Large-scale transport in oceans

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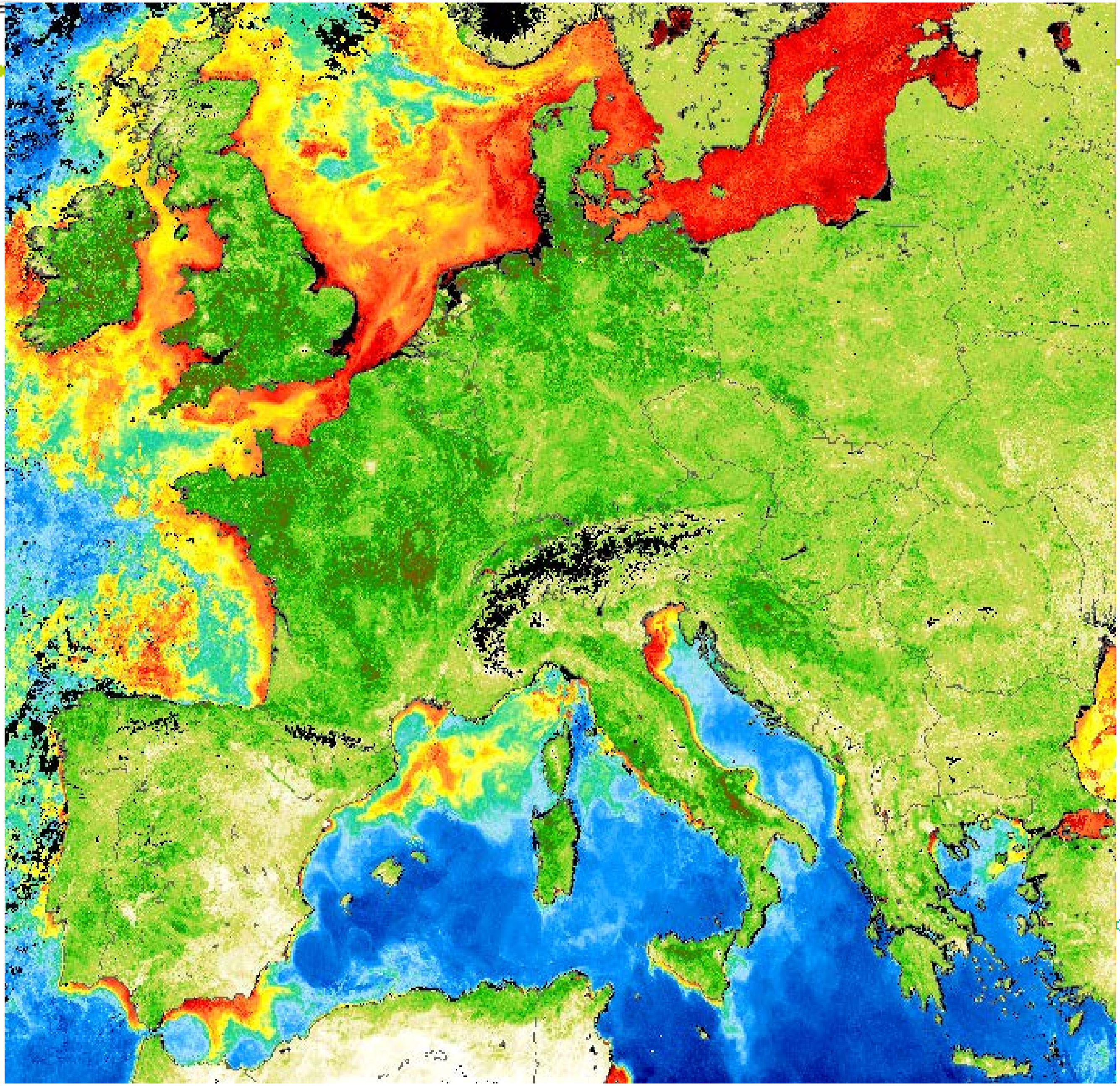
Statistical Physics and Dynamical Systems approaches in Lagrangian Fluid Dynamics

STATISTICAL PHYSICS AND DYNAMICAL SYSTEMS APPROACHES IN LAGRANGIAN FLUID DYNAMICS

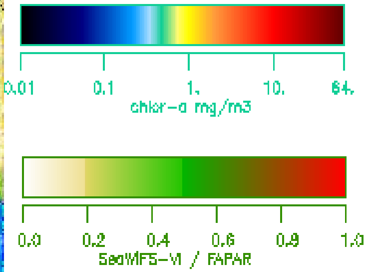
OUTLINE

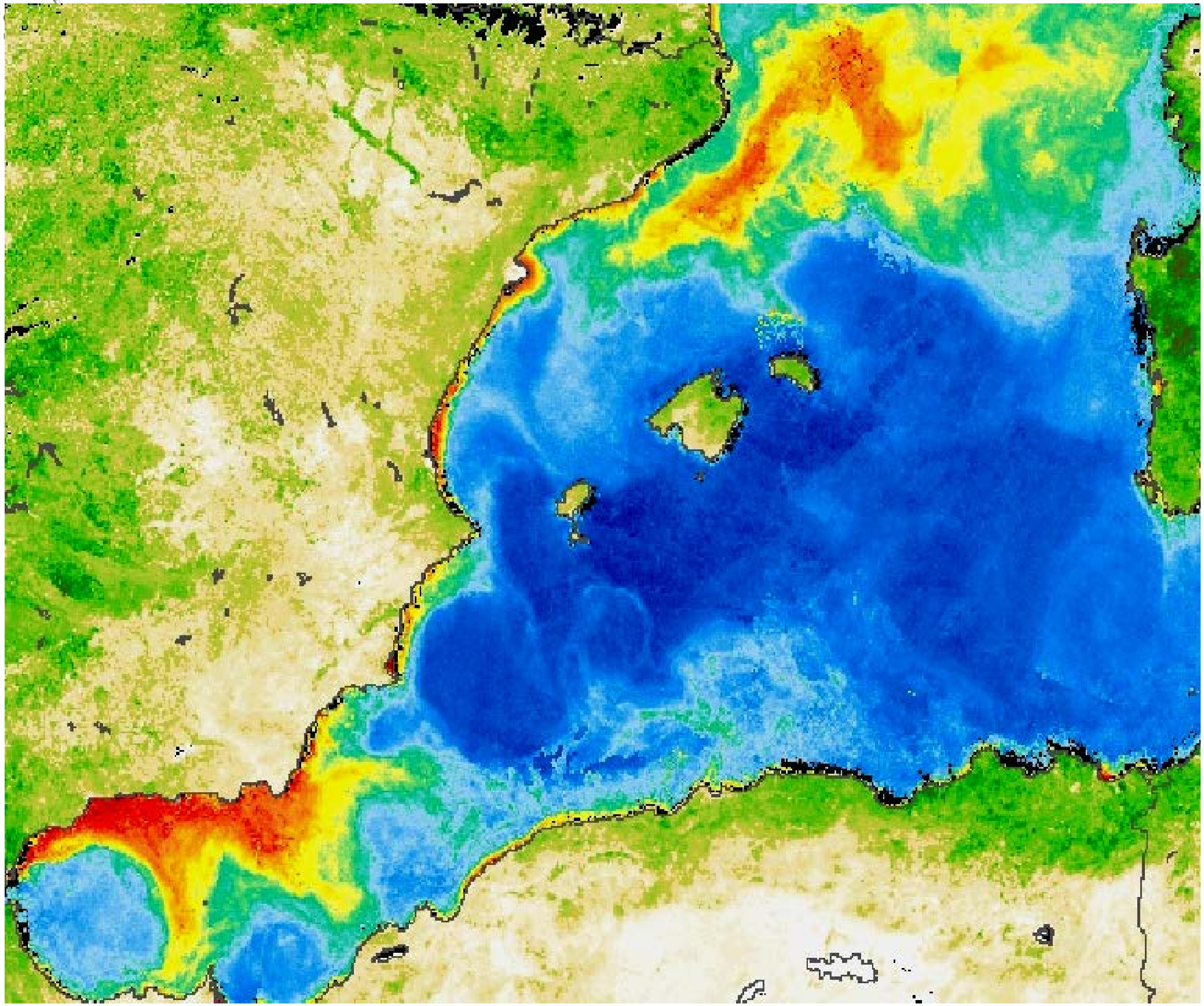
1. Lagrangian fluid dynamics and introduction to chaotic advection. Hamiltonian dynamics, KAM tori, Lyapunov exponents, open flows
2. Dispersion, diffusion and coherent structures in flows. Turbulent, pair and chaotic dispersion, gradient production, FTLE, FSLE, Lagrangian Coherent Structures
3. **Chemical and biological processes in flows.** Fisher and excitable plankton waves, filamental transitions, lamellar approaches, burning manifolds
4. Complex networks of fluid transport. Directed and weighted flow networks. Community detection

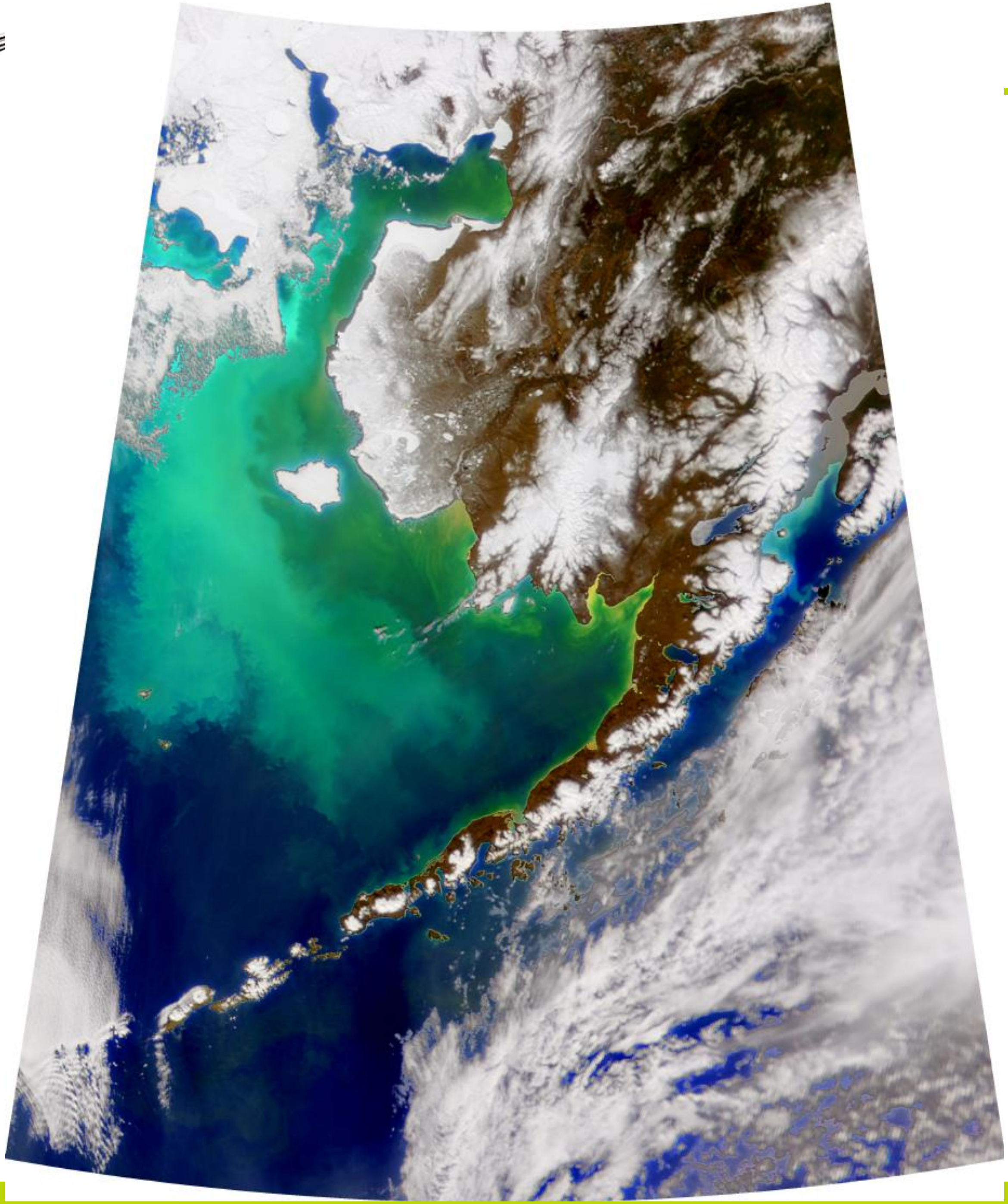


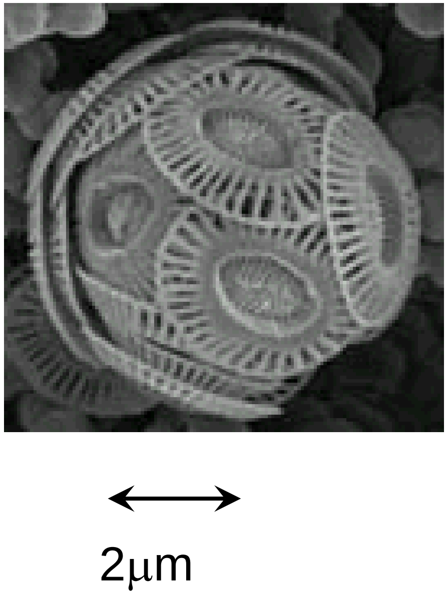
Chlorophyll seen from SeaWiFs

Monthly composite April 2000









Bloom of coccolithophores (Emiliania huxleyi)

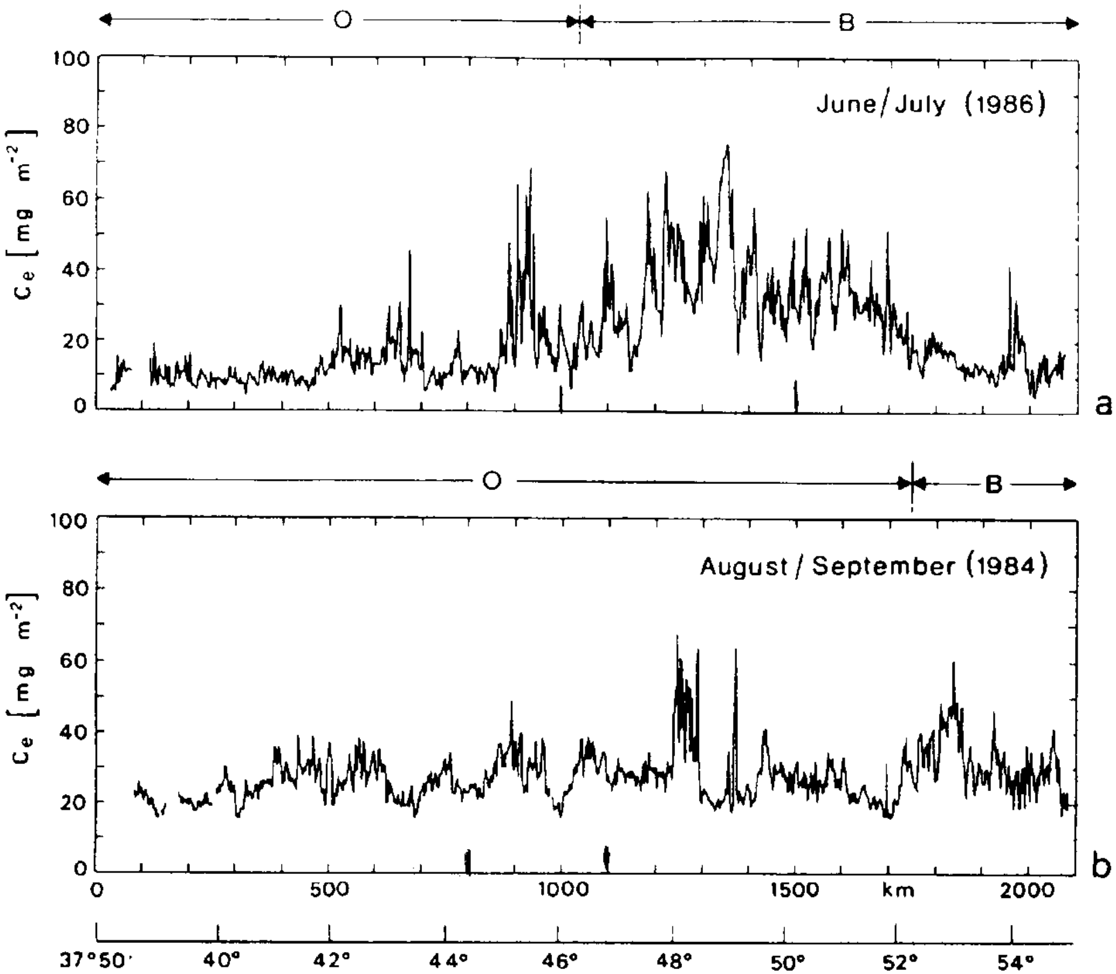
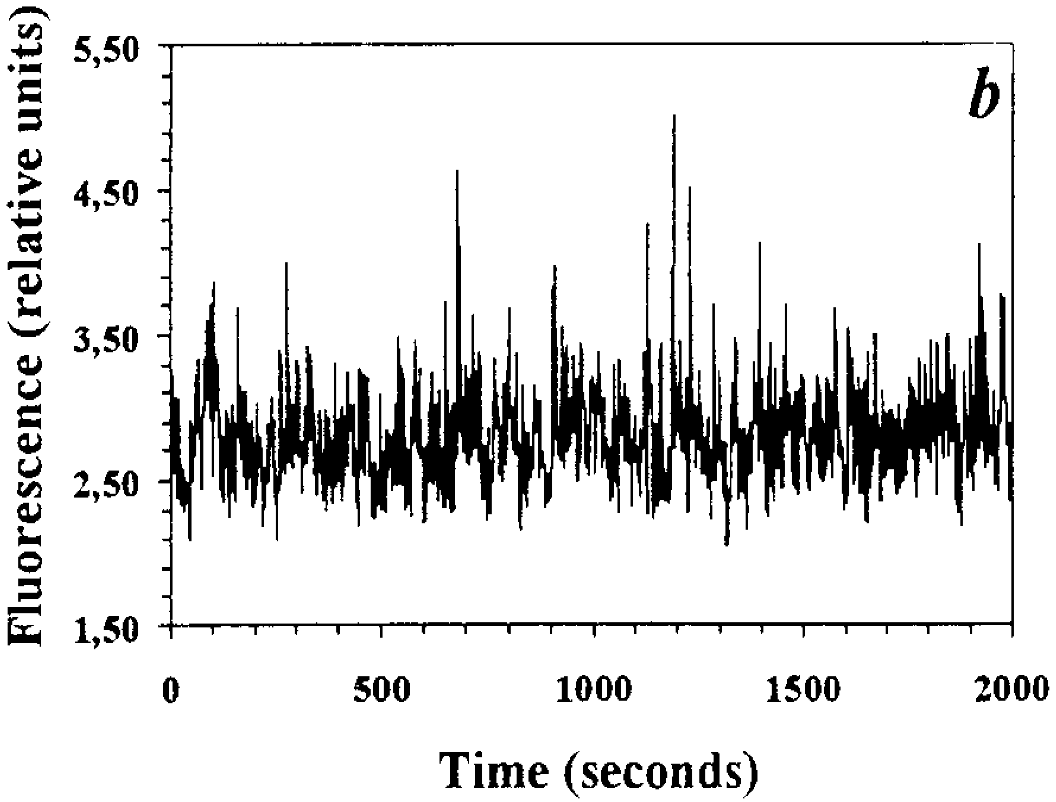


Fig. 4. Horizontal distribution of total euphotic zone chlorophyll content along the standard section in (a) June/July (record B102/NOA'86) and in (b) August/September (B101/NOA'84). The overbar arrows indicate which parts of the section were characterized by oligotrophic conditions with a deep chlorophyll maximum (arrow marked "O") and which by a mixed-layer bloom (arrow marked "B").

V.H.Strass
Deep-Sea Res.
39, 65 (1992)



In vivo fluorescence at a fixed station in the English Channel.
Seuront et al., Nonlinear Proc. In Geophys. 3, 236 (1996).

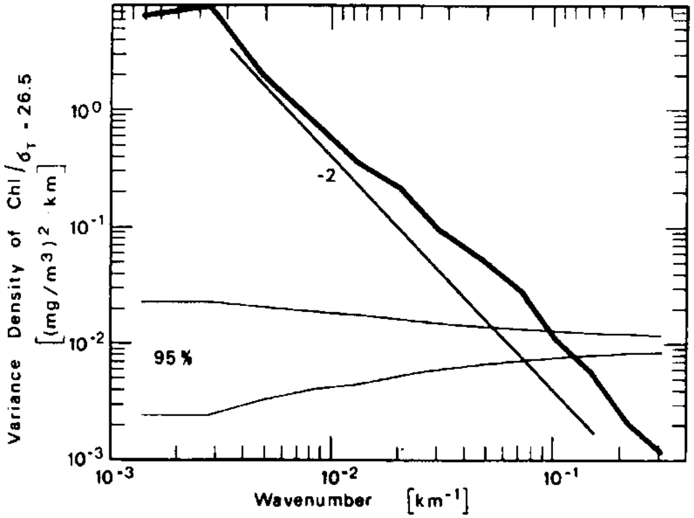


Fig. 12. Variance density spectrum of the variability of salinity and chlorophyll concen along isopycnal $\sigma_T = 26.5$ kg m^{-3} for the late-summer record B101/NOA'84. Lines with s slope of -2 are drawn for comparison.

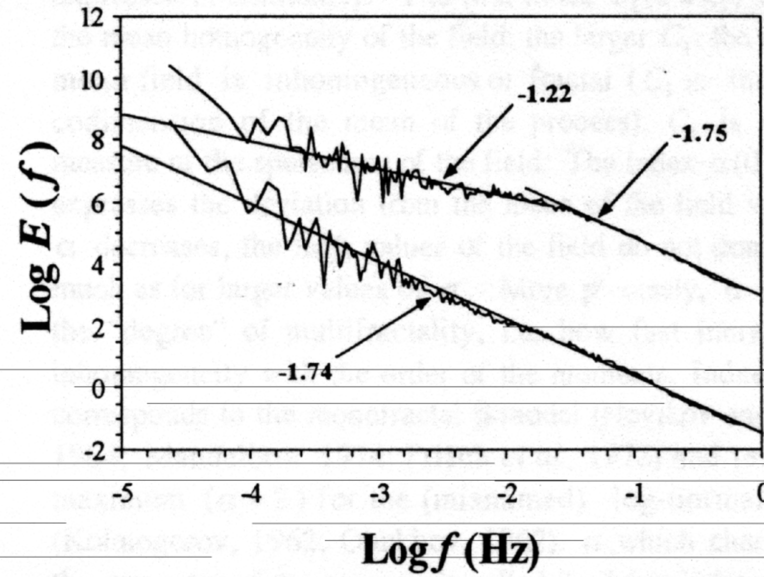
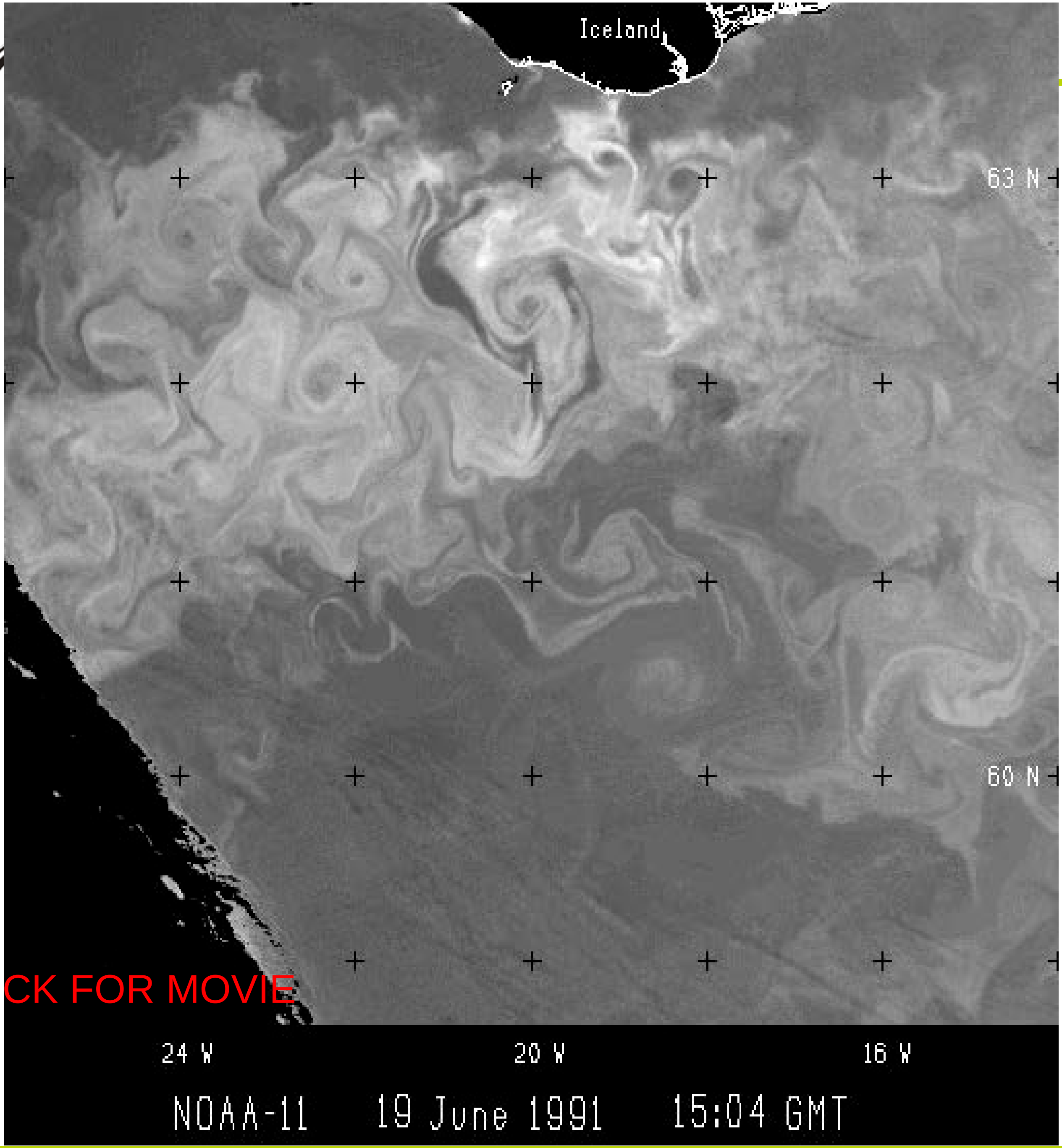


Figure 1. The power spectra $E(f)$ (f is frequency) of the fluorescence and the temperature data, shown in a log-log plot. The fluorescence data are scaling from 0.01 Hz to 1 Hz with a spectral slope $\beta \approx 1.75$ and for frequency smaller than 0.01 Hz with a spectral slope $\beta \approx 1.22$. The temperature data are scaling with $\beta \approx 1.74$.

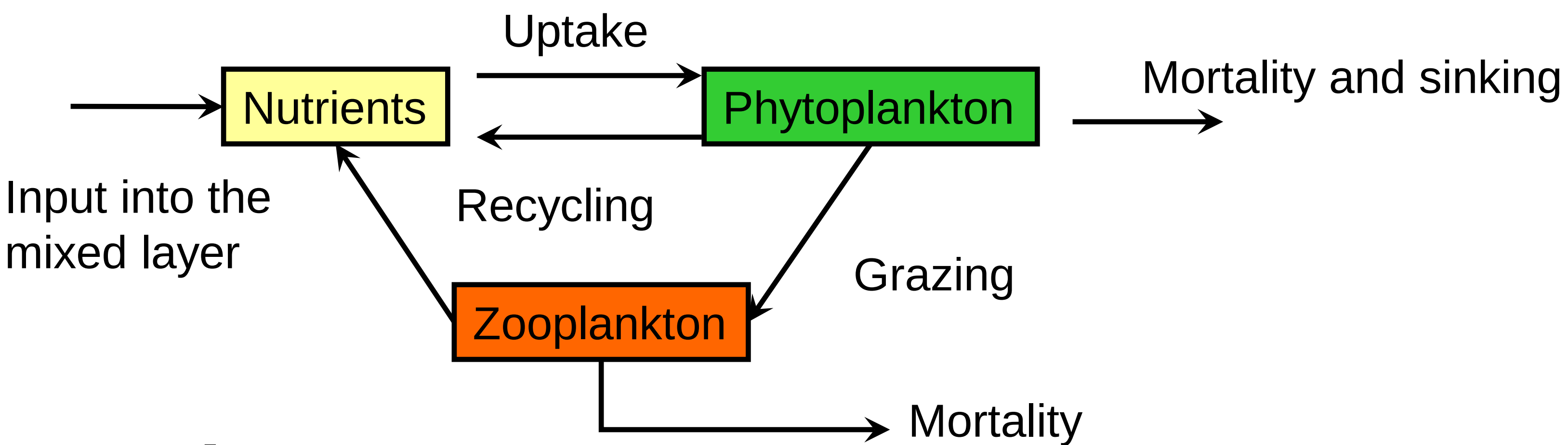


Coccolithophore bloom (Emiliania Huxleyi) in the North Atlantic.

AVHRR images by Steve Groom, RSDAS, Plymouth Marine Lab. UK, and Dundee Satellite Receiving Station, UK

CLICK FOR MOVIE

The NPZ class of models



$$\frac{dN}{dt} = s(N_0 - N) - f(N)P + cP + ag(P)Z + eZ$$

$$\frac{dP}{dt} = f(N)P - qP - g(P)Z$$

Sometimes simplification to a P-Z model is possible

$$\frac{dZ}{dt} = cg(P)Z - fZ \qquad (\text{ó} \quad - fZ^2)$$

TWO IMPORTANT GENERIC CHEMICAL/BIOLOGICAL BEHAVIORS

DECAYING (TO LOCAL EQUILIBRIUM) DYNAMICS

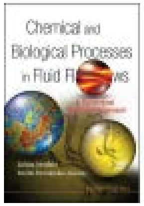
Build-up of gradients, morphological transitions
LAGRANGIAN dynamics

EXCITABLE DYNAMICS

Coherent behavior, persistent patters
Importance of FILAMENTS
Possibility of REDUCED MODELS

Among many others: Bistable, autocatalitic, oscillating, ...

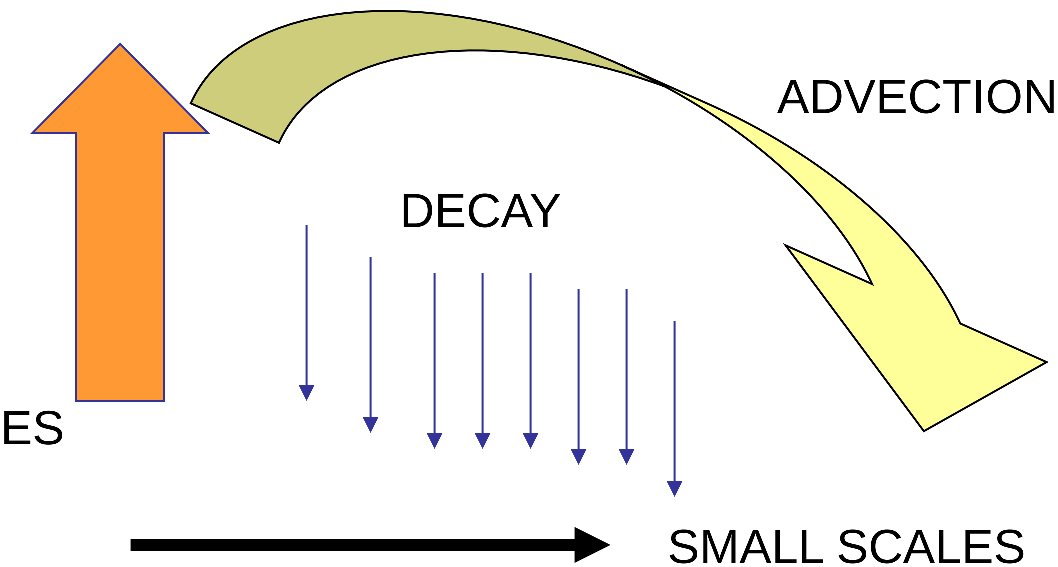
CHEMICAL AND BIOLOGICAL PROCESSES IN FLUID FLOWS: A Dynamical Systems Approach
by Zoltán Neufeld & Emilio Hernández-García



Linear chemical decay (with source): **dC(t)/dt = S(x(t),y(t)) - b C(t),**
or, more generally, any biological or chemical dynamics
with NEGATIVE Lagrangian Lyapunov exponent

$$\frac{dC_i}{dt} = F_i(C_1,\ldots,C_N) + S_i(x(t)) \qquad \delta C_i \approx \exp(\lambda_C t)\,\delta C_i(0) \qquad \lambda_c < 0$$

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$



Decaying biology/chemistry in a simple "alternating sine flow"

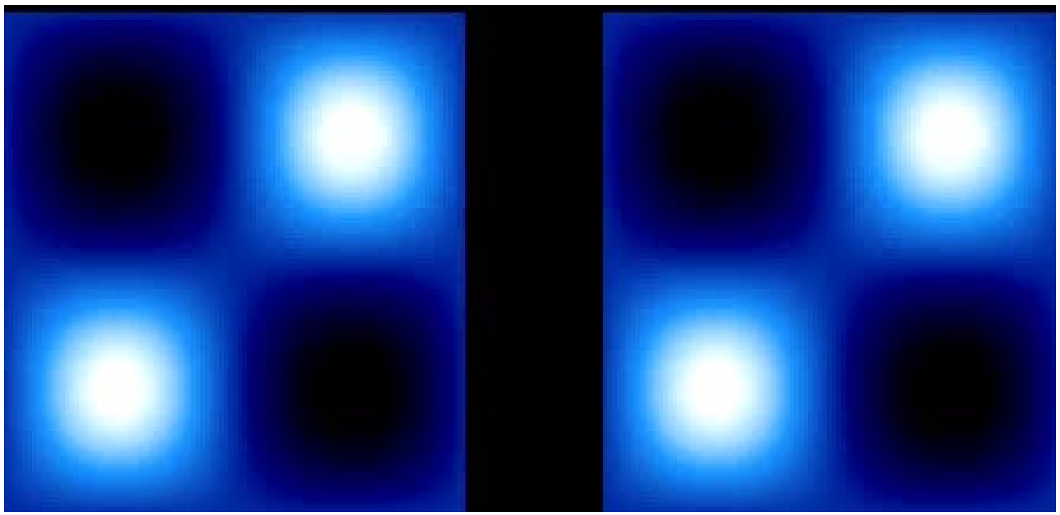
dC(t)/dt = S(x(t),y(t)) - b C(t), S(x,y)= 1+a sin(2πx)sin(2πy)

$$dC(t)/dt = S(x(t),y(t)) - b\,C(t), \quad S(x,y) = 1 + a\sin(2\pi x)\sin(2\pi y)$$

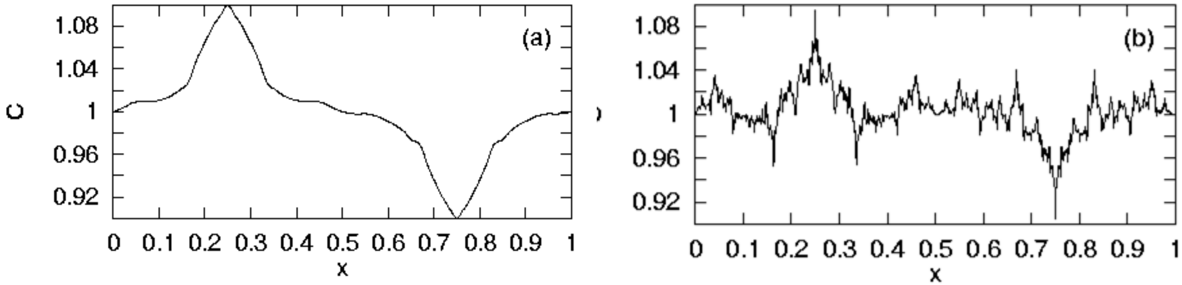
SMOOTH-FILAMENTAL TRANSITION

$b = 4 > \lambda_F = 2.35$ $\qquad$ $b = 0.1 < \lambda_F = 2.35$

CLICK FOR MOVIE

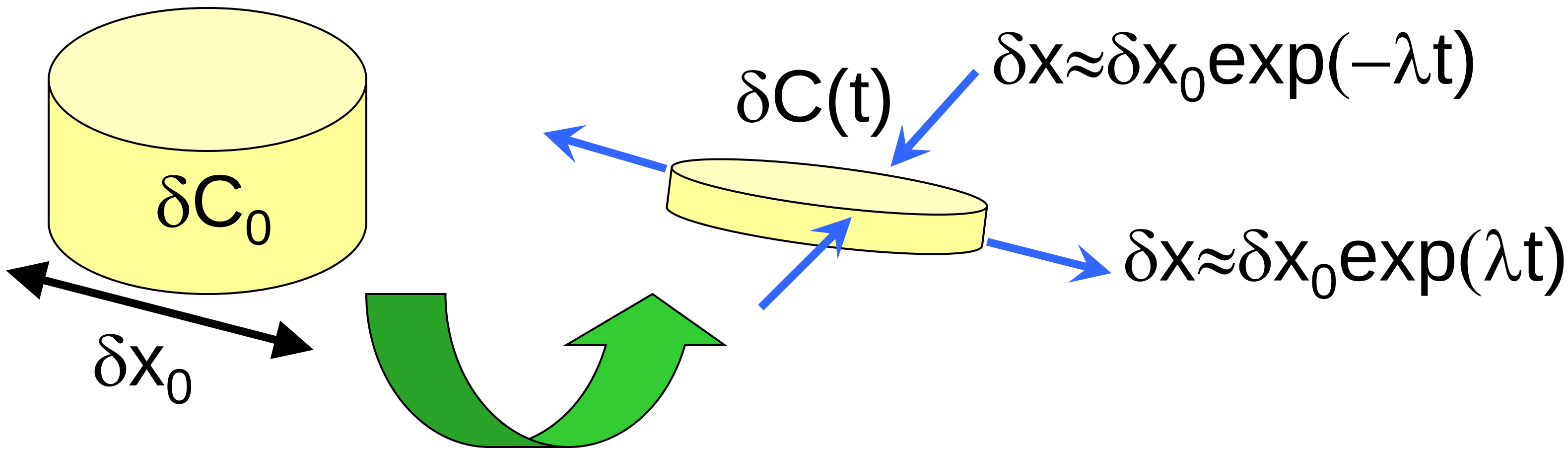


Lagrangian simulation



Characterizing the chemical decay by the (Lagrangian) chemical Lyapunov exponent

$\delta C(t) \approx \delta C_0 \exp(\lambda_C t), \quad \lambda_C < 0$

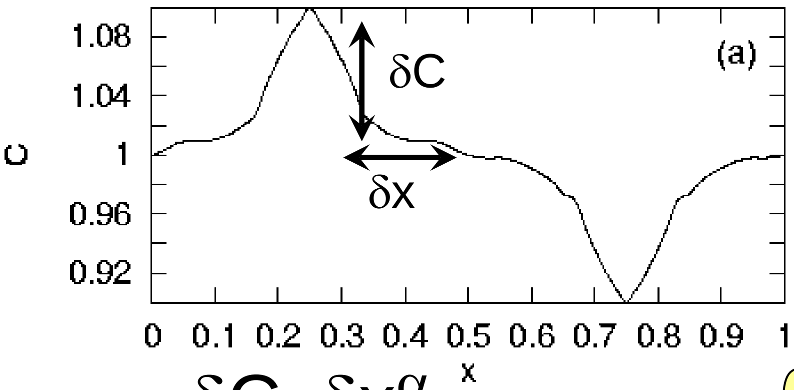


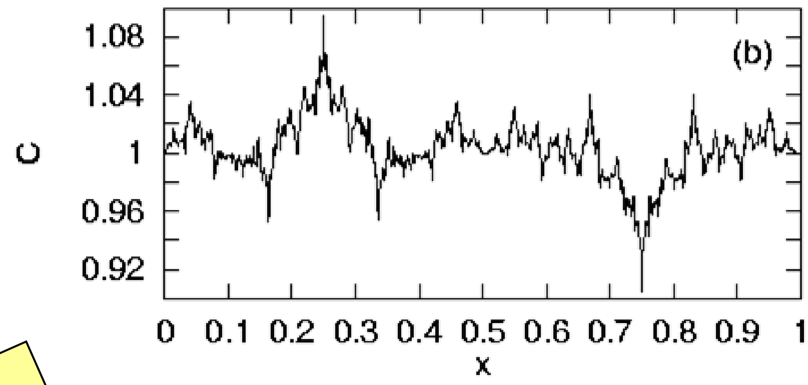
$\nabla C \approx \delta C(t) / \delta x(t)$ → $\exp((\lambda_C + \lambda) t)$

$\nabla C \approx \delta C(t) / \delta x(t)$ → $\exp((\lambda_C - \lambda) t)$

Smooth-nonsmooth (filamental) transition by decreasing $\lambda_C < 0$

Linear decay





$\delta C \approx \delta x^{\alpha}$

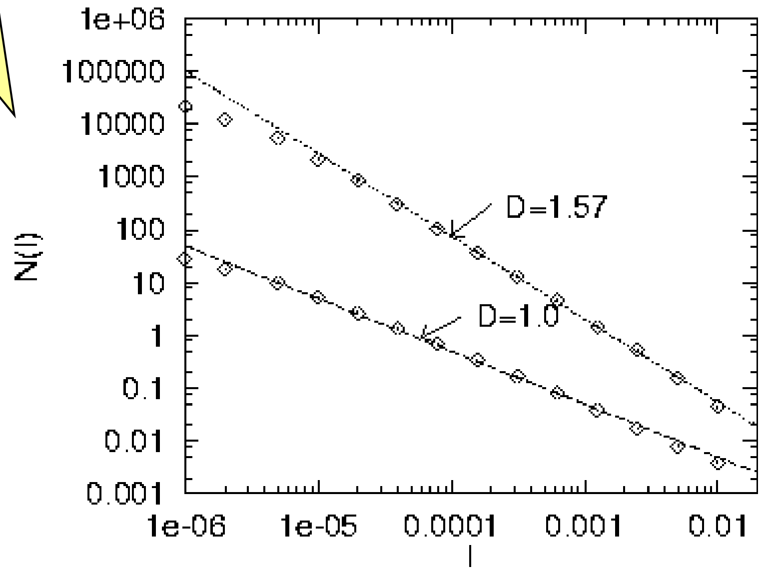
α: Hölder or roughening exponent

Curve box counting dimension: $D=2-\alpha$

Assuming that α is uniform everywhere:

$S(k) \approx k^{-\beta}$,

spectral exponent $\beta=1+2\alpha$



(Batchelor law for the passive scalar under smooth chaotic flow: $\beta=1$)

$\alpha=|\lambda_C|/\lambda$, $D=2-|\lambda_C|/\lambda$, $\beta=1+2|\lambda_C|/\lambda$

λ_C, NOT growth rate!

PLANKTON ON A MEANDERING JET

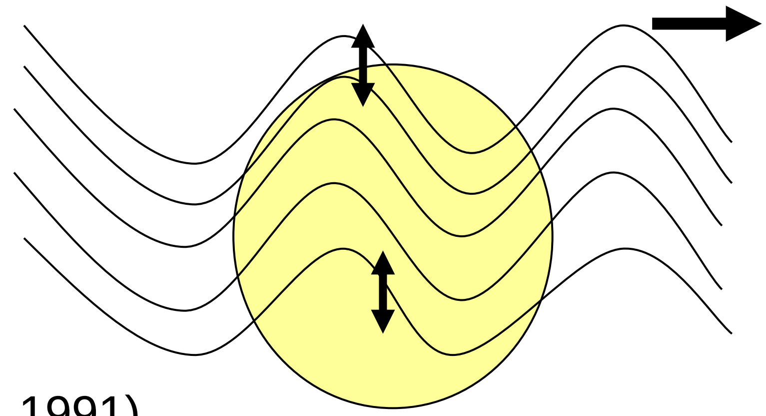
López, Neufeld, Hernández-García, Haynes, Physics and Chemistry of the Earth B 26, 313 (2001).

Plankton dynamics model (Abraham, Nature 391, 577 ('98)):

$$\frac{dN(t)}{dt} = \alpha \left(N_0(\mathbf{x}) - N(t)\right) \equiv F_1(N, P, Z, \mathbf{x})$$

$$\frac{dP(t)}{dt} = P\left(1 - \frac{P}{N}\right) - PZ \equiv F_2(N, P, Z, \mathbf{x})$$

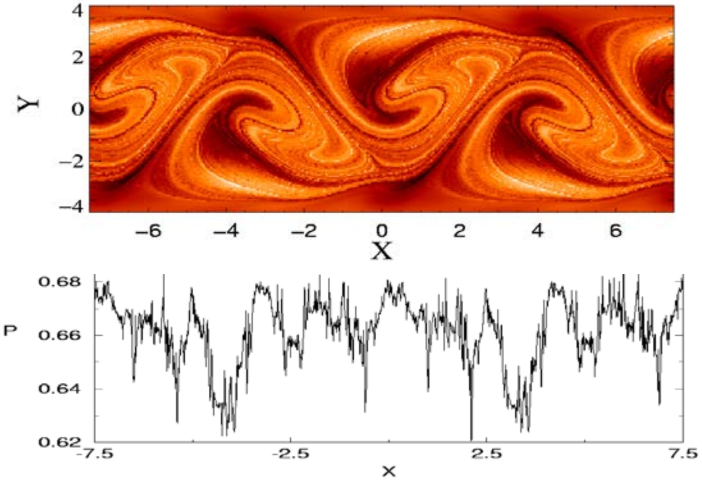
$$\frac{dZ(t)}{dt} = PZ - \delta Z^2 \equiv F_3(N, P, Z, \mathbf{x})$$

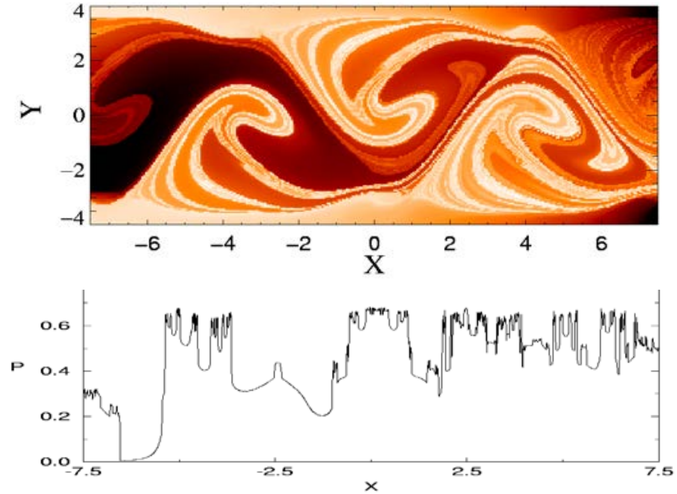


Source: $1 + A \sin(2\pi x/L_x) \sin(2\pi y/L_y)$

Flow: Kinematic model of a unstable jet (Bower, 1991)

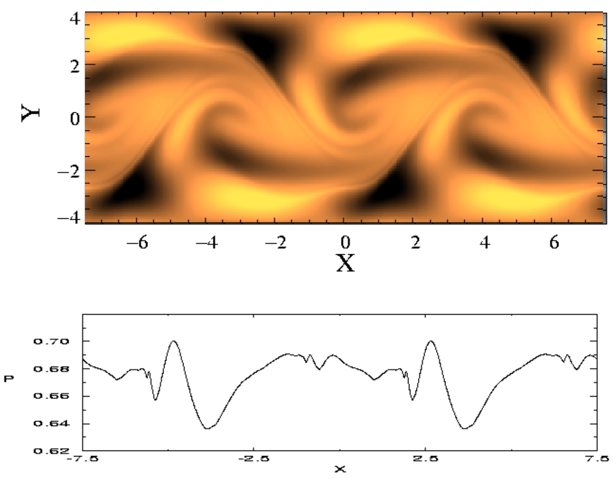
Phytoplankton





Closed Flow.
FILAMENTAL

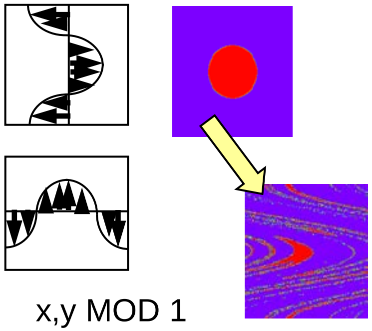
Open flow.
FILAMENTAL



Closed flow.
SMOOTH

Intermittency

Values of λ are arranged in sets that become Fractal (multifractal) at infinite time



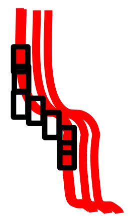
x,y MOD 1

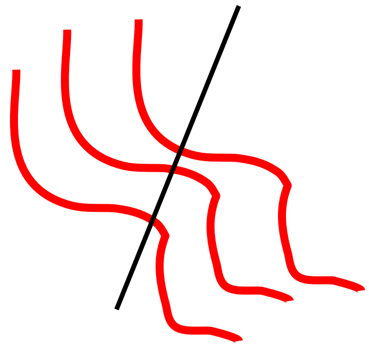
$$N(l) \sim l^{-D_f}$$

$$A_\lambda(t) \sim \exp(-G(\lambda)t)$$

$$w_\lambda(t) \sim \exp(-\lambda t)$$

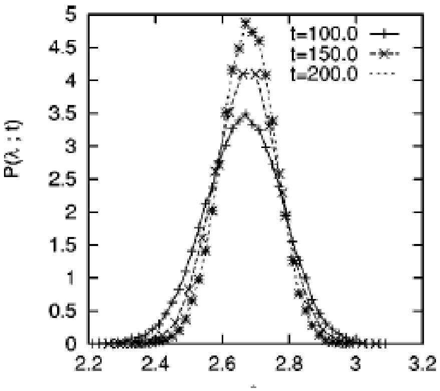
$$N_\lambda(l) \simeq \frac{A_\lambda}{(w_\lambda)^2} \sim e^{[2\lambda - G(\lambda)]t} \sim l^{[G(\lambda)/\lambda]-2}$$





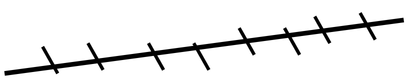
$$D_f(\lambda) = 2 - \frac{G(\lambda)}{\lambda}$$

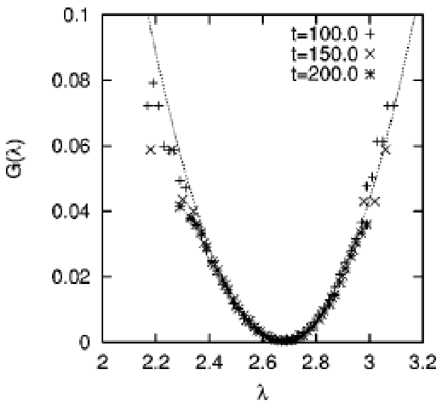
$$D'(\lambda) = D(\lambda) - 1$$



Prob to find λ in a segment made of M pieces =

$$\frac{N(\lambda)}{M} \approx \frac{l^{-D'(\lambda)}}{l^{-1}} = l^{1-D'(\lambda)} = l^{2-D(\lambda)}$$





Intermittency : STRUCTURE FUNCTIONS

$$S_q(\delta r) = \langle |\delta C_\infty(\mathbf{r};\delta r)|^q \rangle \sim \int_{\lambda_{min}}^{\lambda_{max}} \delta r^{2-D(\lambda)} |\delta C_\infty(r(\lambda),\delta r)|^q d\lambda \sim$$

q>0

$$\sim \int_{\lambda_{min}}^{b} \delta r^{2-D(\lambda)} \delta r^q d\lambda + \int_b^{\lambda_{max}} \delta r^{2-D(\lambda)} \delta r^{qb/\lambda} d\lambda$$

$$S_q(\delta r) \sim \delta r^{\zeta_q}$$

anomalous scaling

$$\zeta_q = \min_\lambda \left\{ q, \frac{qb}{\lambda} + 2 - D(\lambda) \right\} = \min_\lambda \left\{ q, \frac{qb + G(\lambda)}{\lambda} \right\}$$

POWER SPECTRUM: $G(k) \sim k^{-\beta}$,

$\beta = 1 + \zeta_2 = 1 + \min(2, 2b/\lambda_0) +$ intermittency corrections

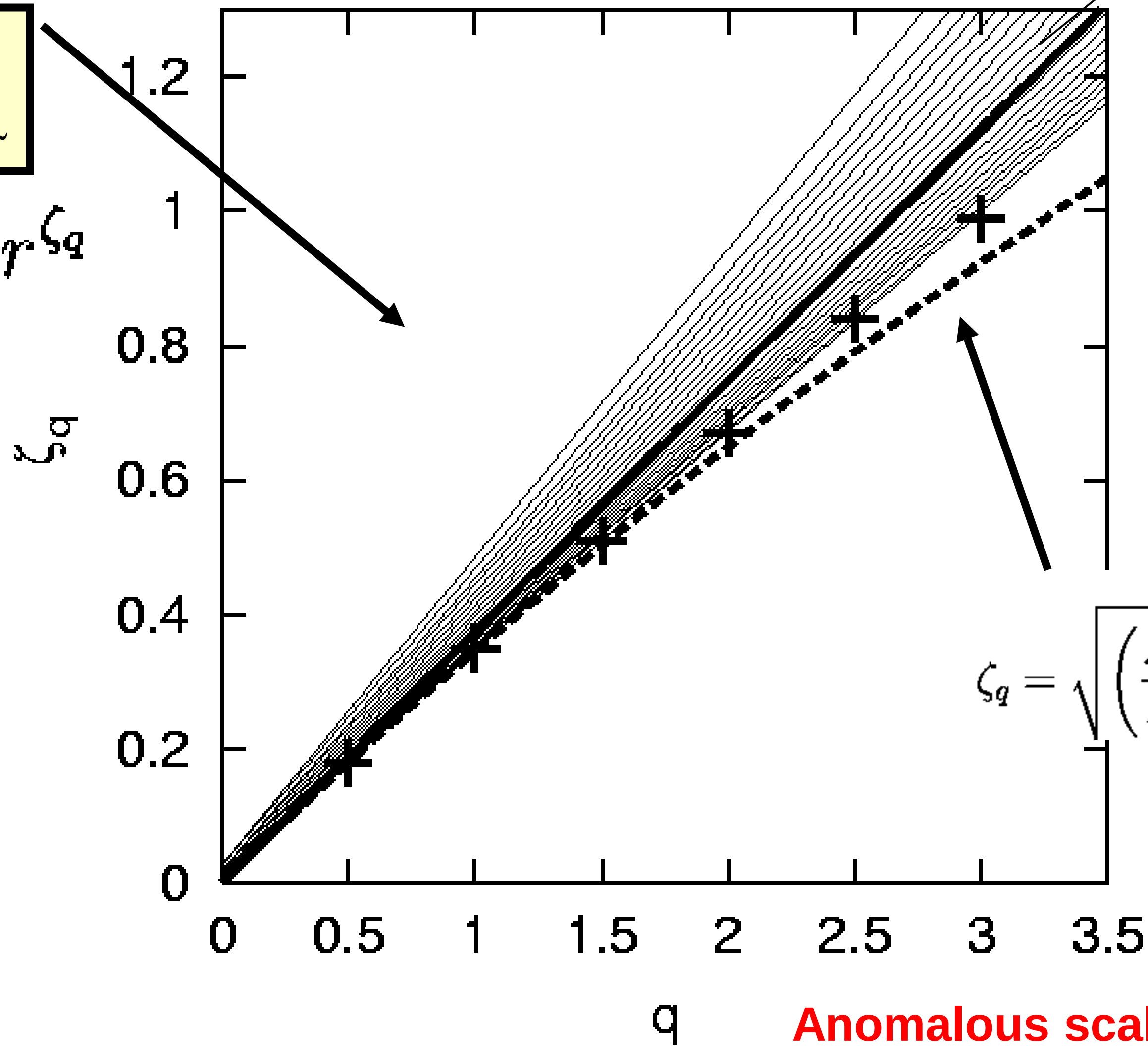
Biological dynamics: linear relaxation at rate b

qb/λ_0

$\zeta_q = q$

$\zeta_q = (qb + G(\lambda))/\lambda$

$$S_q(\delta r) \sim \delta r^{\zeta_q}$$



$$\zeta_q = \sqrt{\left(\frac{\lambda_0}{\Delta}\right)^2 + \frac{2qb}{\Delta}} - \frac{\lambda_0}{\Delta}$$

Anomalous scaling: MULTIFRACTALITY

+: Direct numerical measurements

ANOMALOUS SCALING AND MULTIFRACTALITY IN REAL PLANKTON

M. Pascual, J. Plankton Research 17, 1209 (1995)

L. Seuront, F. Schmitt, Y. Lagadeuc, D.. Schertzer, S. Lovejoy, S. Frontier
Geophys. Res. Lett. 23, 3591 (1996)

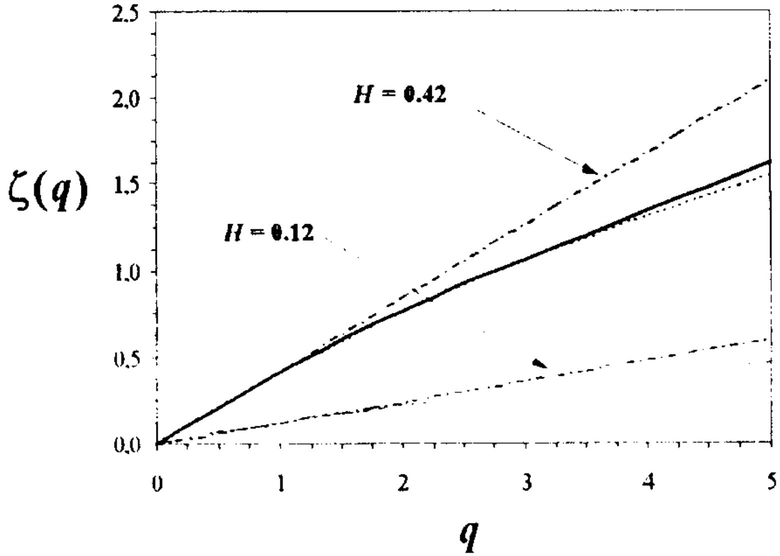


Figure 3. The empirical curves of scaling exponent structure functions $\zeta(q)$ for temperature (thick continuous line), small-scale (dashed line) and large-scale fluorescence (thin continuous line) compared to the theoretical monofractal linear curve $\zeta(q) = qH$ with $H = 0.42$ and $H = 0.12$ (discontinuous lines). The nonlinearity of the empirical curves indicates multifractality.

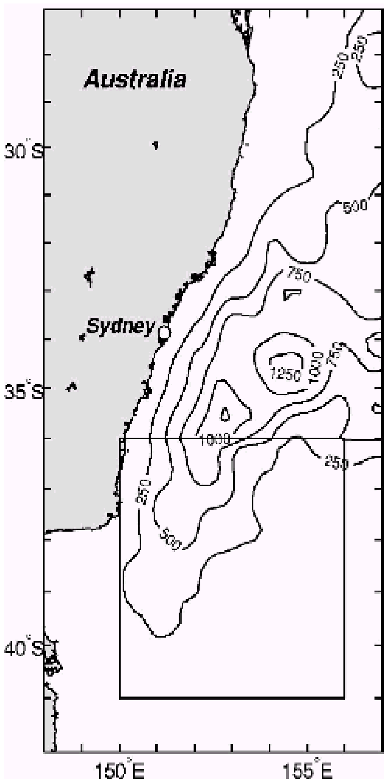
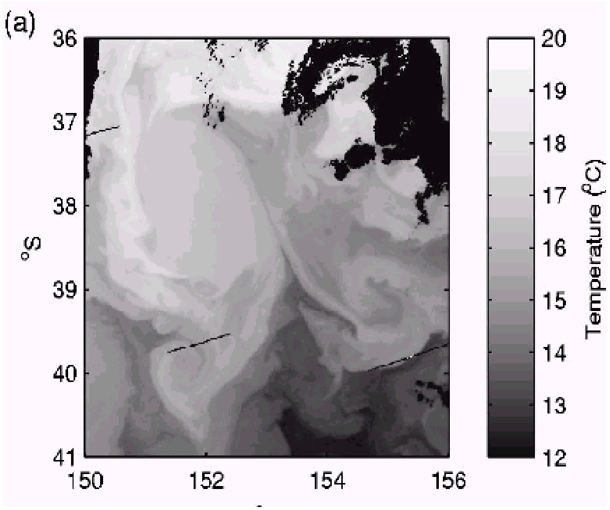


FIG. 1. The study region in the south-west Tasman Sea, showing the region over which the satellite derived velocity data are available. The contours show the average eddy kinetic energy (half the velocity variance, $cm^2\ s^{-2}$) from the 1997 data. The small box marks the area over which stirring rates are derived.



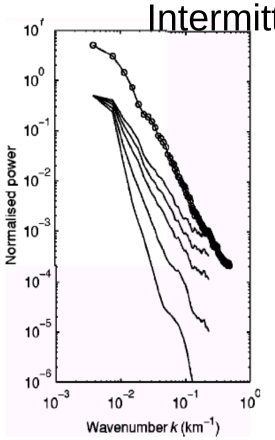


FIG. 6. Power spectra of the actual and modeled SST data shown in Fig. 5. The spectra are the mean of the 1-D power spectra taken along 250 km long sections of constant latitude which contain no invalid data. The normalization is arbitrary, having been chosen to aid comparison of the spectra. The spectrum of the actual SST data is shown by the line with circles. The other lines correspond to the modeled SST, with the following values of α (day^{-1}) from the steepest to the flattest: 0.2, 0.1, 0.05, 0.025 and 0.

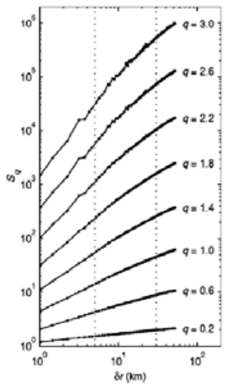


FIG. 7. Structure functions, $S_q(\delta r)$, calculated using Eq. (14) from the 22 November 1997 SST image [Fig. 5(a)]. The scaling exponents, $\zeta(q)$, are calculated from a least-squares fit to the structure functions within the range 5 to 30 km, shown by vertical dotted lines. The structure functions are approximately power-law over this range, but roll off toward larger separations, δr.

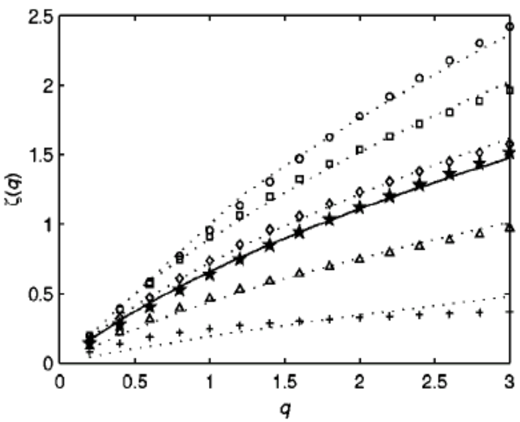


FIG. 8. Multifractal scaling exponents of sea-surface temperature. The solid stars (⋆) are the exponents calculated from the SST data shown in Fig. 5(a). The solid line is a least-squares fit of Eq. (18) to the exponents. The other symbols and the dotted lines mark the scaling exponents calculated from the modeled data and the associated best-fit curve, with the following values of α (day^{-1}): 0.2 (○), 0.1 (□), 0.05 (◇), 0.025 (△), 0 (+).

Abraham and Bowen,
CHAOS 12, 373 (June 2002).

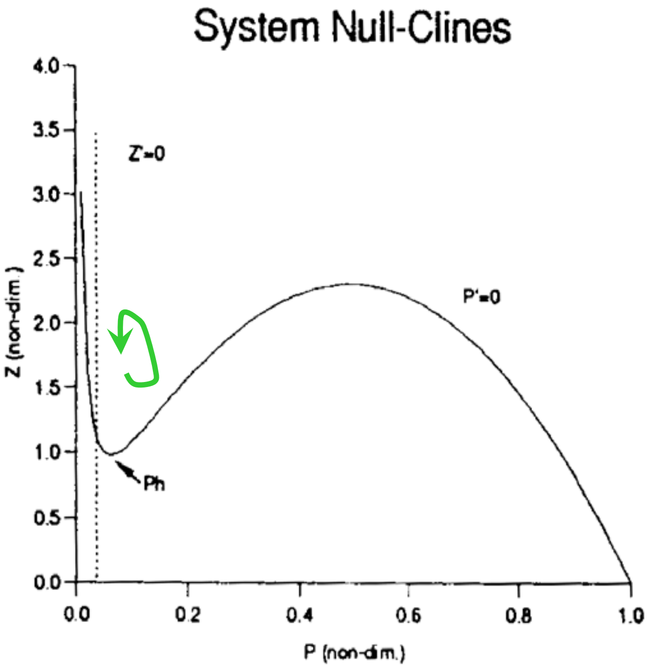
EXCITABLE DYNAMICS

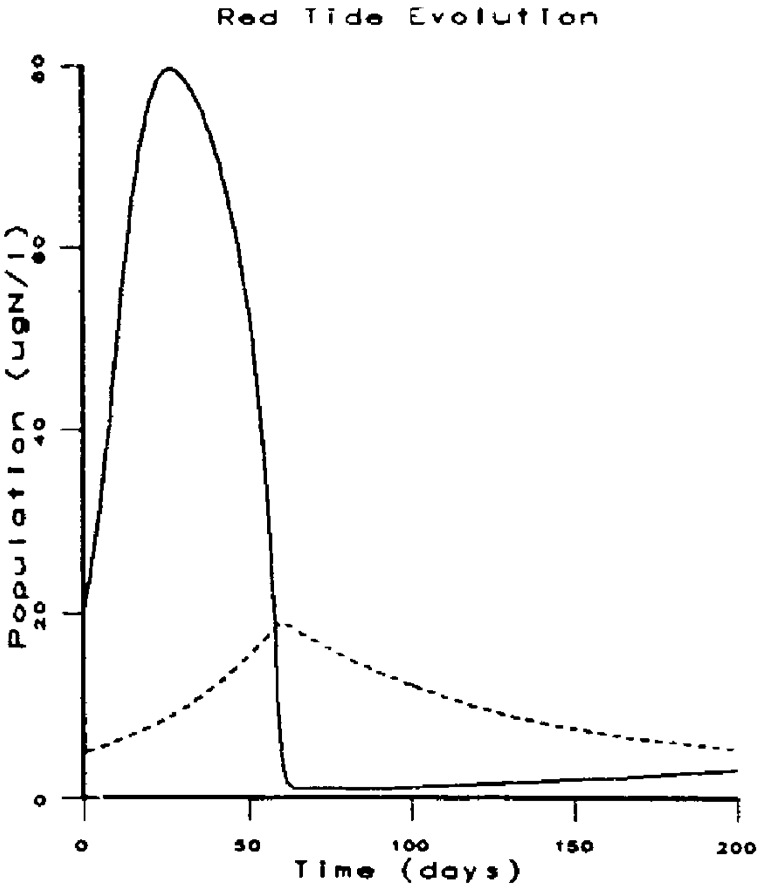
$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right) - \alpha\frac{P^2}{P_0^2+P^2}Z$$

$$\frac{dZ}{dt} = c\alpha\frac{P^2}{P_0^2+P^2}Z - fZ$$

J.E. Truscott and J. Brindley, Bull. Math. Biol. 56, 981 (1994)

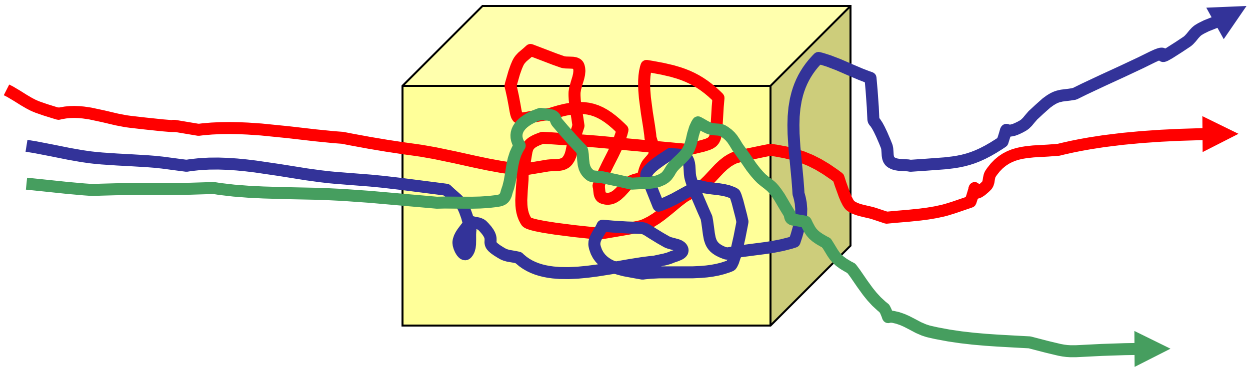
(Any Hollings III zooplankton grazing will do the job)



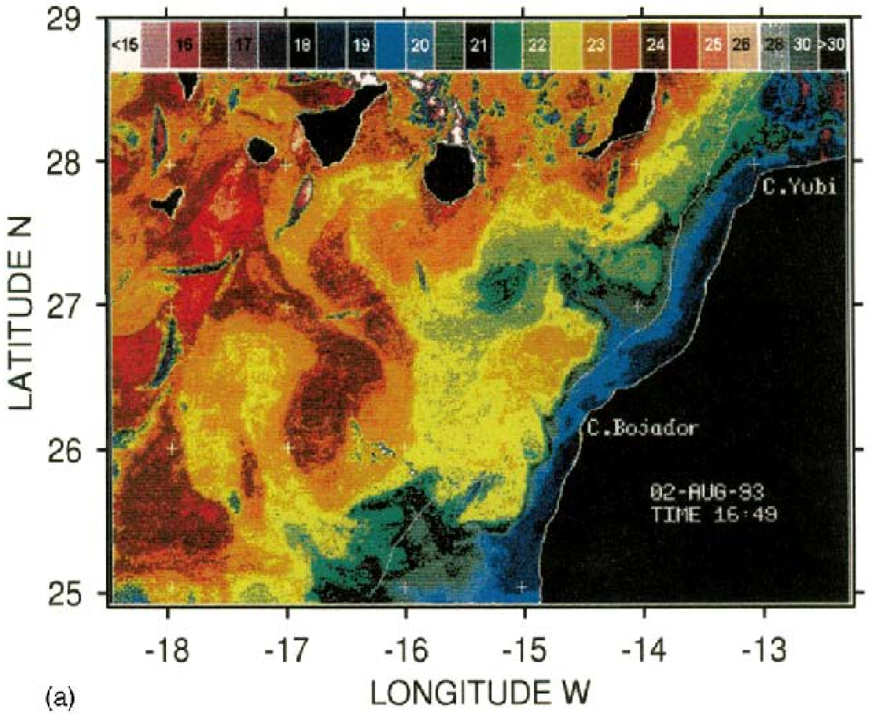


OPEN CHAOTIC FLOWS

transient chaos (or chaotic scattering)



472 E. Barton et al./Progress in Oceanography 41 (1998) 455–504



(a)

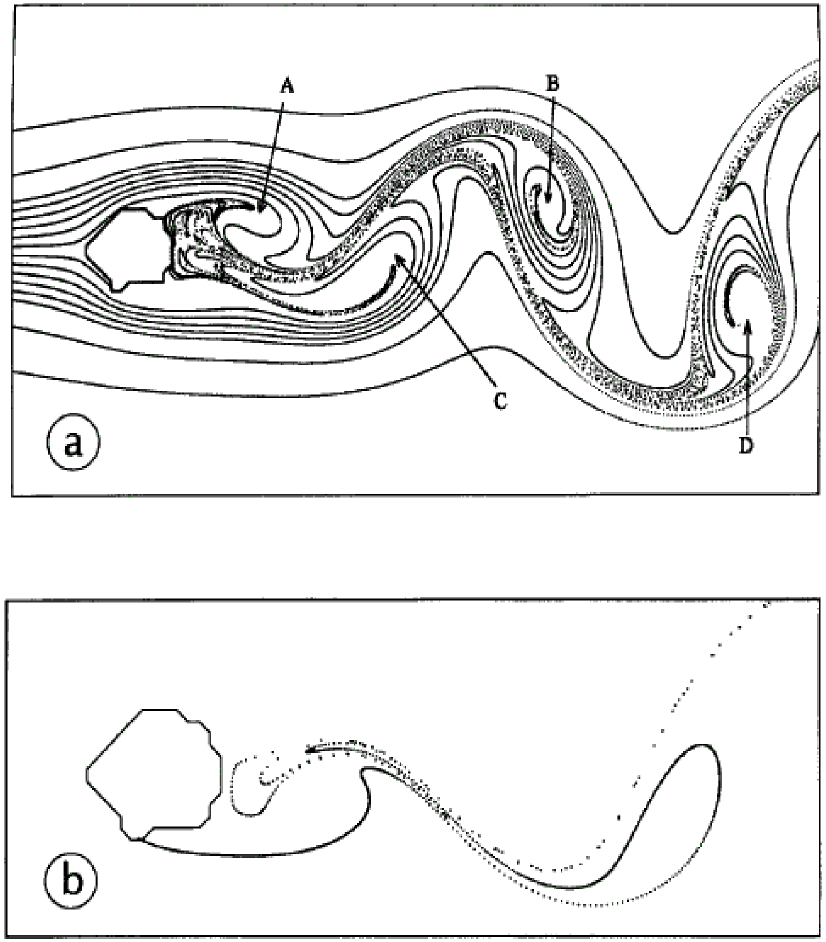


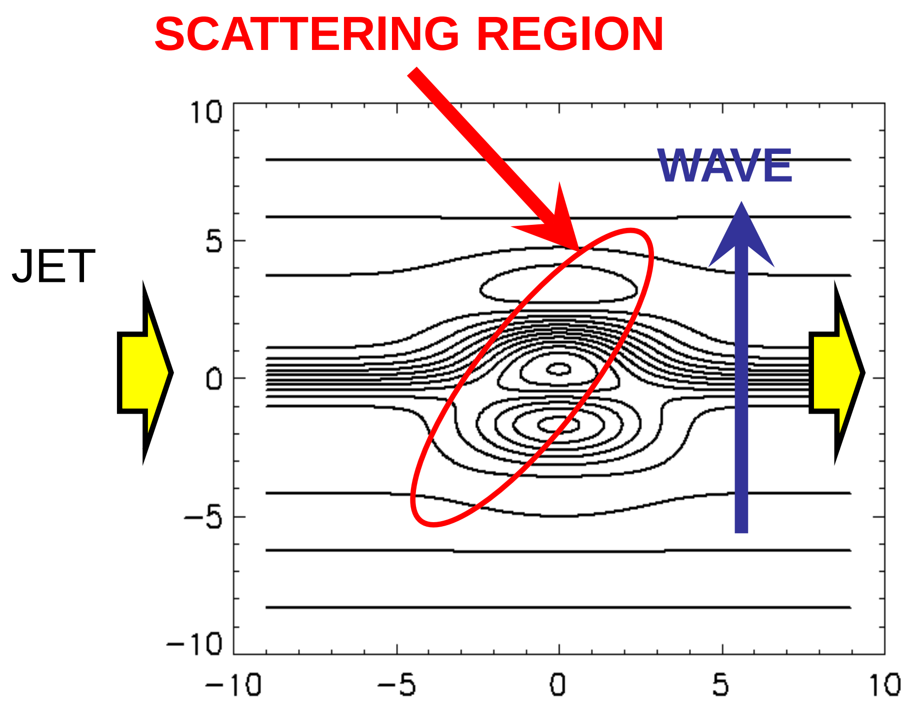
Fig. 10. (a) Simulated streak-line eddies street downstream of Gran Canaria for Reynolds number = 100. (A) Anticyclonic eddy in stage of formation; (B) mature anticyclonic eddy; (C) cyclonic eddy being shed by the obstacle; (D) mature cyclonic eddy. (b) Structure of one streak line that originated on the west side of the island at a different instant than in (a). The incident flow comes from the left side of the picture (northeast). The density of dots is a function of the residence time of the tracer. Regions with dispersion of dots indicate shorter residence times (after Sangrá, 1995).

A MODEL STREAMFUNTION for a jet perturbed by a wave:

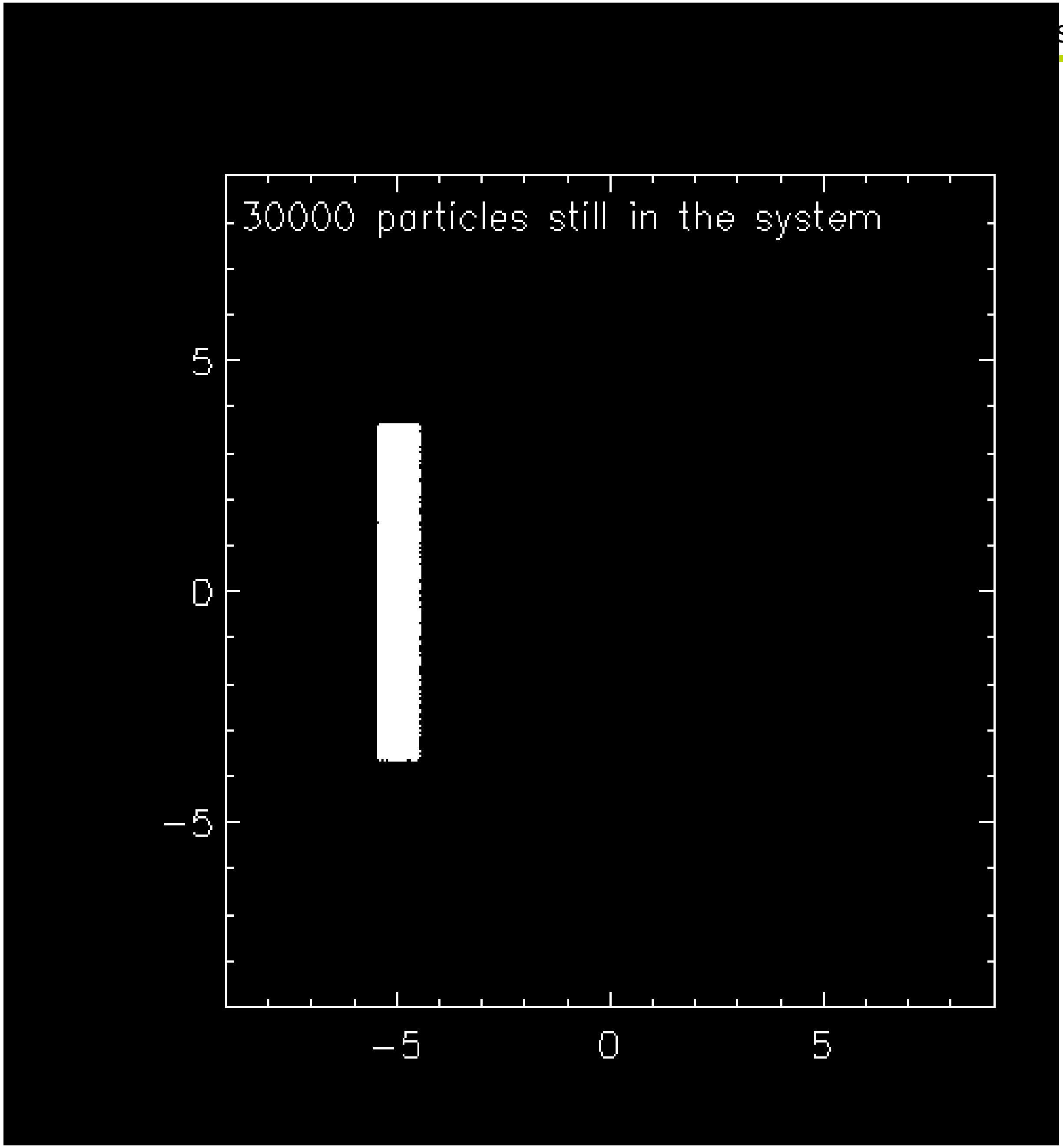
$$\Psi(x,y,t) = \Psi_0 \tanh\left(\frac{x}{w}\right) + \varepsilon \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\cos\left(k(y-vt)\right)$$

$$v_x = \frac{\partial \Psi(x,y,t)}{\partial y}$$

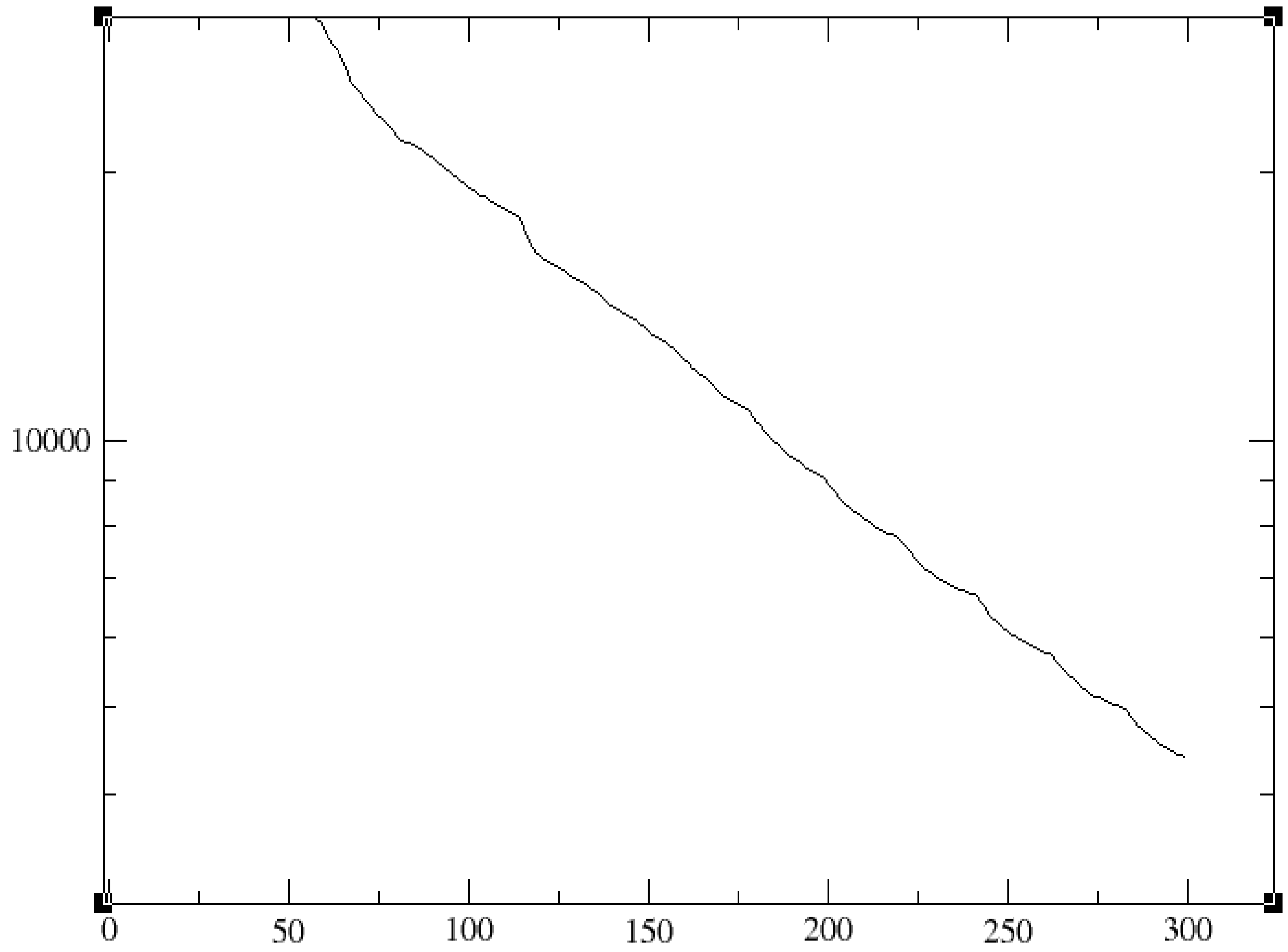
$$v_y = -\frac{\partial \Psi(x,y,t)}{\partial x}$$



CLICK FOR MOVIE



N(t)



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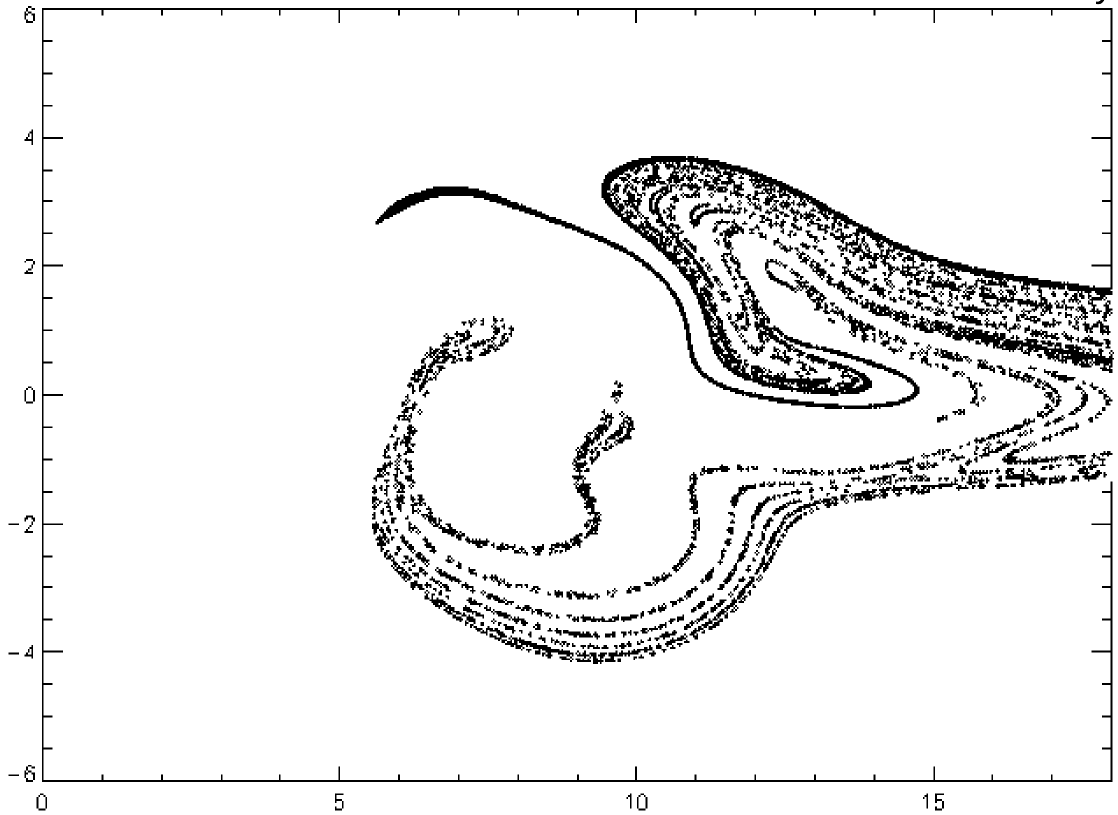


CHAOTIC SADDLE:

set of trajectories never living the chaotic area

Lyapunov exponent $\lambda > 0$, escape tate $\kappa>0$

Fractal set of zero measure. Dimension D=2(1-κ/λ)

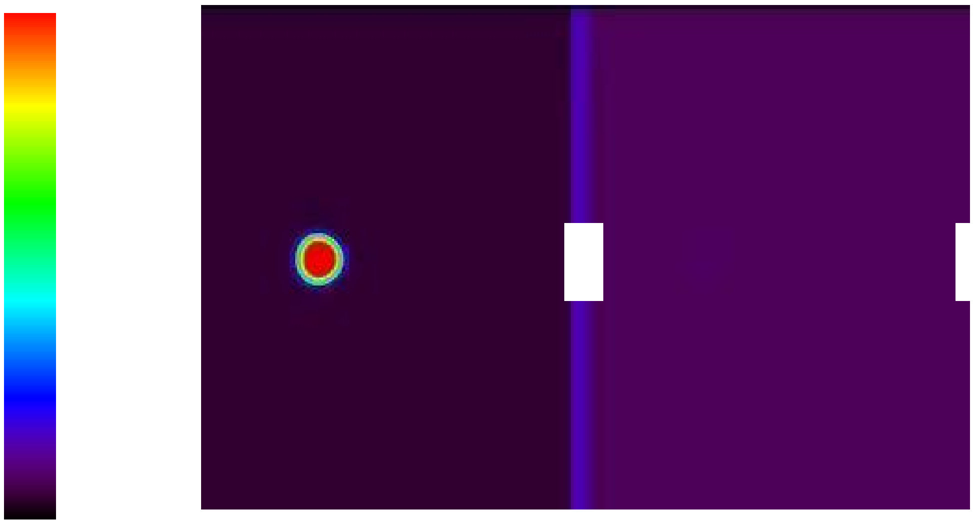
Tracers accumulate at its UNSTABLE MANIFOLD (dimension 2-κ/λ)

$$\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P = rP\left(1 - \frac{P}{K}\right) - \alpha \frac{P^2}{P_0^2 + P^2} Z + D\nabla^2 P$$

$$\frac{\partial Z}{\partial t} + \vec{v} \cdot \nabla Z = c\alpha \frac{P^2}{P_0^2 + P^2} Z - fZ \qquad + D\nabla^2 Z$$

Advection-Reaction-Diffusion dynamics.

SIMPLE JET WITHOUT SCATTERING PERTURBATION



CLICK FOR MOVIE

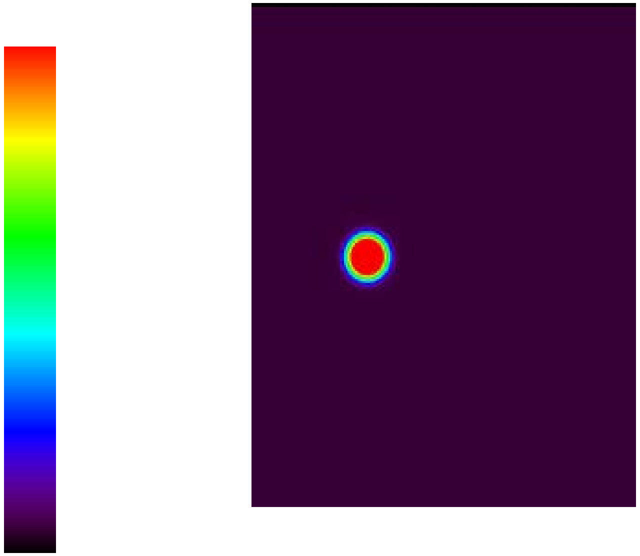
Phytoplankton Zooplankton

$$\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P = rP\left(1 - \frac{P}{K}\right) - \alpha \frac{P^2}{P_0^2 + P^2} Z + D\nabla^2 P$$

$$\frac{\partial Z}{\partial t} + \vec{v} \cdot \nabla Z = c\alpha \frac{P^2}{P_0^2 + P^2} Z - fZ \qquad + D\nabla^2 Z$$

Advection-Reaction-Diffusion dynamics.

Phytoplankton



Fast stirring-slow growth

Quenching by fast stretching

CLICK FOR MOVIE

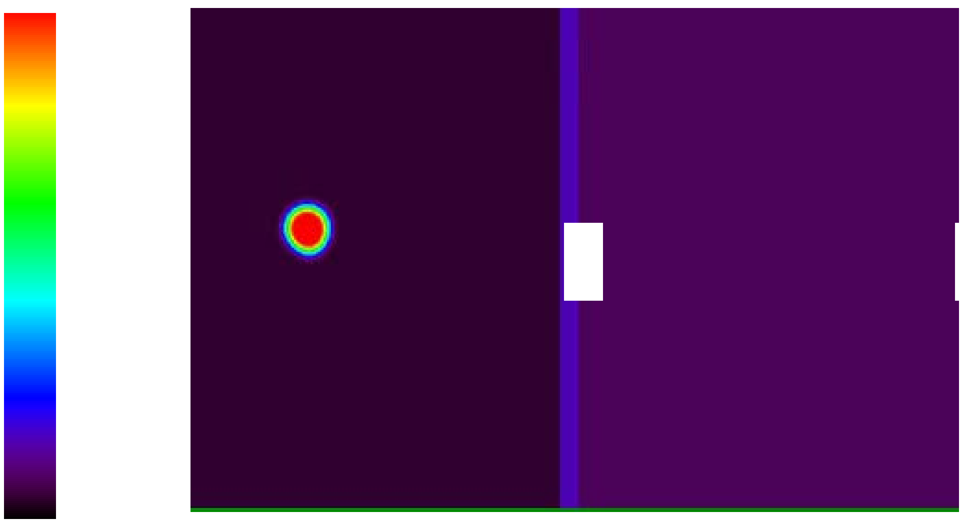
$$\frac{\partial P}{\partial t}+\vec{v}\cdot\nabla P = rP\left(1-\frac{P}{K}\right)-\alpha\frac{P^2}{P_0^2+P^2}Z + D\nabla^2 P$$

$$\frac{\partial Z}{\partial t}+\vec{v}\cdot\nabla Z = c\alpha\frac{P^2}{P_0^2+P^2}Z - fZ \qquad + D\nabla^2 Z$$

Advection-Reaction-Diffusion dynamics.

JET WITH CHAOTIC SCATTERING REGION

CLICK FOR MOVIE



Phytoplankton Zooplankton

Intermediate stretching values

- In the absence of flow, excitable blooms are transient phenomena: at each spatial point, excitation occurs and dissapears

- Chaotic scattering is a transient phenomenon: almost all particle trajectories leave the system soon or later

- TOGETHER: Permanent pattern of excitation: sustained productivity with species permanently at a much higher concentration than the stable equilibrium value.

WHY ?

HOW GENERAL ? → Excitable dynamics in open flows

A DIFFERENT A SIMPLER MODEL

Neufeld, López, Hernández-García, Piro, Phys. Rev. E 66, 066208 (2002)

$$\frac{DP}{Dt} = \mathrm{Da}\left[P(P-a)(1-P)-Z\right]+D\nabla^2 P$$

$$\frac{DZ}{Dt} = \varepsilon \mathrm{Da}\left(P-fZ\right)+D\nabla^2 Z$$

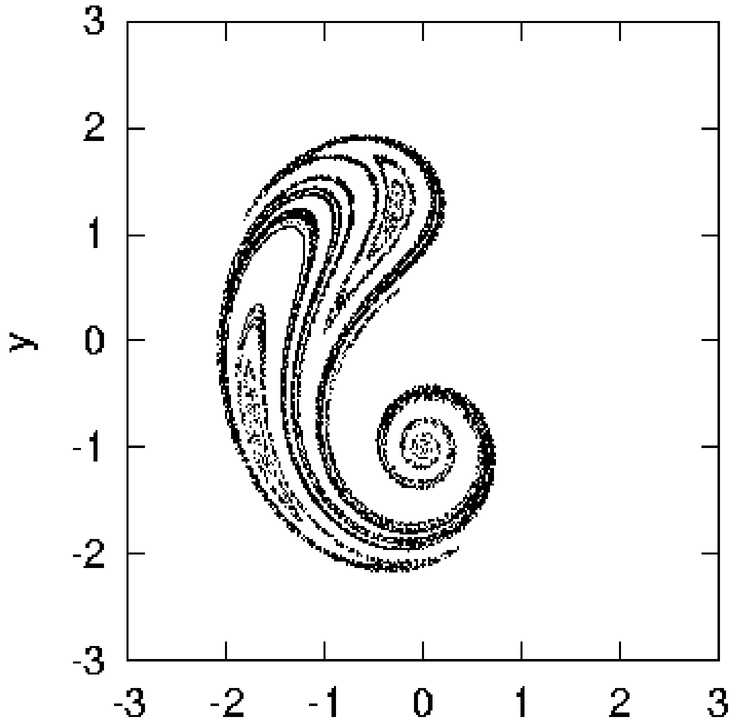
A simple two-species-competition **excitable** dynamics: FitzHugh-Nagumo model

+

+ blinking vortex-sink **open** flow (Károlyi & Tél, Phys. Rep. 290, 125 (1997))

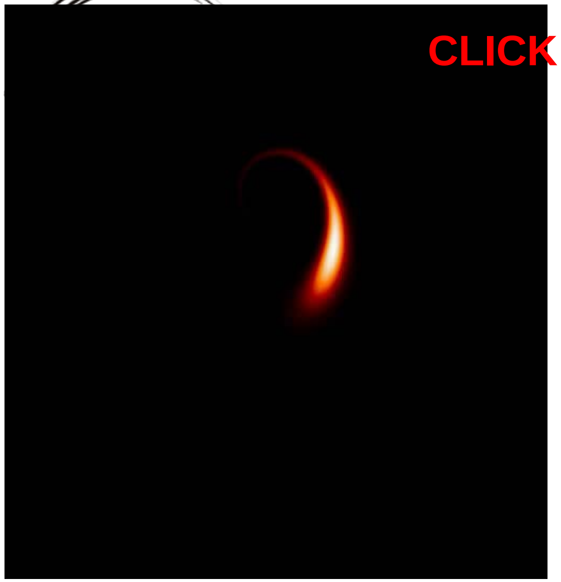
+

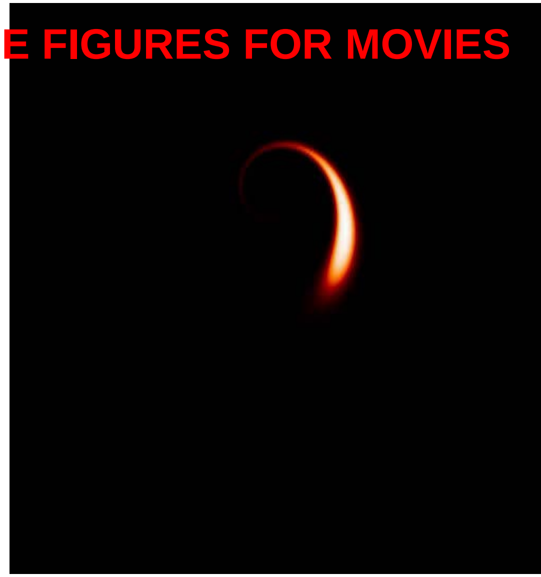
Diffusion

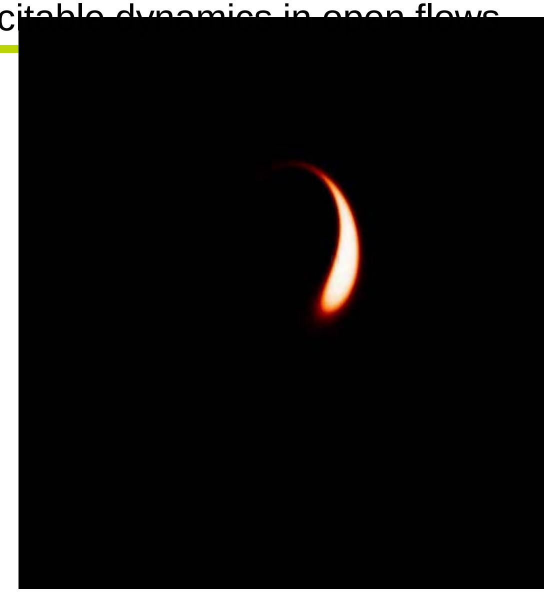


Da=Damköhler number, ratio of the activator (phytoplankton) growth rate to a strain rate in the flow

CLICK THE FIGURES FOR MOVIES



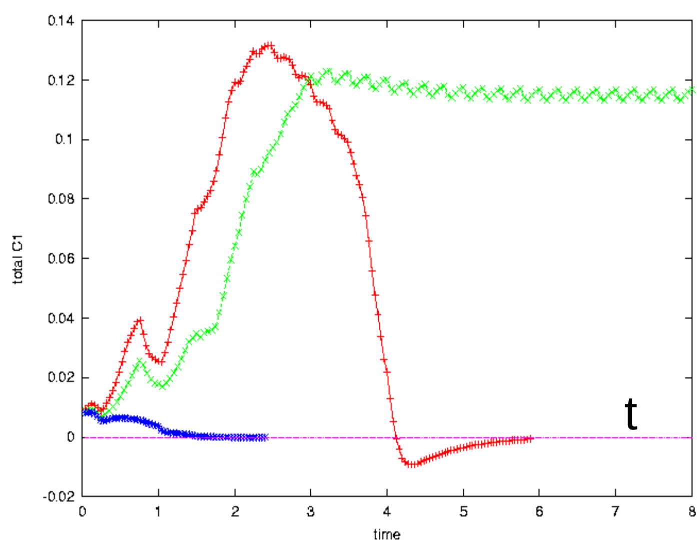


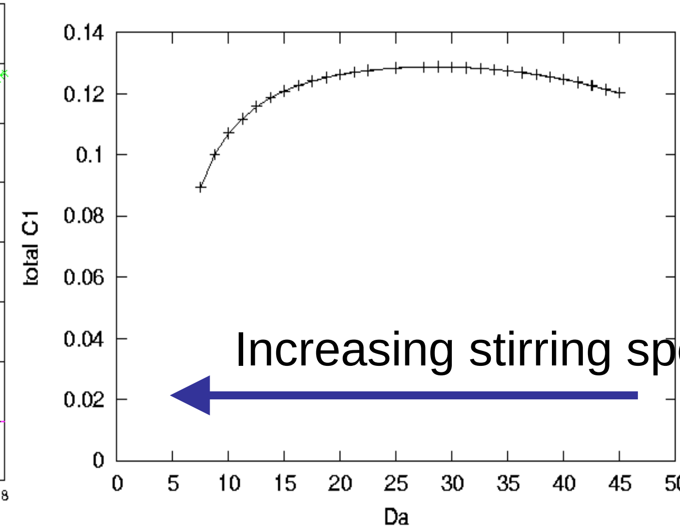


Fast stirring

Intermediate stirring

Slow stirring





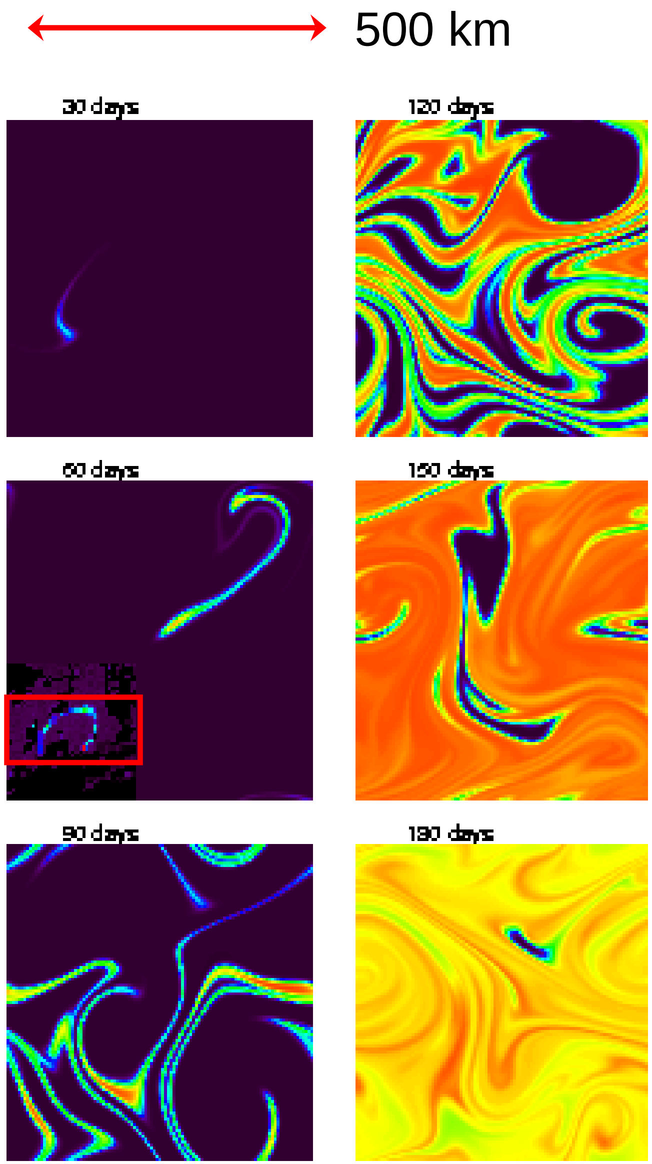
OPEN BLINKING VORTEX-SINK FLOW

Neufeld, Haynes, Garçon, Sudre (2001)

$$\frac{\partial P}{\partial t} + \mathbf{v}(\mathbf{r},t)\cdot\nabla P = r(F)P\left(1-\frac{P}{K}\right) - gZ\frac{P^2}{\alpha^2+P^2} + \kappa\Delta P$$

$$\frac{\partial Z}{\partial t} + \mathbf{v}(\mathbf{r},t)\cdot\nabla Z = \gamma gZ\frac{P^2}{\alpha^2+P^2} - \mu Z + \kappa\Delta Z$$

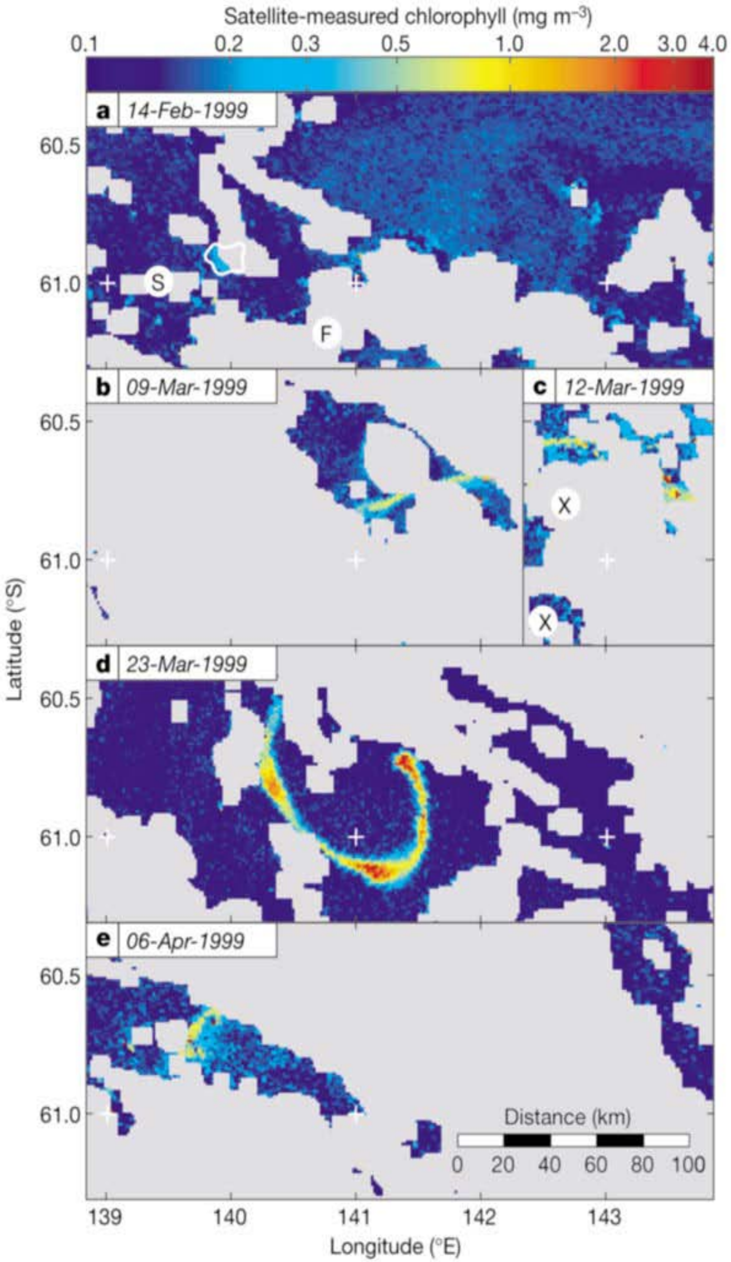
$$\frac{\partial F}{\partial t} + \mathbf{v}(\mathbf{r},t)\cdot\nabla F = \kappa\Delta F$$

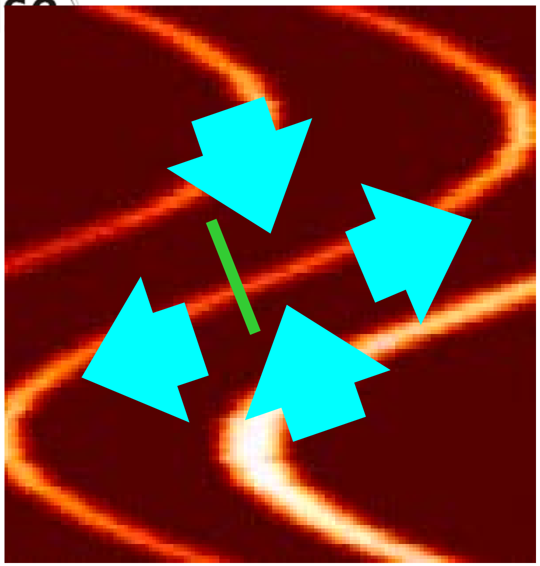


Flow: random seeded eddy model

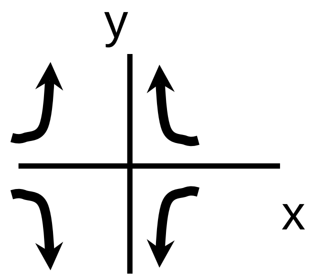
$r(F) = r_0(1 + F/F_0)$

The SOIRÉE fertilization experiment. Nature 407, 695 (2000)





Focus in hyperbolic regions:



$v_x = -\lambda x$

$v_y = +\lambda y$

Local strain flow

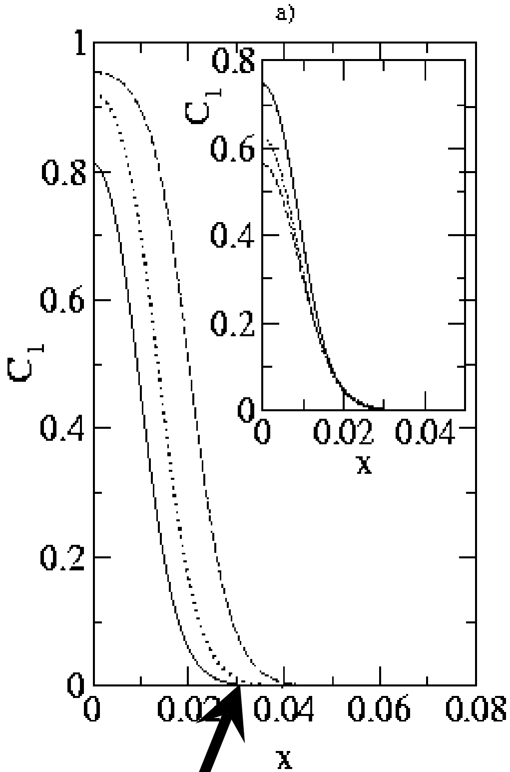
Tracers accumulate in the UNSTABLE FOLIATON

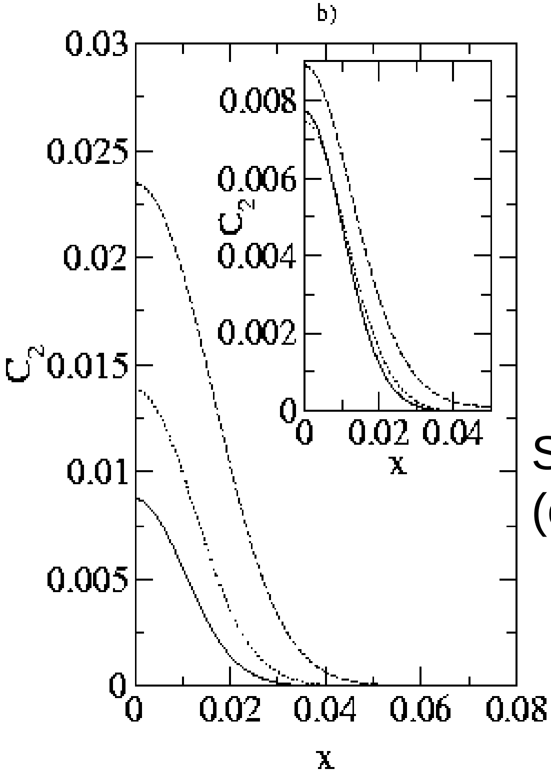
After a transient, the expanding direction becomes homogeneized and analysis is simplified by focussing on the onedimensional transverse structure of the filaments:

$$\partial P/\partial t - \lambda x \partial P/\partial x = f(P,Z) + D\partial^2 P/\partial x^2$$
$$\partial Z/\partial t - \lambda x \partial Z/\partial x = g(P,Z) + D\partial^2 Z/\partial x^2$$

FILAMENT MODEL

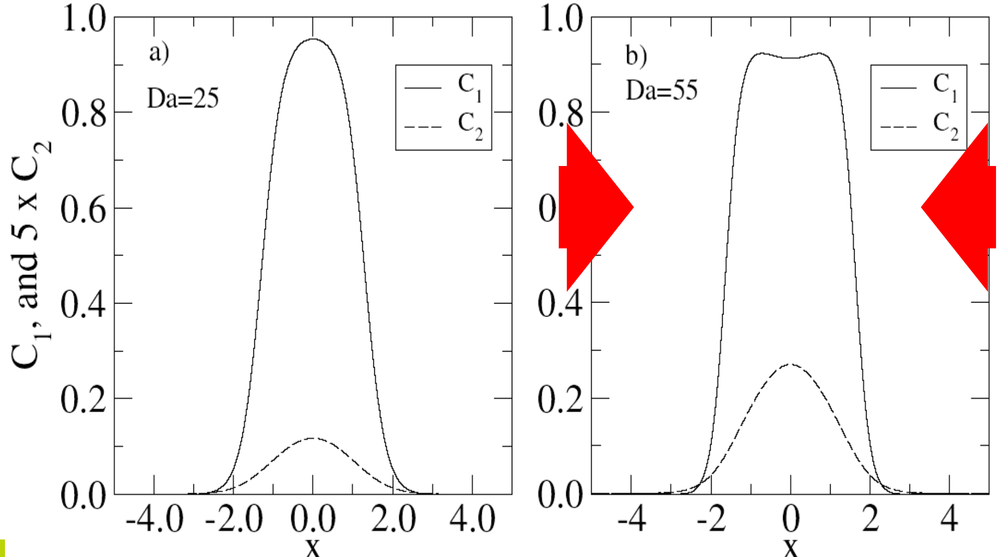
zooplankton





Stable and unstable solutions (obtained by shooting, FN model)

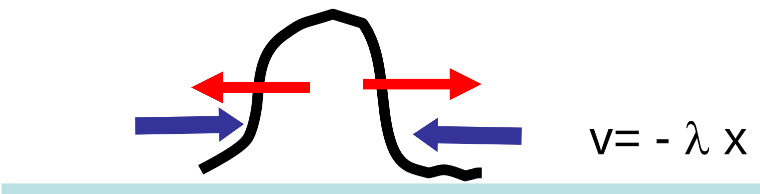
phytoplankton



Flow

speed=(1-a) √D/2 ≈ width . λ

$$width \approx \frac{1-a}{\lambda}\sqrt{\frac{D}{2}}$$



v= - λ x

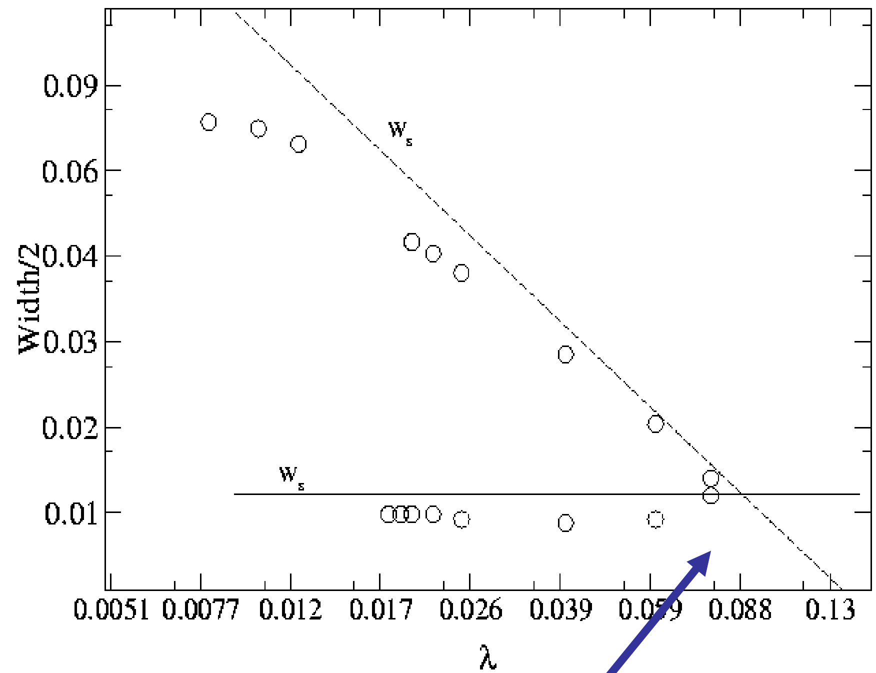
Unstable filament

$$kC_1(a - C_1)(C_1 - 1) + D\frac{\partial^2}{\partial x^2}C_1 = 0$$

$$C_1^u(x) = C_+ - \frac{C_+ - C_-}{1 - \frac{C_-}{C_+}\tanh^2\left(\frac{x}{w_u}\right)}$$

$$C_{\pm} = \frac{2}{3}(1+a) \pm \sqrt{\frac{4}{9}(1+a)^2 - 2a}\ .$$

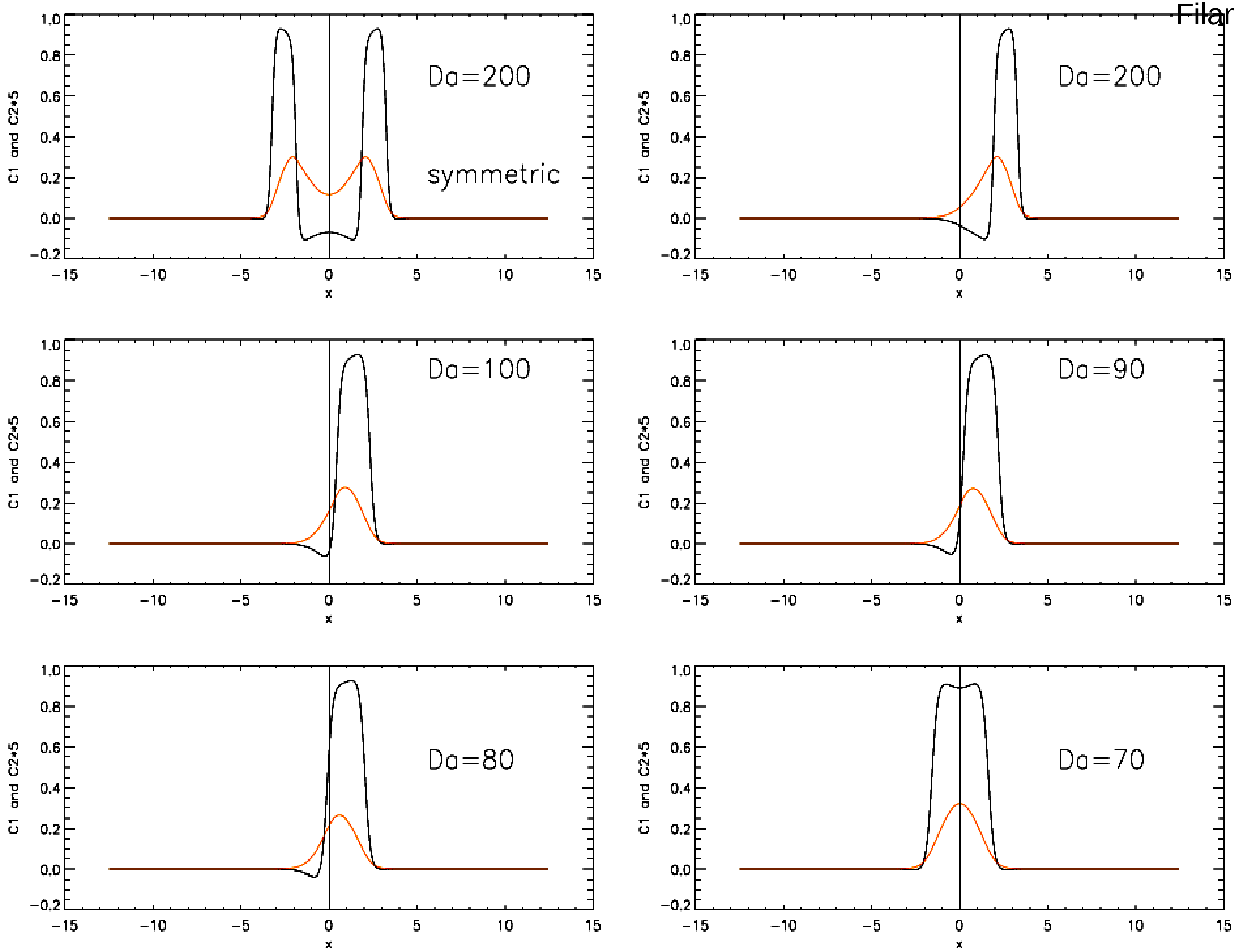
$$w_u = 2\sqrt{\frac{D}{ak}}$$



If λ =1/T

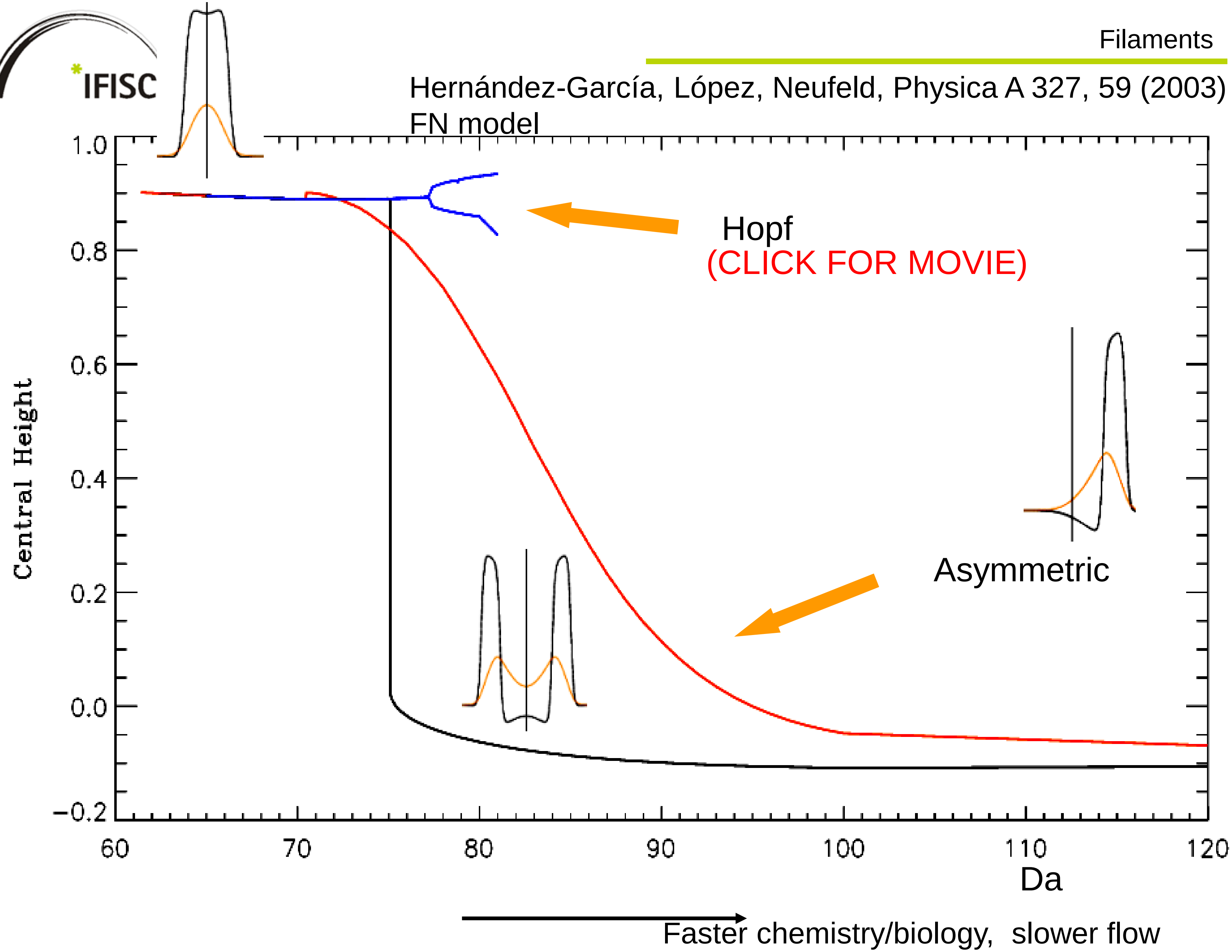
λ=√a (1–2a)/2

Gives ‘correctly’ the *extinction transition* of the 2d flow



Asymmetric filaments

Hernández-García, López, Neufeld, Physica A 327, 59 (2003)
FN model



In summary:

- Chaotic advection + simple biological dynamics present a variety of scenarios that (hopefully) may help to classify and interpret environmental observations.

- The models are simple enough to allow efficient numerics and even analytic understanding.

- Linear relaxational dynamics + chaotic advection = intermittent distributions of substances, with calculable spectral properties.

- Transient excitable dynamics + transient chaotic motion = permanent patterns of excitation. The responsible: filament structures.

There is much more …

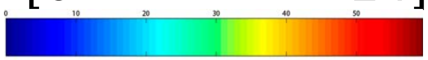
e.g. Relationship with LCS: burning invariant manifolds:
Mitchell & Mahoney, Chaos 22, 037104 (2012), Chaos 23,043106 (2013)

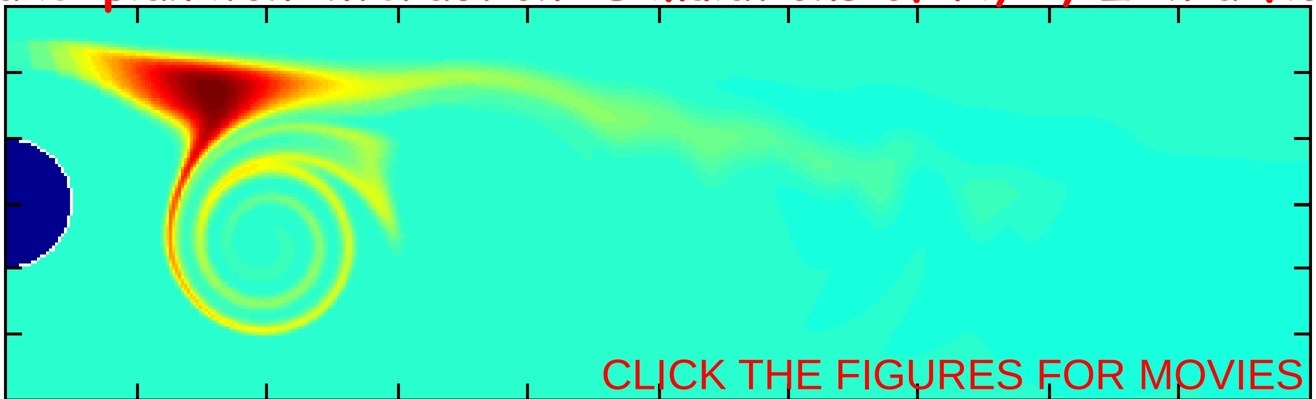
Wake-plankton interaction: Simulations of N, P, Z in a flow

steady state inflow concentrations (N, P, Z)

phytoplankton

[0 14]

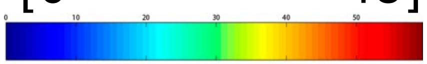


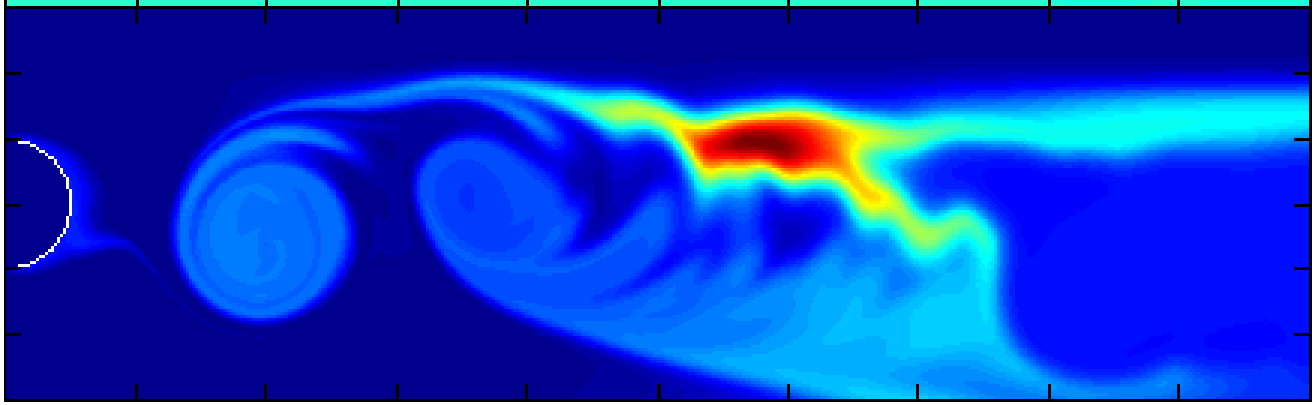


low inflow concentrations (N, P, Z)

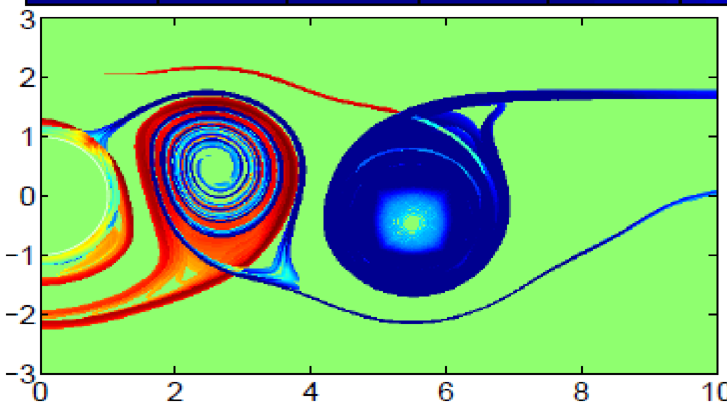
phytoplankton

[0 45]





FSLE field



Sandulescu et al. *Nonlinear Processes in Geophysics* **14**, 443-454 (2007)

Sandulescu et al. *Ecological Complexity* **5**, 228-237 (2008)

Hernández-Carrasco, Rossi, Hernández-García, Garçon and López

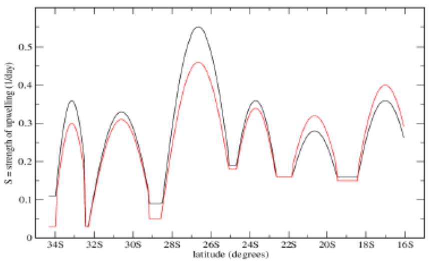
The reduction of plankton biomass induced by mesoscale stirring: A modeling study in the Benguela upwelling.

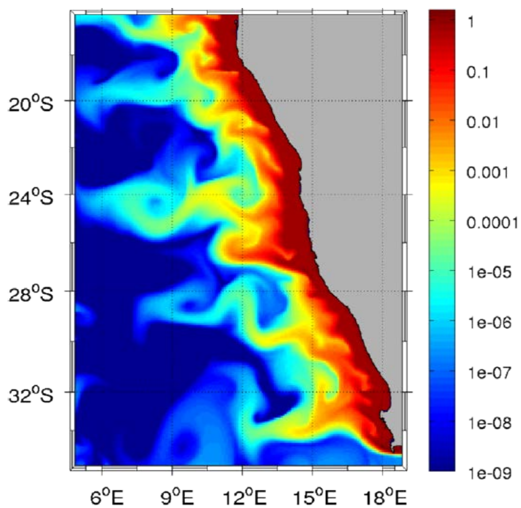
Deep-Sea Research I, 83, 65-80 (2013)

Advection-Reaction-Diffusion Equations

$$\frac{\partial N}{\partial t} + \mathbf{v}\nabla N = F_N + D\nabla^2 N$$
$$\frac{\partial P}{\partial t} + \mathbf{v}\nabla P = F_P + D\nabla^2 P$$
$$\frac{\partial Z}{\partial t} + \mathbf{v}\nabla Z = F_Z + D\nabla^2 Z$$

$$F_N = \Phi_N - \beta\frac{N}{\kappa_N + N}P + +\mu_N\left((1-\gamma)\frac{\alpha\eta P^2}{\alpha+\eta P^2}Z + \mu_P P + \mu_z Z^2\right)$$
$$F_P = \beta\frac{N}{\kappa_N + N}P - \frac{\alpha\eta P^2}{\alpha+\eta P^2}Z - \mu_P P$$
$$F_Z = -\gamma\frac{\alpha\eta P^2}{\alpha+\eta P^2}Z - \mu_Z Z^2$$





P (mg/m³)

Conclusion: Mesoscale turbulence greatly enhances nutrient flux out of upwelling cells. More turbulence -> less nutrients available