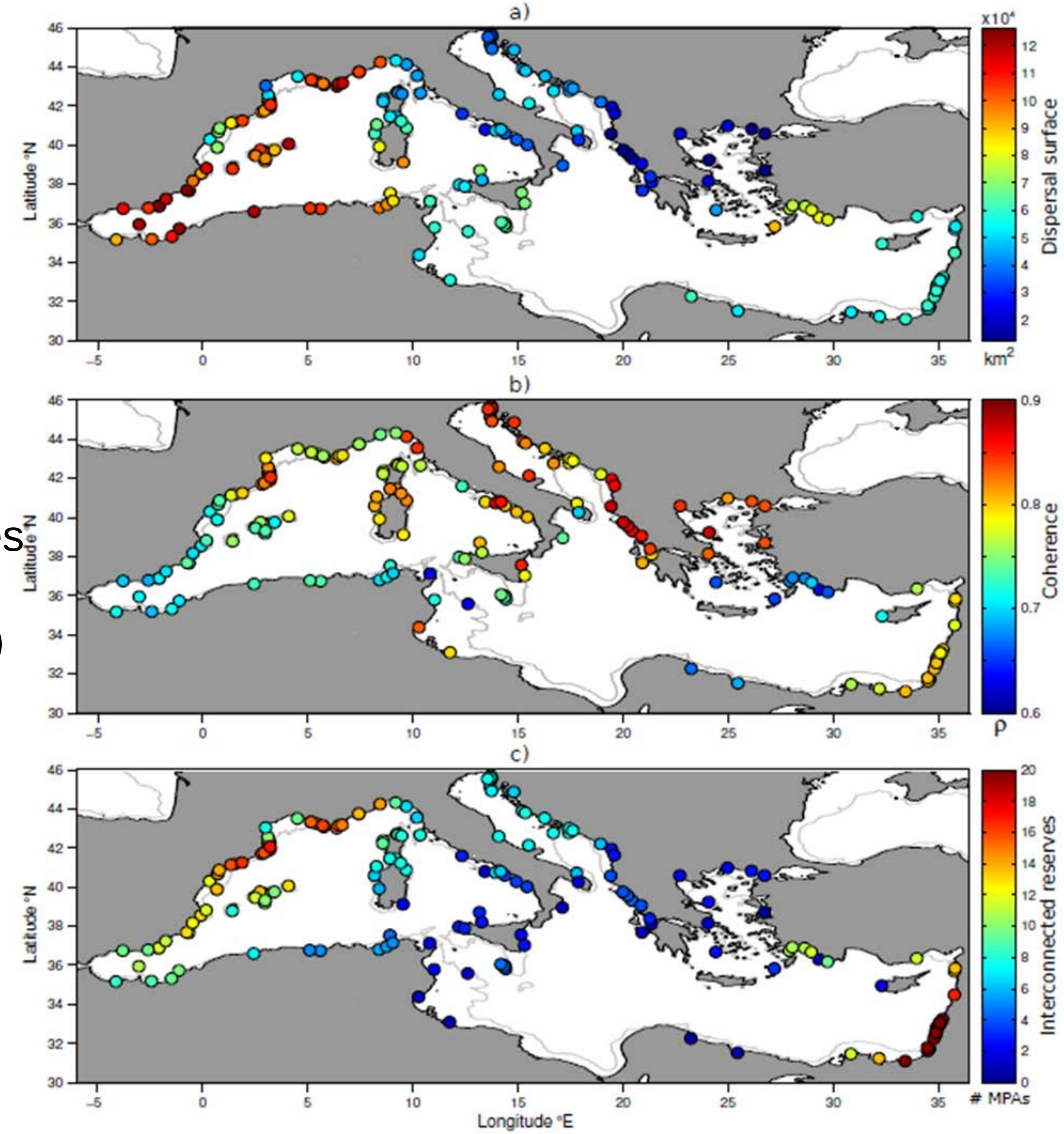
mostly in winter

mostly in summer

Marine Protected Areas (from MedPan)

τ=30

Values averaged over the 20 values of t_0 (across 10 years)



Area of province

Coherence ratio ρ of the province

Number of reserves in the same province

SUMMARY

- Fluid transport can be described in the language of network theory by means of transfer matrix methodology
- A family of entropies can be defined for any weighted network, quantifying dispersion and mixing
- Tools of network theory can then be applied to describe transport and mixing. In particular the Infomap algorithm of community detection provides identification of well-mixed almost-invariant sets
- Application to marine dynamics provides insight into environmental decision and planning tasks

Rossi, Ser-Giacomi, Lopez, Hernandez-Garcia (2014), *Hydrodynamic provinces and oceanic connectivity from a transport network help designing marine reserves,* Geophys. Res. Lett. 41, 2883-2891

Ser-Giacomi, Rossi, Lopez, Hernandez-Garcia (2014), *Flow networks: A characterization of geophysical fluid transport,* `http://ifisc.uib-csic.es/publications/publication-detail.php?indice=2556`