IV Summer School on Statistical Physics of Complex and Small Systems
Palma de Mallorca, 8-19 September 2014

Large-scale transport in oceans

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Statistical Physics and Dynamical Systems approaches in Lagrangian Fluid Dynamics

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Universitat de les Illes Balears

STATISTICAL PHYSICS AND DYNAMICAL SYSTEMS APPROACHES IN LAGRANGIAN FLUID DYNAMICS

OUTLINE

1. Lagrangian fluid dynamics and introduction to chaotic advection. Hamiltonian dynamics, KAM tori, Lyapunov exponents, open flows
2. **Dispersion, diffusion and coherent structures in flows.** Turbulent, pair and chaotic dispersion, gradient production, FTLE, FSLE, Lagrangian Coherent Structures
3. Chemical and biological processes in flows. Fisher and excitable plankton waves, filamental transitions, lamellar approaches, burning manifolds
4. Complex networks of fluid transport. Directed and weighted flow networks. Community detection

Diffusion equation: $\frac{\partial C}{\partial t} = D\nabla^2 C.$

$$C(\mathbf{x},t) = \int d\mathbf{x}' G(\mathbf{x}-\mathbf{x}',t) C_0(\mathbf{x}') \ , \qquad G(\mathbf{x},t) = \frac{1}{(4\pi D t)^{d/2}} e^{-\frac{\mathbf{x}^2}{4Dt}}.$$

$$w^2 \equiv \frac{\int \mathbf{x}^2 G(\mathbf{x},t) d\mathbf{x}}{\int G(\mathbf{x},t) d\mathbf{x}}, \qquad w \approx (2dDt)^{1/2} \ ,$$

Turbulent diffusion- diffusion-like behavior by advection:

$$\frac{d}{dt}\langle(\mathbf{r}-\mathbf{r}_0)^2\rangle = 2\langle(\mathbf{r}-\mathbf{r}_0)\cdot\mathbf{v}[\mathbf{r}(t)]\rangle.$$

$$\langle(\mathbf{r}-\mathbf{r}_0)^2\rangle = 2\int_0^t dt' \int_0^{t'} dt'' \langle \mathbf{v}[\mathbf{r}(t')]\cdot\mathbf{v}[\mathbf{r}(t'-t'')]\rangle$$

$$f_L(t',t'') \equiv \frac{1}{\langle v^2\rangle}\langle \mathbf{v}[\mathbf{r}(t')]\cdot\mathbf{v}[\mathbf{r}(t'-t'')]\rangle \qquad T_L \equiv \int_0^\infty d\tau f(\tau) \ .$$

Lagrangian correlation time

$$\langle(\mathbf{r}-\mathbf{r}_0)^2\rangle \simeq \langle v^2\rangle t^2 \qquad t \ll T_L \qquad\qquad \langle(\mathbf{r}-\mathbf{r}_0)^2\rangle = \langle v^2\rangle T_L t \qquad t \gg T_L.$$

$$\langle(\mathbf{r}-\mathbf{r}_0)^2\rangle = \langle v^2\rangle T_L t \qquad\qquad \boxed{D_T \simeq \frac{1}{2d}\langle v^2\rangle T_L.}$$

Taylor dispersion

$$\frac{d}{dt}\langle(\mathbf{r}_2(t)-\mathbf{r}_1(t))^2\rangle = 2\langle(\mathbf{r}_2(t)-\mathbf{r}_1(t))(\mathbf{v}(\mathbf{r}_2)-\mathbf{v}(\mathbf{r}_1))\rangle$$

Small separations – similar to infinitesimal dispersion:

$$\langle(\mathbf{r}_2(t)-\mathbf{r}_1(t))^2\rangle \approx \exp(\lambda t)$$

Large separations – similar to Taylor turbulent dispersion: $D=2D_T$

In between … correlated dispersion.
For example, in the inertial range of 3d turbulence:

$$\frac{\langle[\mathbf{v}(\mathbf{r}_2)-\mathbf{v}(\mathbf{r}_1)](\mathbf{r}_2-\mathbf{r}_1)\rangle}{|\mathbf{r}_2-\mathbf{r}_1|} = C\epsilon^{1/3}|\mathbf{r}_2-\mathbf{r}_1|^{1/3}$$

Kolmogorov scaling

$$\frac{d}{dt}\langle(\mathbf{r}_2(t)-\mathbf{r}_1(t))^2\rangle = 2C\epsilon^{1/3}|\mathbf{r}_2-\mathbf{r}_1|^{4/3}$$

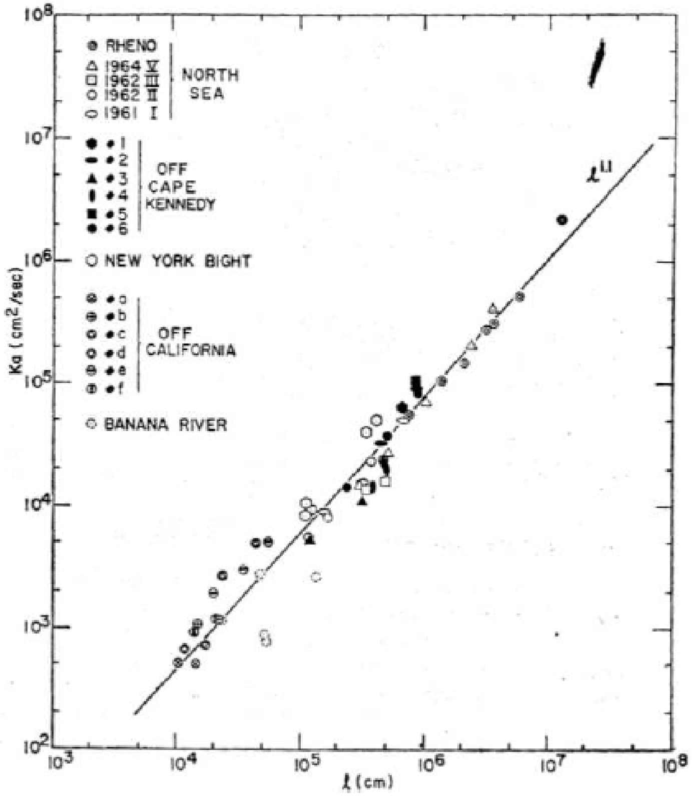
$$\langle|\mathbf{r}_2-\mathbf{r}_1|^2\rangle = \left(\frac{2}{3}C\right)^3\epsilon t^3$$

Richardson law

This is somehow equivalent to a scale-dependent diffusivity: $D = C\epsilon^{1/3}|\mathbf{r}_2 - \mathbf{r}_1|^{4/3}$

Empirical effective (pair) diffusivity
Okubo, Dee Sea Res. 18, 789 (1971)

$$D_{eff}(l) \sim l^{1.15}$$

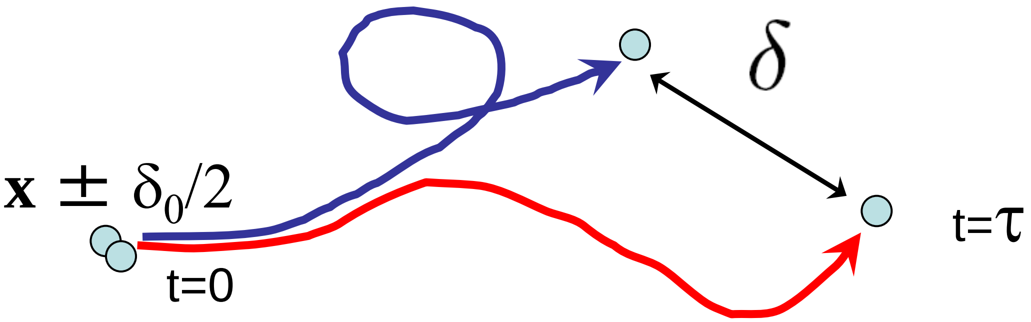


$$\lambda(t) = \lim_{\|\delta(0)\|\to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t\to\infty} \lambda(t)$$

Lyapunov exponent



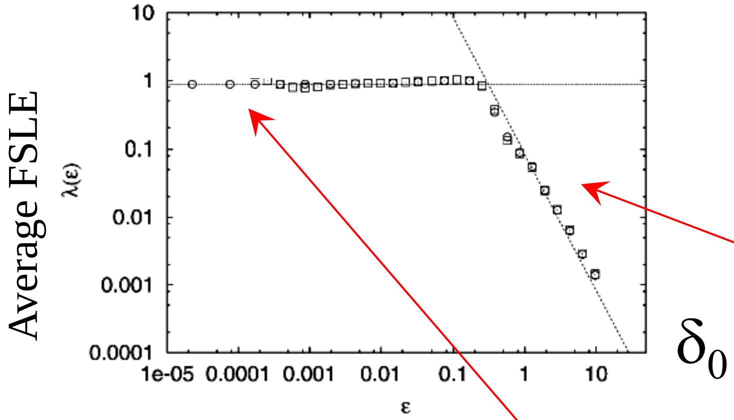
$$\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

Finite-size Lyapunov exponent

FSLE

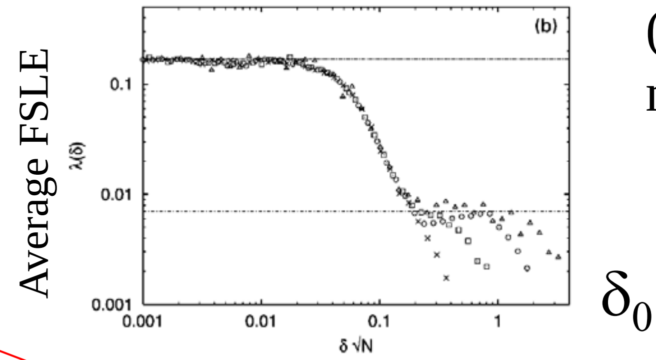
All the quantities are also functions of the initial position and time: $\lambda(\mathbf{x}, t, \delta_0, \delta_f)$

A chaotic map



System with several time scales

(coupled maps)

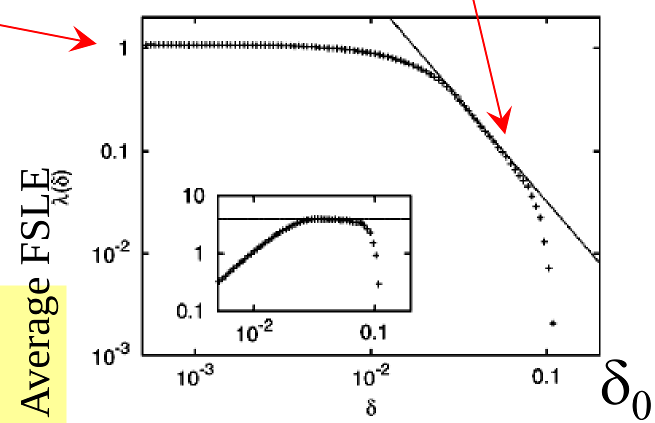


Subexponential growth (diffusion regime)

$\lambda(\delta) \sim \delta^{-2}$

Exponential growth of separations (chaotic regime)

G. Boffetta et al. / Physics Reports 356 (2002) 367–474

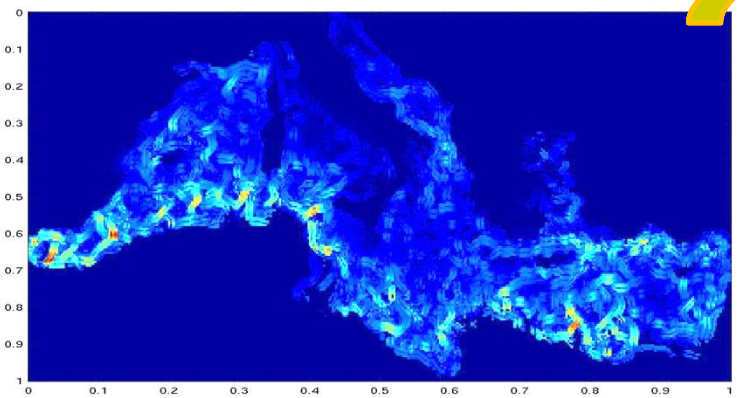


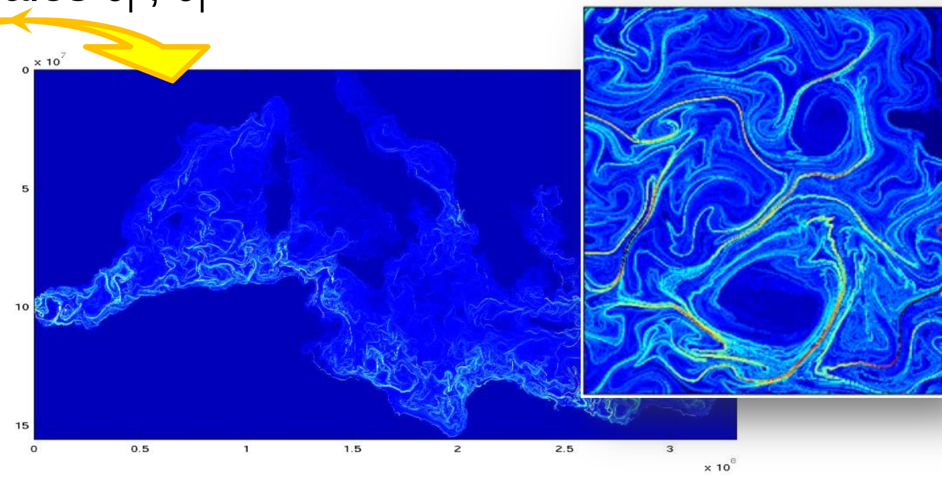
2D turbulence

When $\delta_0 \to 0$,
FSLE→Lyapunov
and when $t \to \infty$,
FTLE→Lyapunov

The FSLE was originally introduced to quantify dispersion from non-infinitesimal initial separations (Aurell et al. 1997)

Reducing scales δ_i , δ_f



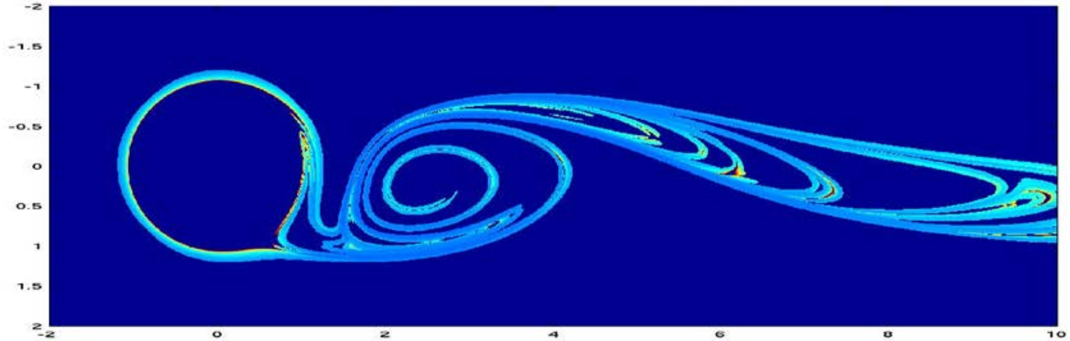


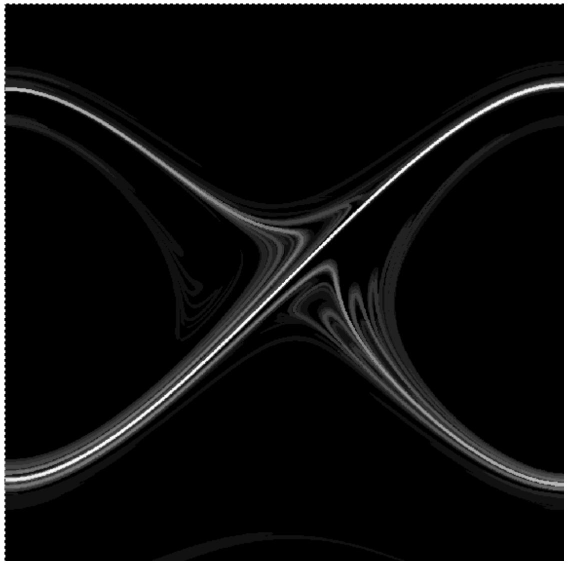
FSLE for small enough scales, $\leftrightarrow$ **FTLE** for large enough times

Forward in time: repelling manifolds

Backward in time: attracting manifolds

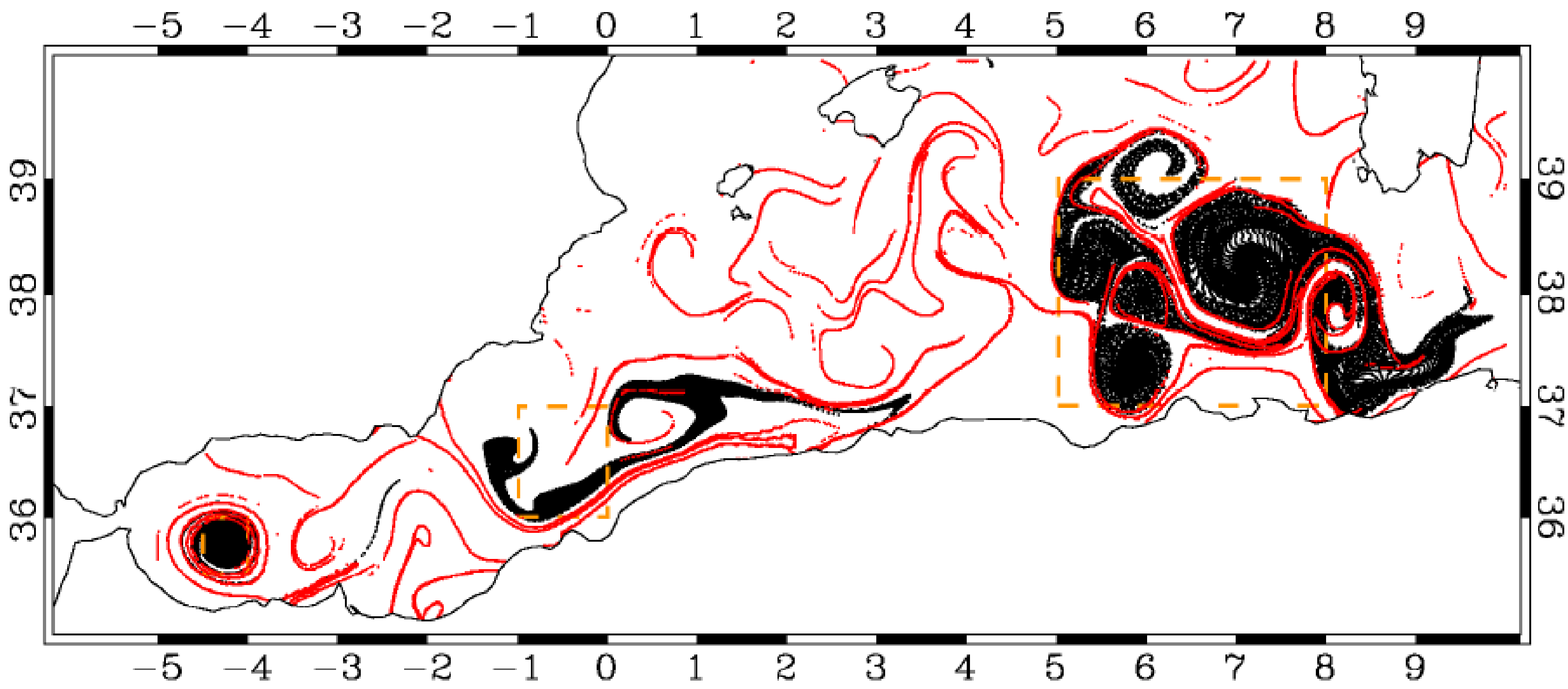
LAGRANGIAN COHERENT STRUCTURES





Lines: FSLE > 0.2 $days^{-1}$

Tracer advection for 2 or 1 weeks



FSLE are Lagrangian, but not direct advection:

- shorter simulations
- no problems with exponentially increasing line lengths
- exhaustive consideration of initial conditions

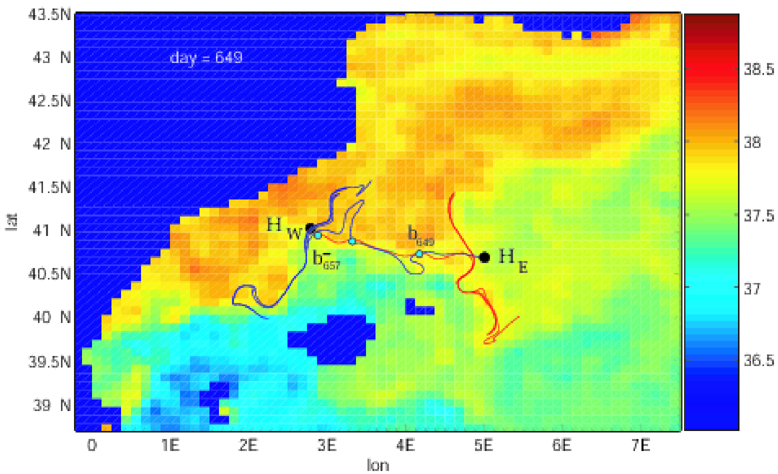
Lagrangian approaches to transport and mixing

- Geometric, local, … : FTLE, FSLE, geodesics, variational theory, M function, …
- Set-oriented, probabilistic ,…: Transfer operator, coherent sets, eigenvectors and singular vectors, networks, …

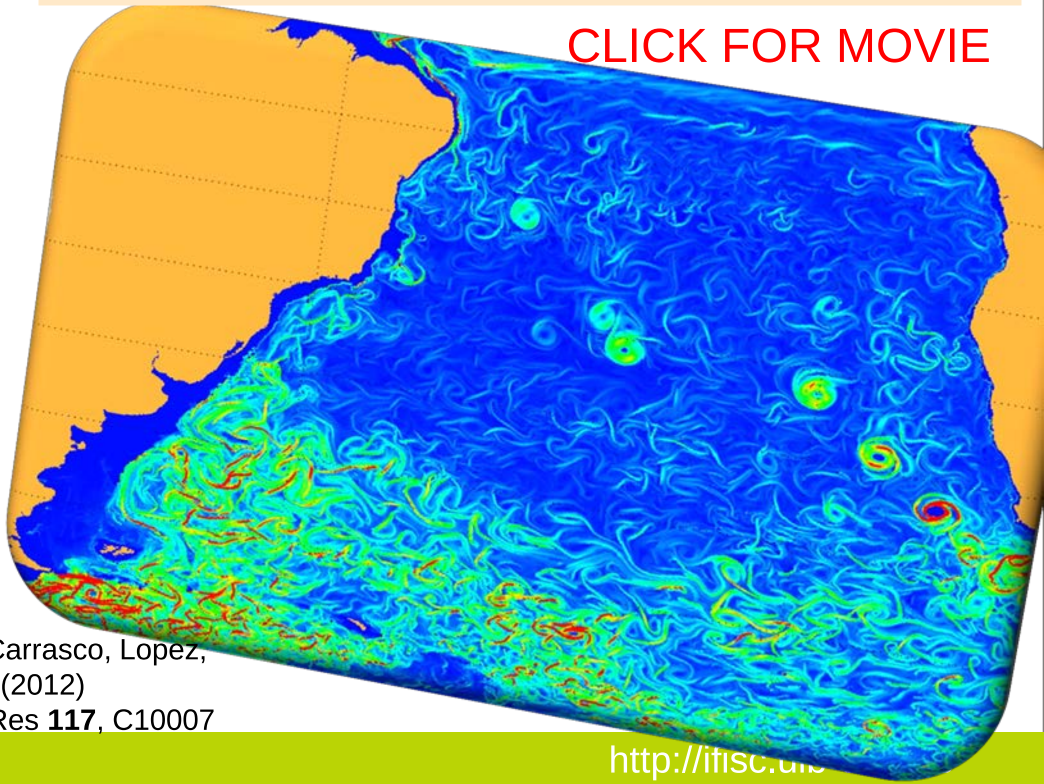
- Detailed view of single events
- Statistical (climatological) descriptions

BIBLIOGRAPHY at 'Resources' for the School: www.gefenol.es/school2014/resources/

CLICK FOR MOVIE

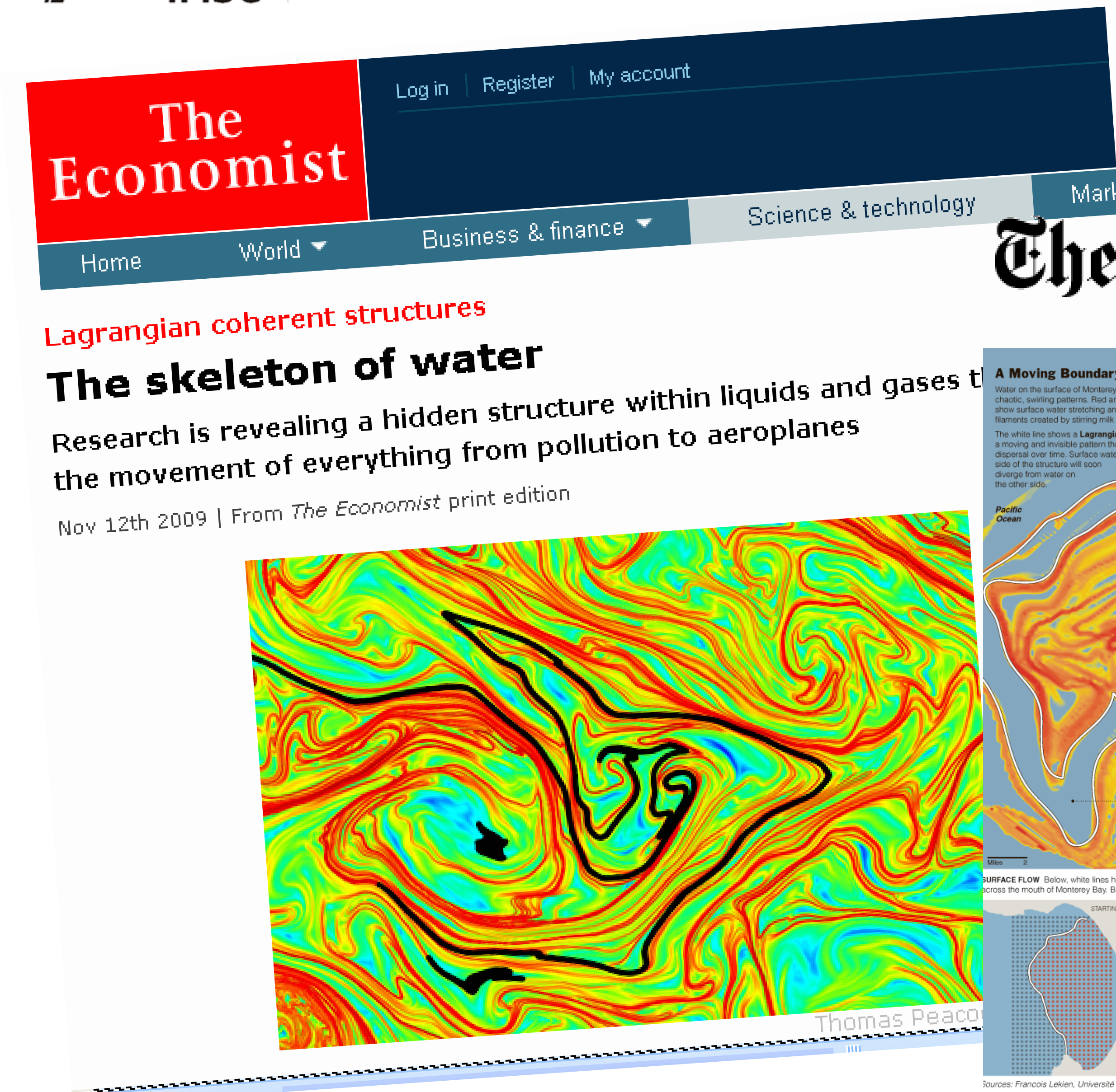


A.M. Mancho, E. Hernandez-Garcia, D. Small, S. Wiggins, V. Fernandez, J. Physical Oceanography **38**, 1222-1237 (2008).



Hernandez-Carrasco, Lopez, EHG, Turiel, (2012) J. Geophys Res **117**, C10007

Lagrangian Coherent Structures


The Economist

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Lagrangian coherent structures

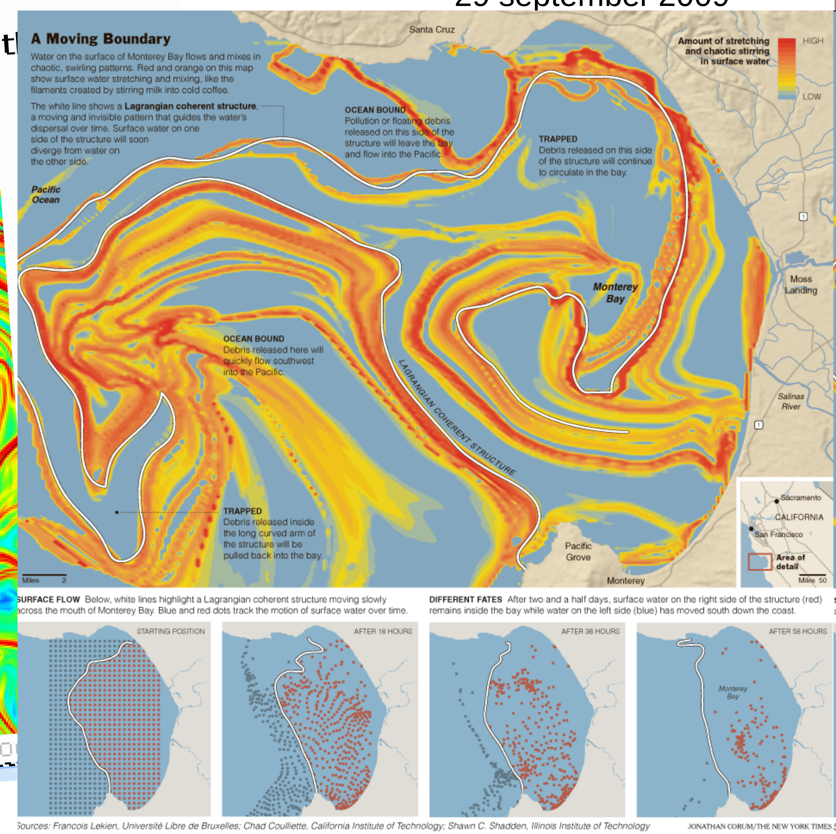
The skeleton of water

Research is revealing a hidden structure within liquids and gases t[...] the movement of everything from pollution to aeroplanes

Nov 12th 2009 | From *The Economist* print edition

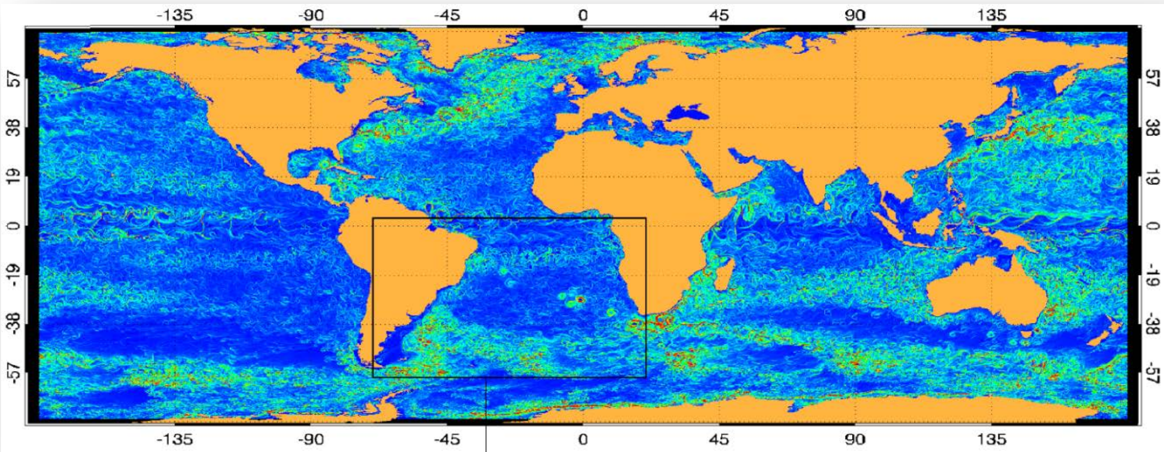
The New York Times

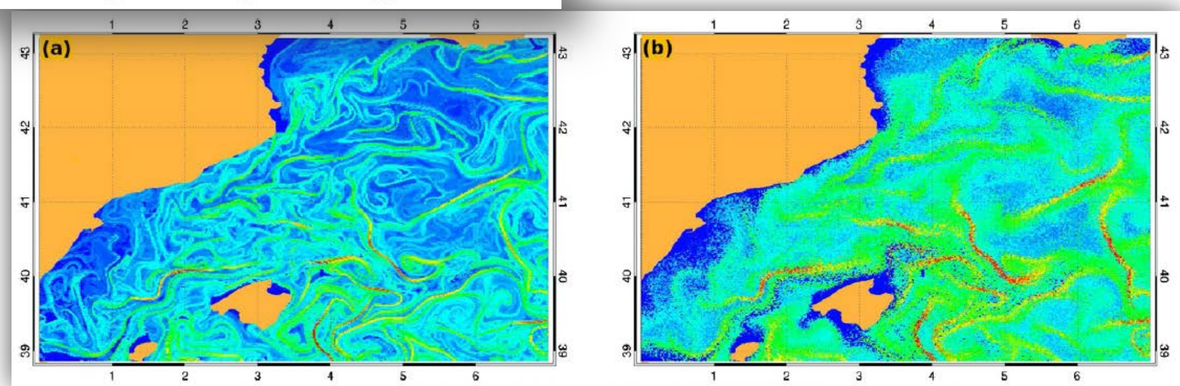
29 september 2009



Any advantage in using FSLE to locate LCS?

In oceanographic contexts it is usually straightforward to identify the relevant spatial scales: Rossby radius, coastal features



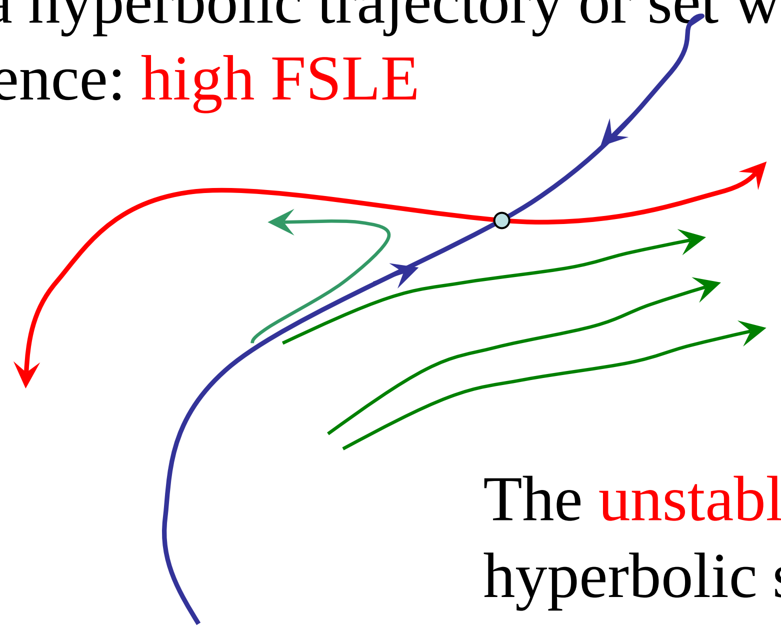


Trajectories can be nonsmooth (noise …)
I. Hernández-Carrasco et al. Ocean Mod. 36, 208 (2011)

Disadvantage:

No theorems …

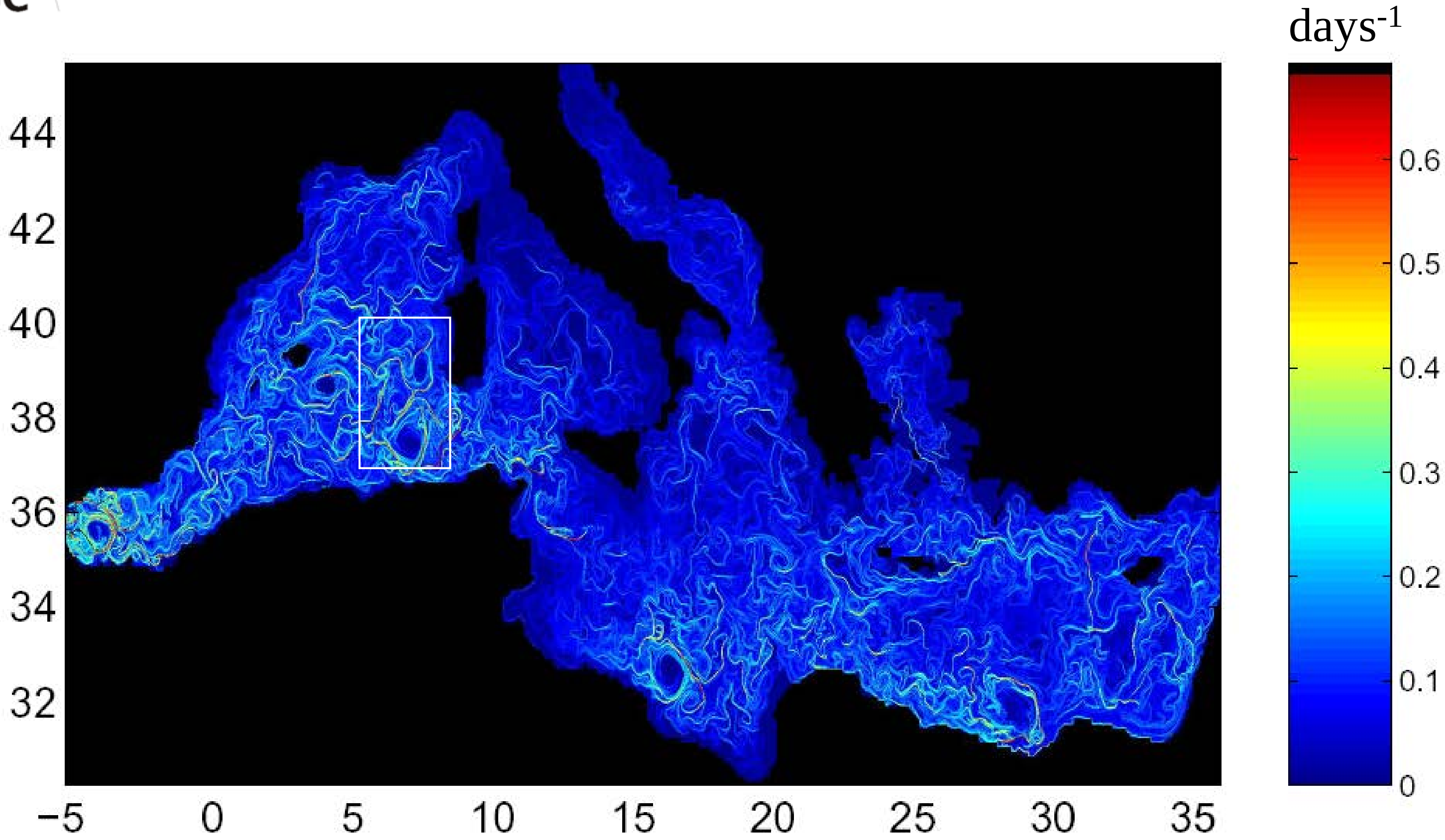
The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE



The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

Other types of Lyapunov exponents would display similar information, but FSLE is less afected by saturation

REMARK: these are heuristic consideration. Theorems needed (some available for FTLE)



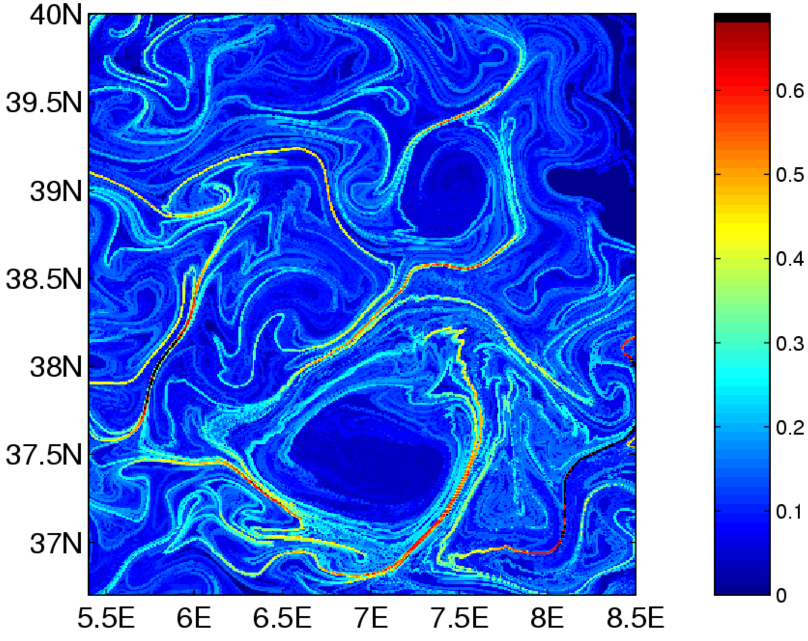
DieCAST model for the full MediterraneanPrimitive equations, 48 vertical levels, 1/8 º horizontal resolution, climatological forcings … → 5 years of daily velocity fields

$\delta_0 = 0.02$ º → $\delta_f = 1$ º (mesoscale transport)

$\delta_0 \approx 2$ km → $\delta_f \approx 110$ km twodimensional

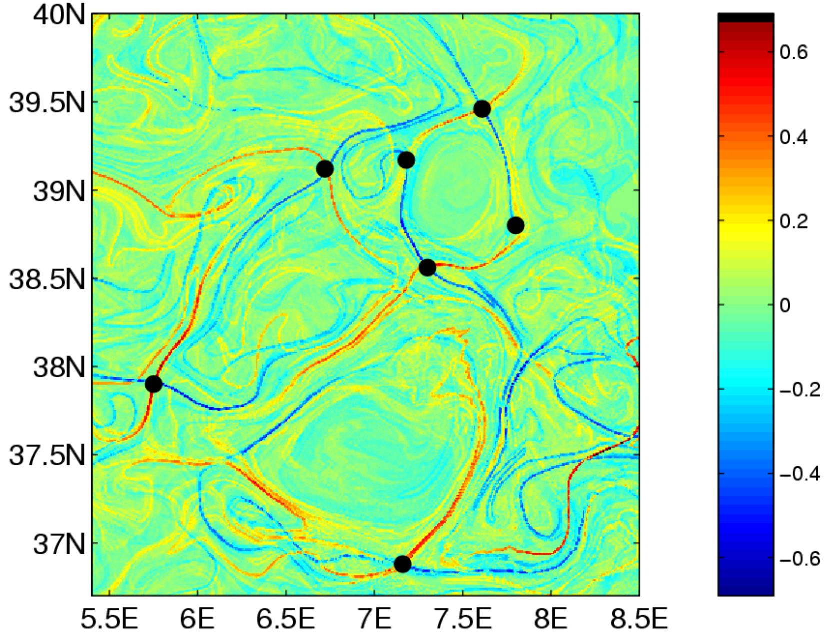
d'Ovidio, Fernández, Hernández-García, López, Geophys. Res. Lett. 31, L17203 (2004)

CLICK THE FIGURES FOR MOVIES



FSLE from time-backwards Integrations.
Are they really unstable manifolds of hyperbolic trajectories?

FSLE from forward and backwards integrations

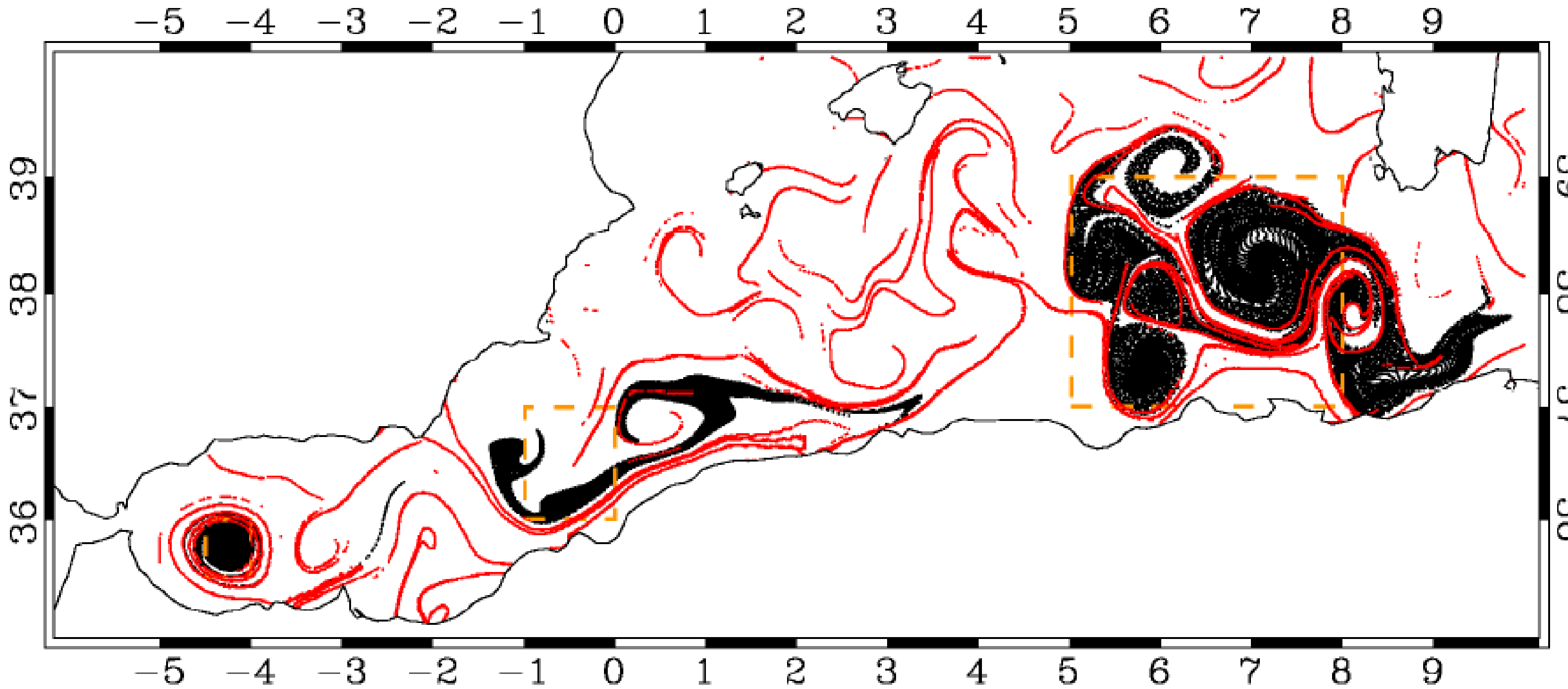


The strongest lines are seen to organize tracer flow

G, ShR

Lines: FSLE > 0.2 $days^{-1}$

Tracer advection for 2 or 1 weeks



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- exhaustive consideration of initial conditions

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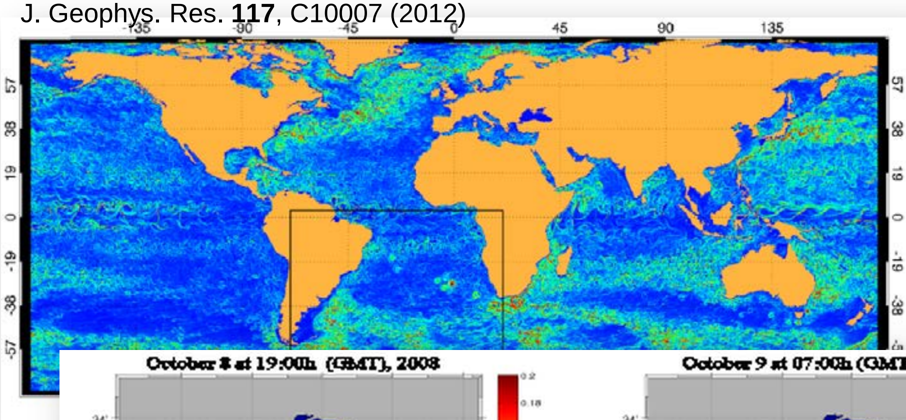
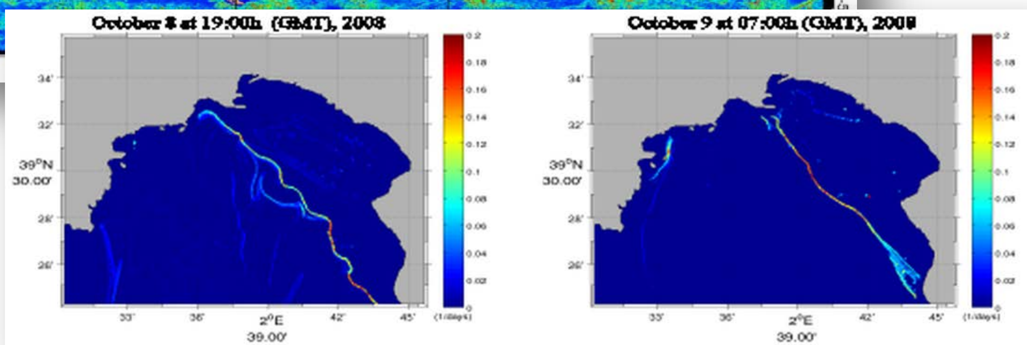
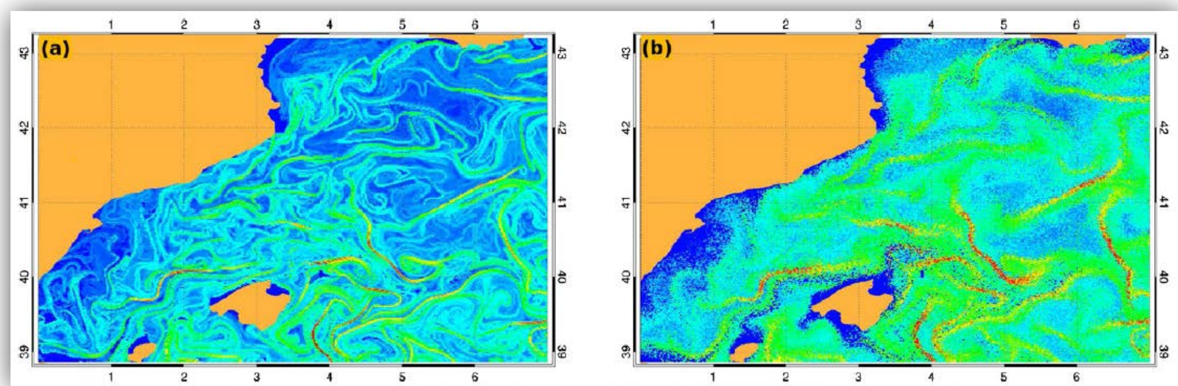
Hernández-Carrasco, López, Hernández-García, Turiel,
J. Geophys. Res. **117**, C10007 (2012)





Hernández-Carrasco, López,Orfila, Hernández-García,
Nonlinear Processes in Geophysics **20**, 921-933 (2013)

Bahía de Palma



Any advantage in using FSLE in Lagrangian studies?

- Easy switching between local and statistical approaches
- In oceanographic contexts it is usually straightforward to identify the relevant spatial scales: Rossby radius, coastal features
- Trajectories can be nonsmooth (noise …)

Disadvantages:

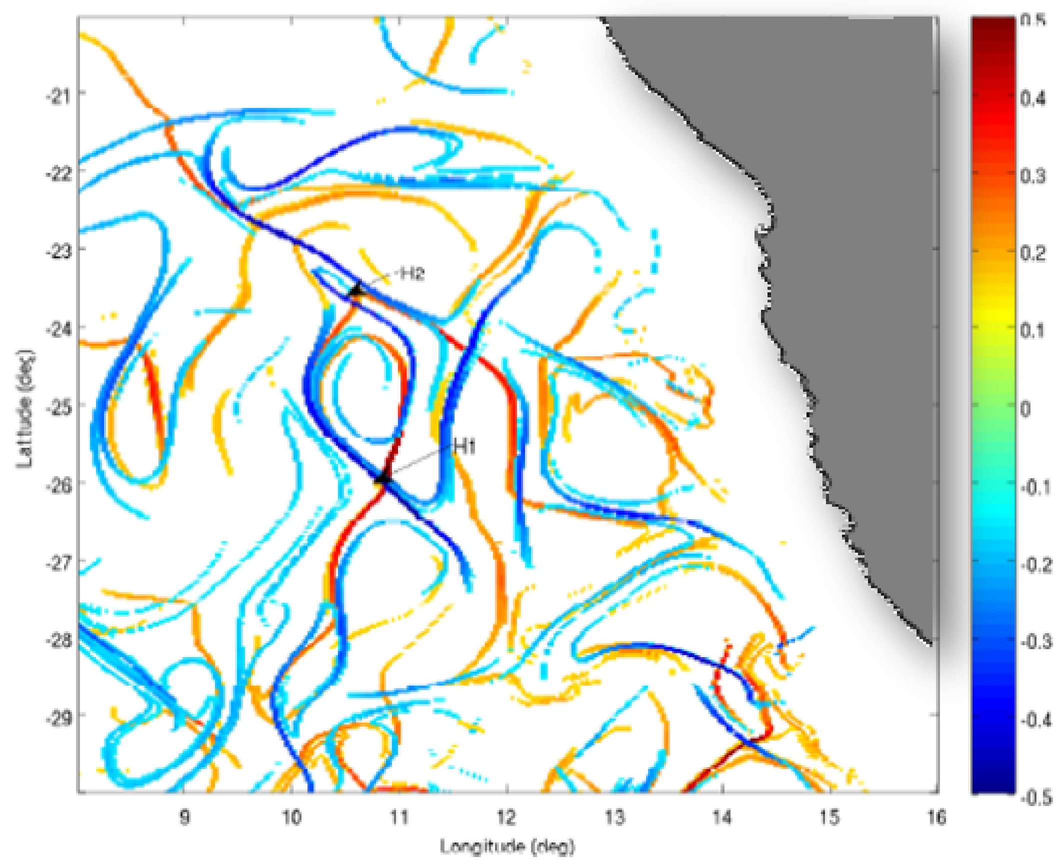
- No distinction between hyperbolic, shear, … structures
- Lack of analytical approaches (but see Tzella and Haynes, PRE 2010, Karrasch and Haller, Chaos 2013)
- As for FTLE, not all high FSLE structures have a clear impact on flows. Need to check with actual particle trajectories

Hernández-Carrasco et al.
Ocean Mod. 36, 208 (2011)

Some examples of recent Lagrangian studies in the ocean using Finite Size Lyapunov Exponents

Three-dimensional characterization flow and eddies in Benguela

J.H. Bettencourt, C. Lopez, E. Hernandez-Garcia, Ocean Modelling 51 (2012) 73–83

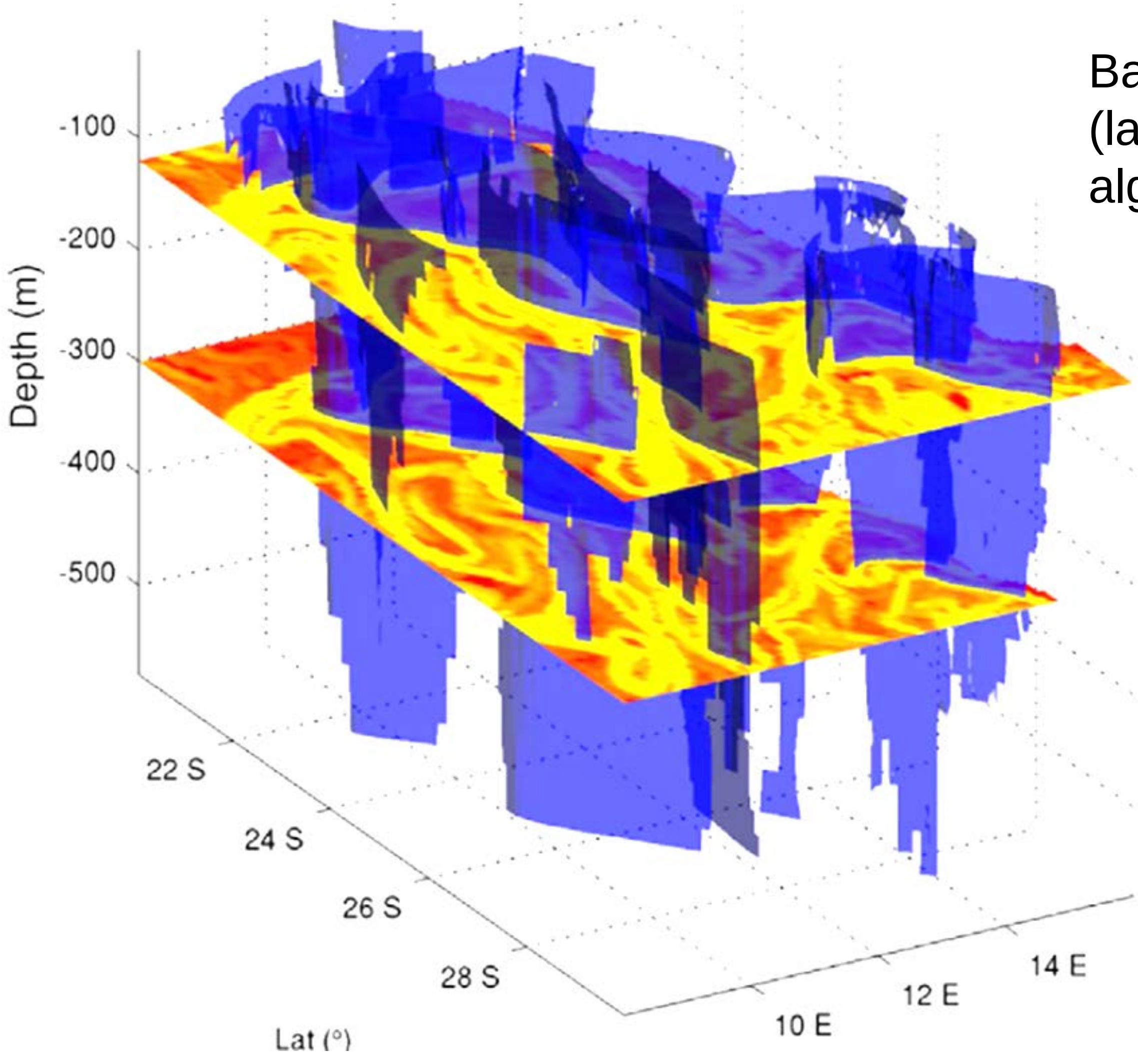


ROMS model:
(from Gutknecht et al.(2013). and Le Vu et al.)
2 years of simulation, climatologically forced.

Horizontal resolution 1/12 degrees (8 km)
32 vertical terrain-following levels

Forward and backward FSLE fields
δ_0=2 km ; δ_f=100 km

Particles released in horizontal planes every 20 m and integrated in 3D

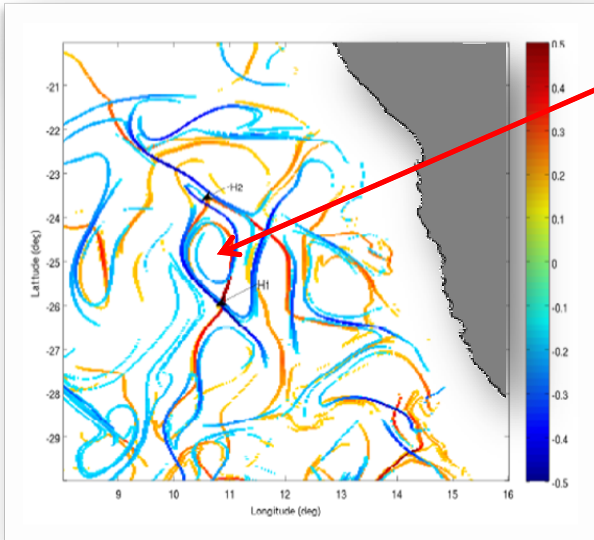


Backward FSLE from a (largest) ridge extracting algorithm

Curtain-like structure as arising when vertical shear of horizontal velocities much smaller than horizontal velocities

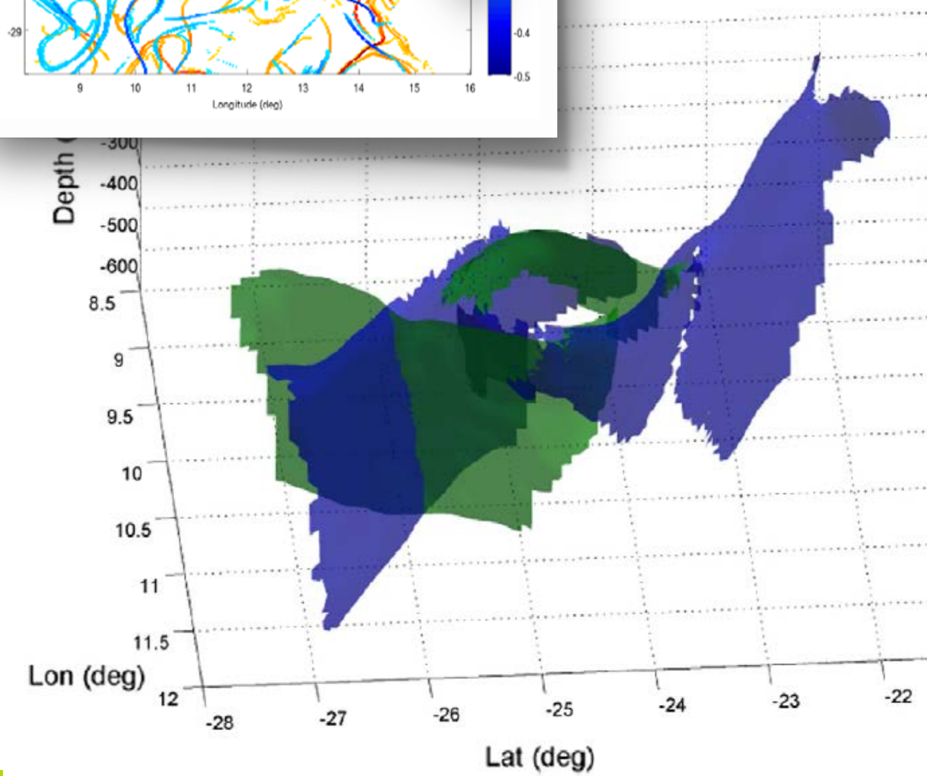
(Branicki, Mancho, Wiggins, Physica D 240 (2011) 282–304)

BACKWARDS FSLE
FORWARD FSLE
Q-criterion isosurface

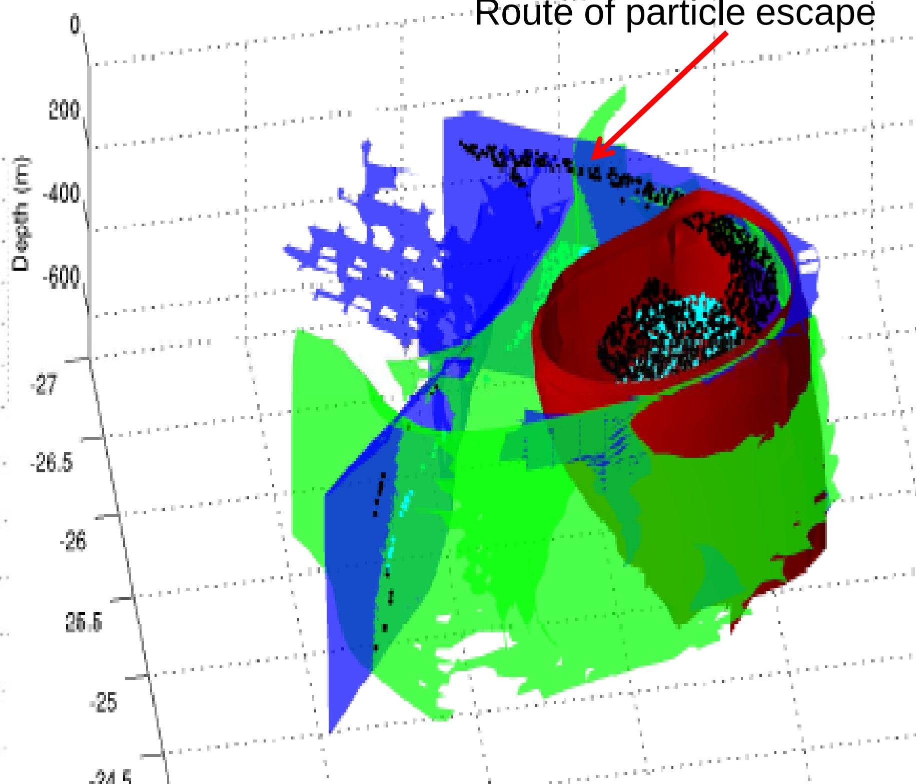


Particular eddy enclosed by hyperbolic manifolds

FSLE methodology is giving the hyperbolic filamentation region, not the coherent core



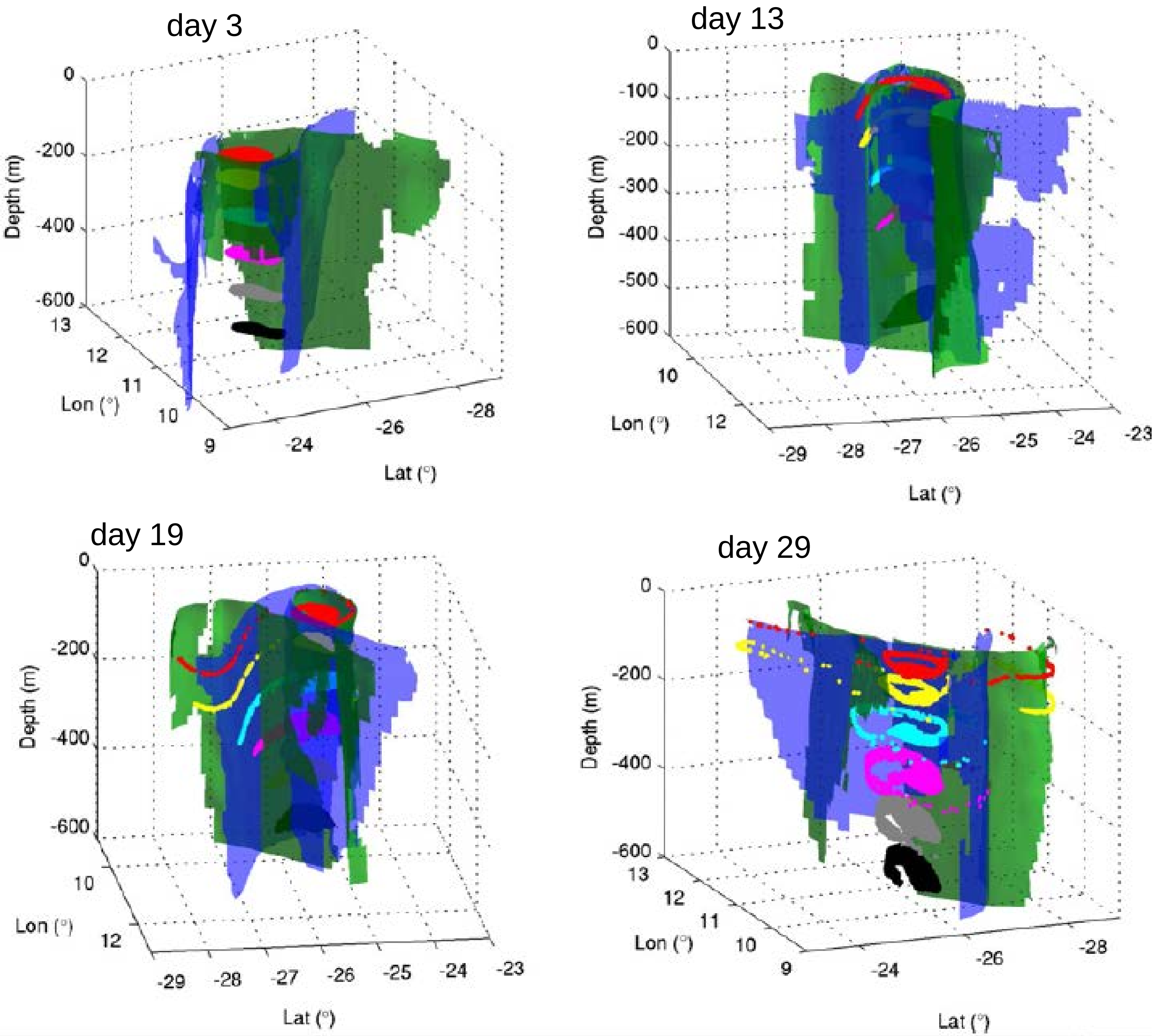
Route of particle escape



J.H. Bettencourt, C. Lopez, E. Hernandez-Garcia
Ocean Modelling 51 (2012) 73–83
J. Phys. A 46 (2013), 254022

BACKWARDS FSLE
FORWARD FSLE

Red: 40 m
yellow: 100 m
cyan: 200 m
magenta: 300 m
grey: 400 m
black: 500 m



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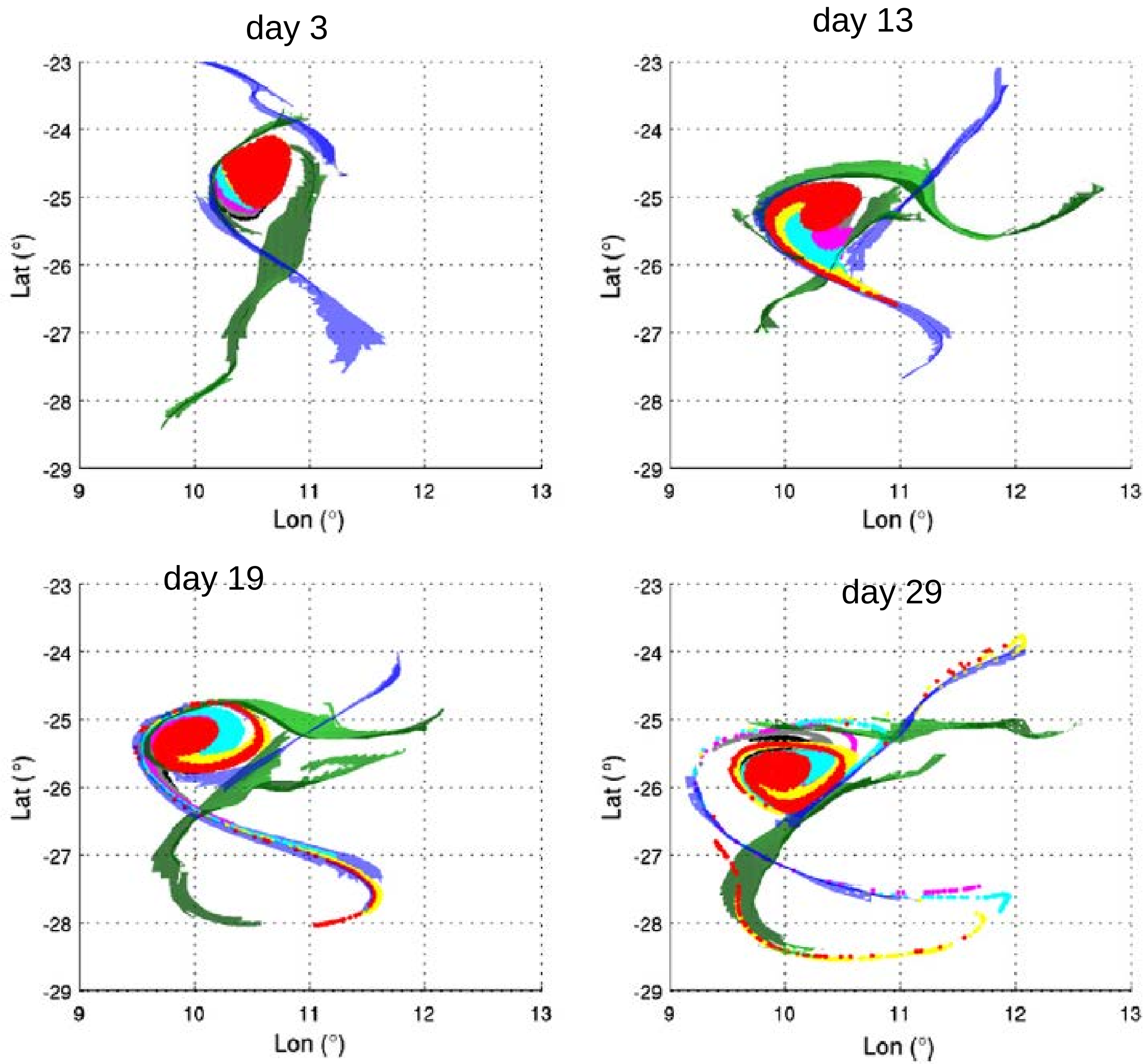
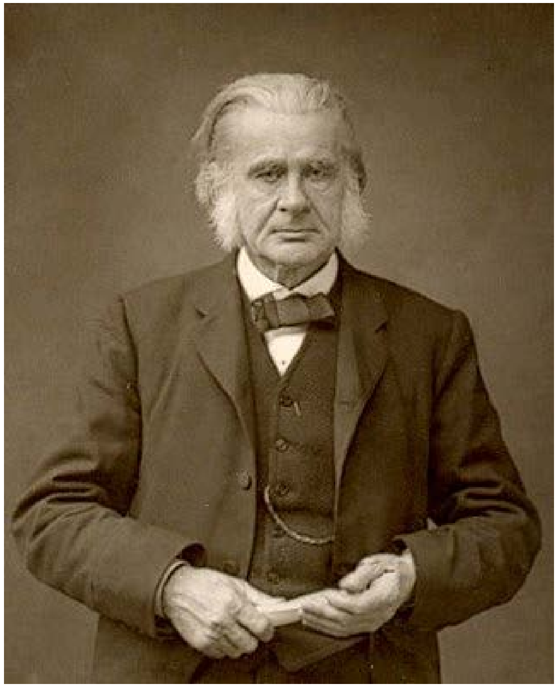


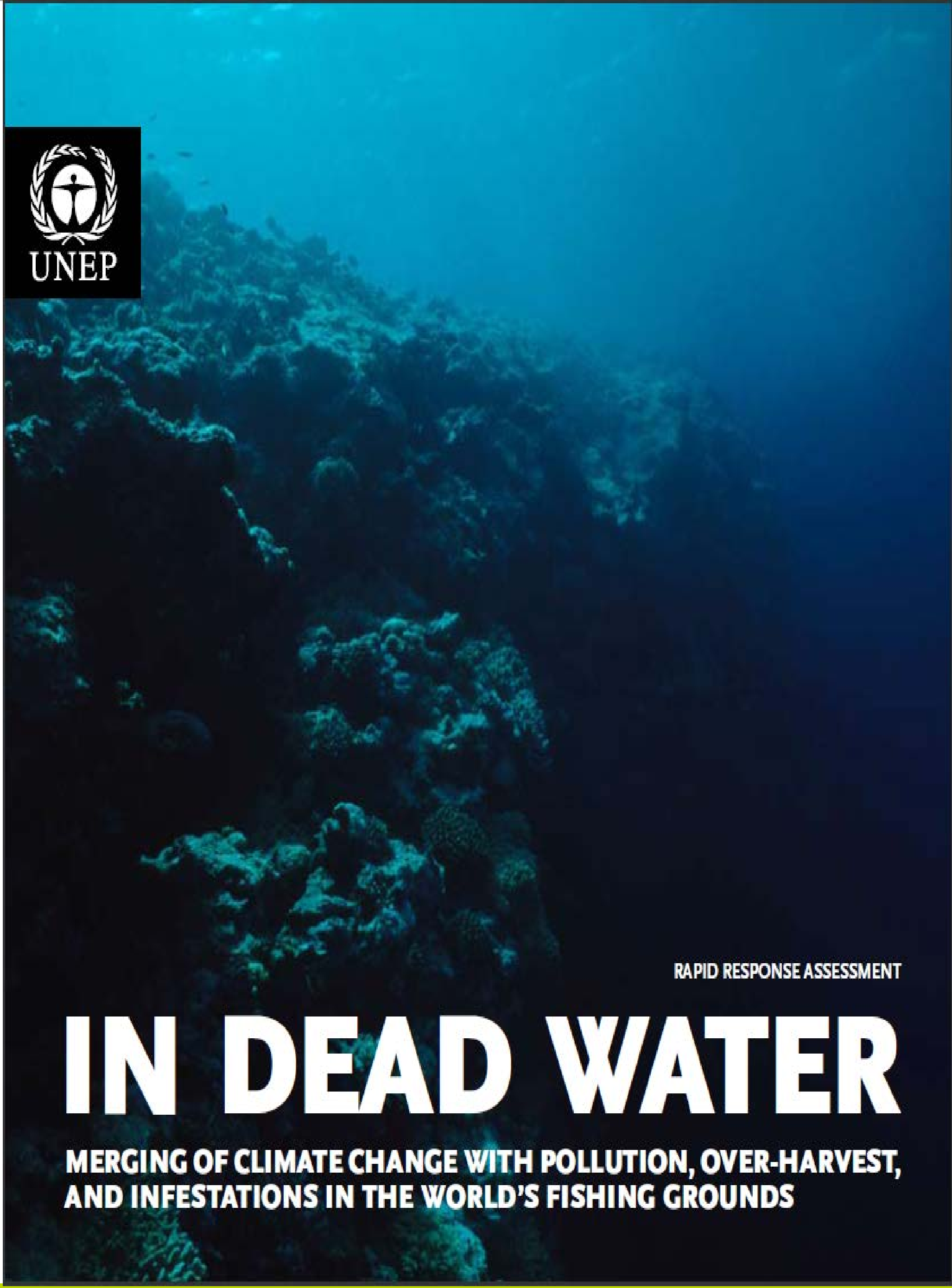


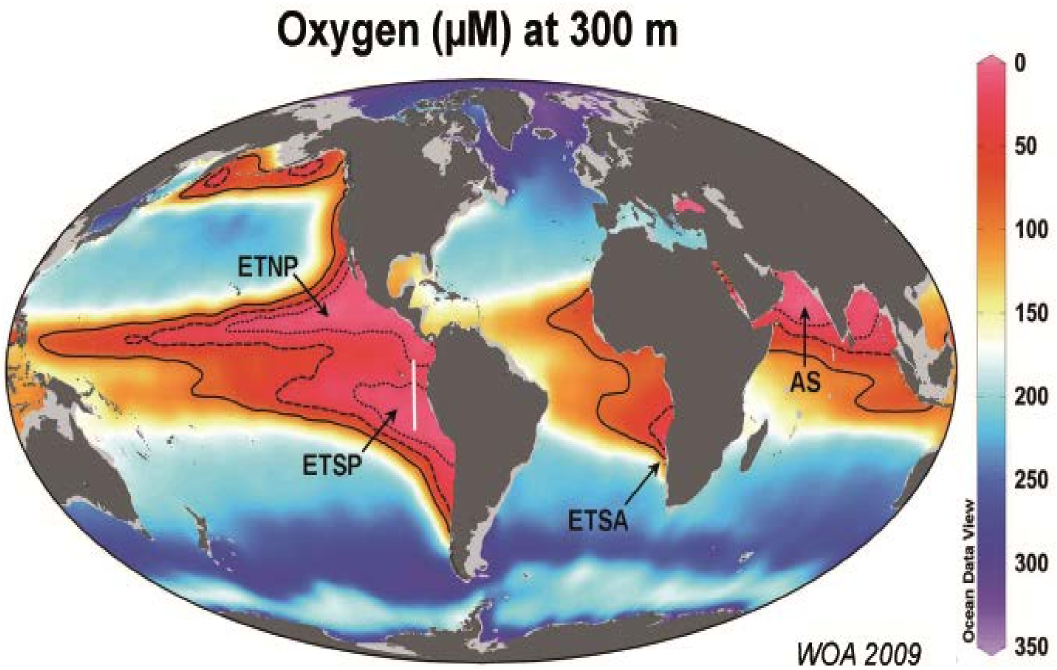
"I believe, then, that the cod fishery, the herring fishery, the pilchard fishery, the mackerel fishery, and probably all the great sea fisheries, are inexhaustible; that is to say, that nothing we do seriously affects the number of the fish. And any attempt to regulate these fisheries seems, consequently, from the nature of the case, to be useless."

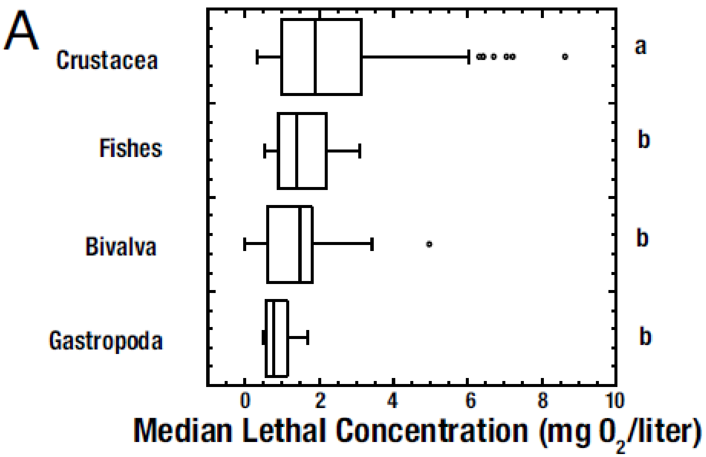
Thomas H. Huxley, Intern. Fisheries Exhibition, London (1883)

"Increased development, coastal pollution and climate change impacts on ocean currents will accelerate the spreading of marine dead zones, many around or in primary fishing grounds."

United Nations Environmental Programme (2008)







Hypoxic levels - O2 < 88 μM

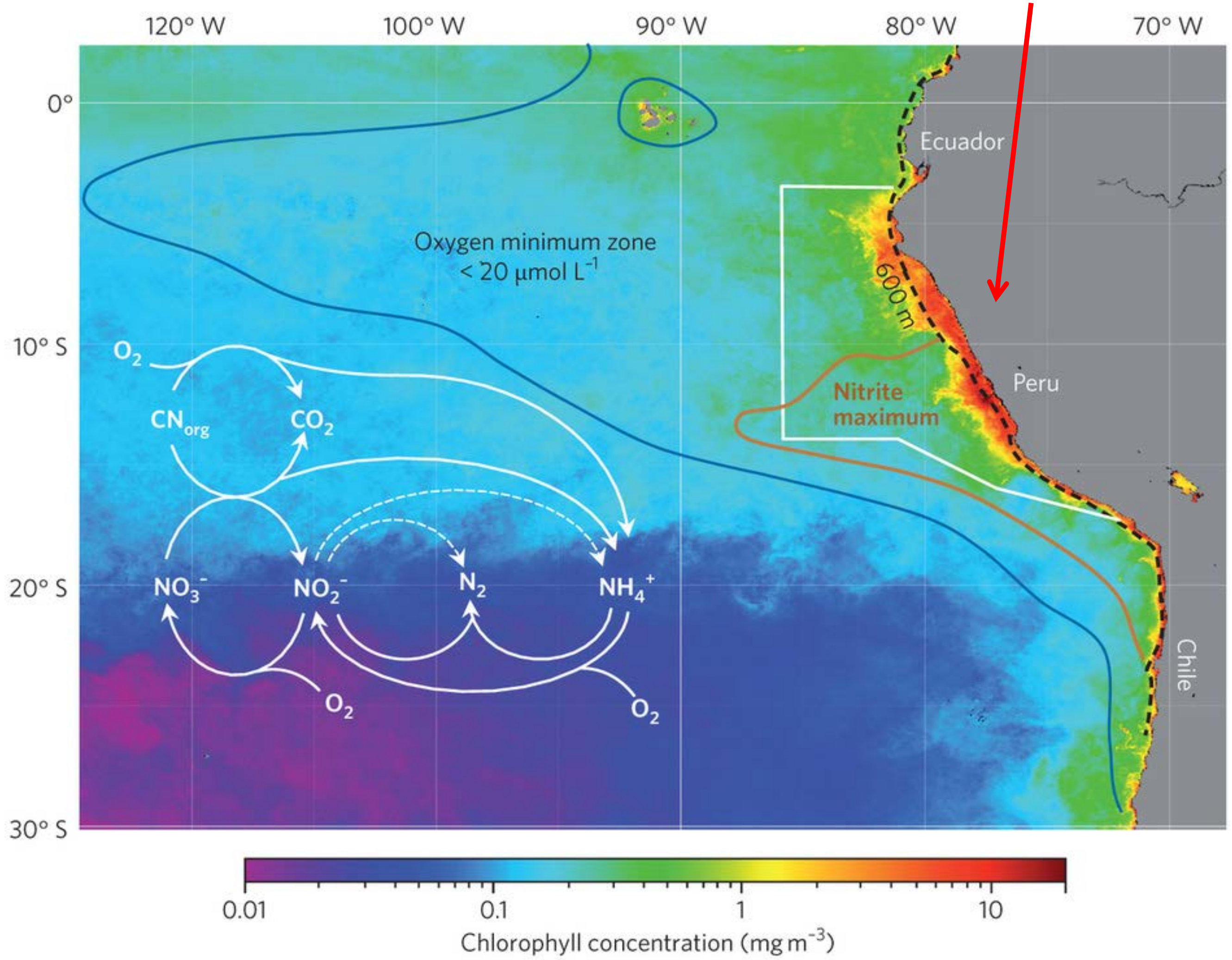
Vaquer-Sunyer & Duarte (2008)

Respiration and nitrification consume oxigen

Increased stratification associated to global warming will make things worse

Role of flow:
Large scale patterns induce low ventilation areas.
What about horizontal stirring and mixing?

Coastal upwelling



Thamdrup (2013)

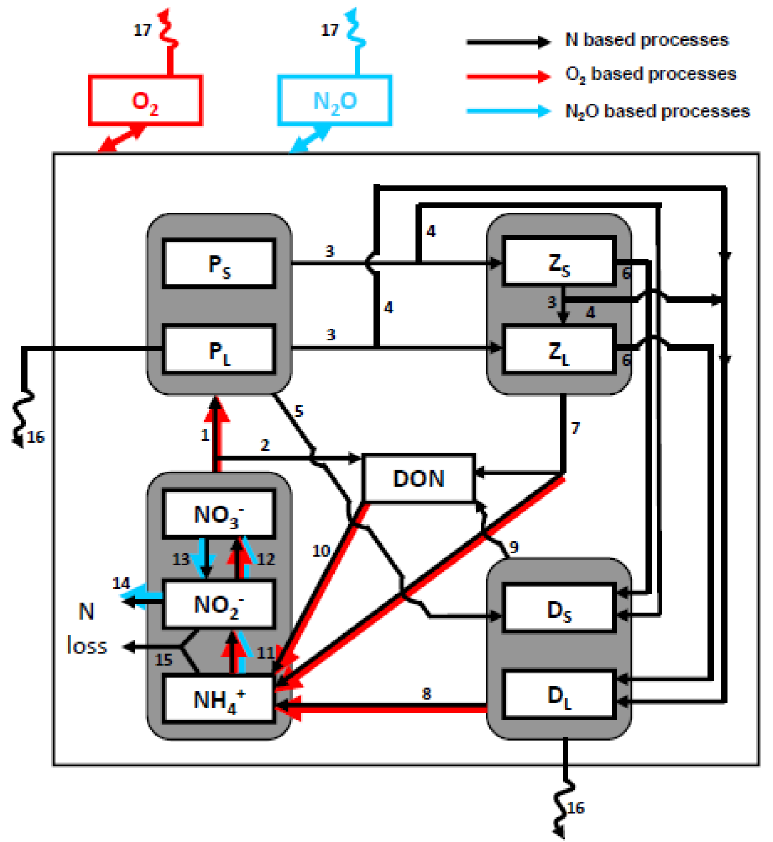
ROMS hydrodynamic model:

- 3D primitive equations
- Hydrostatic
- Terrain following
- Forced by climatology
- horizontal resolution of 1/9 degrees (~ 12 km)
- 32 terrain-following vertical levels

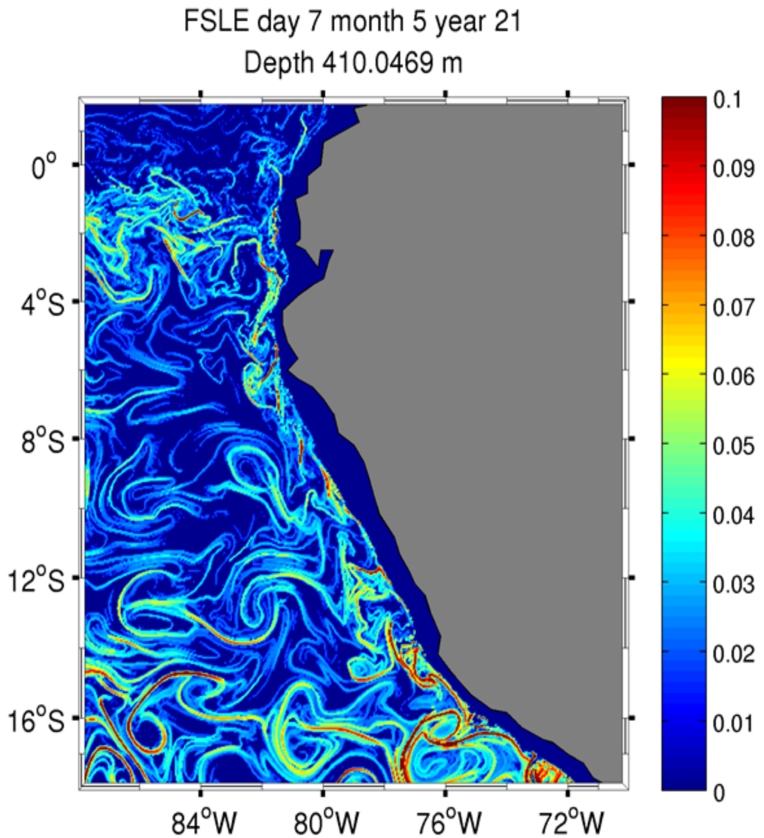
BioEBUS biogeoch. model:

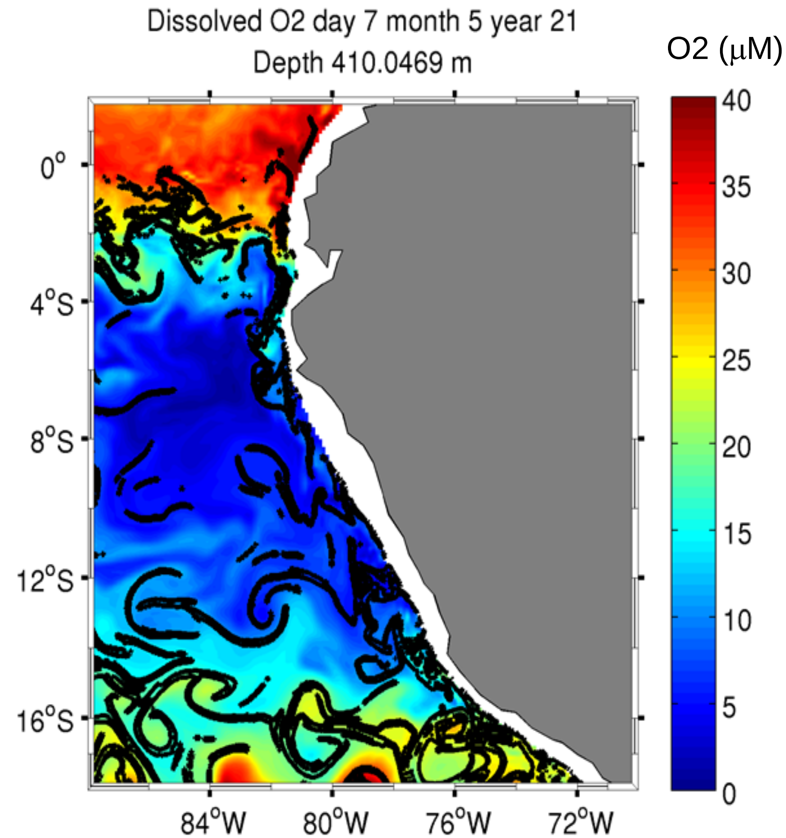
$$\frac{\partial C_i}{\partial t} = -\nabla \cdot (\mathbf{u}C_i) + K_h \nabla^2 C_i + \frac{\partial}{\partial z}\left(K_z \frac{\partial C_i}{\partial z}\right) + SM$$

(Gutknecht et al, 2013)



1. Assimilation of nutrients
2. Exudation
3. Grazing
4. Fecal pellets
5. Mortality of phytoplankton
6. Mortality of zooplankton
7. Excretion
8. Decomposition of detritus
9. Hydrolysis
10. Decomposition of DON
11,12. Nitrification
13,14. Denitrification
15. Anammox
16. Vertical sinking
17. Sea-air flux

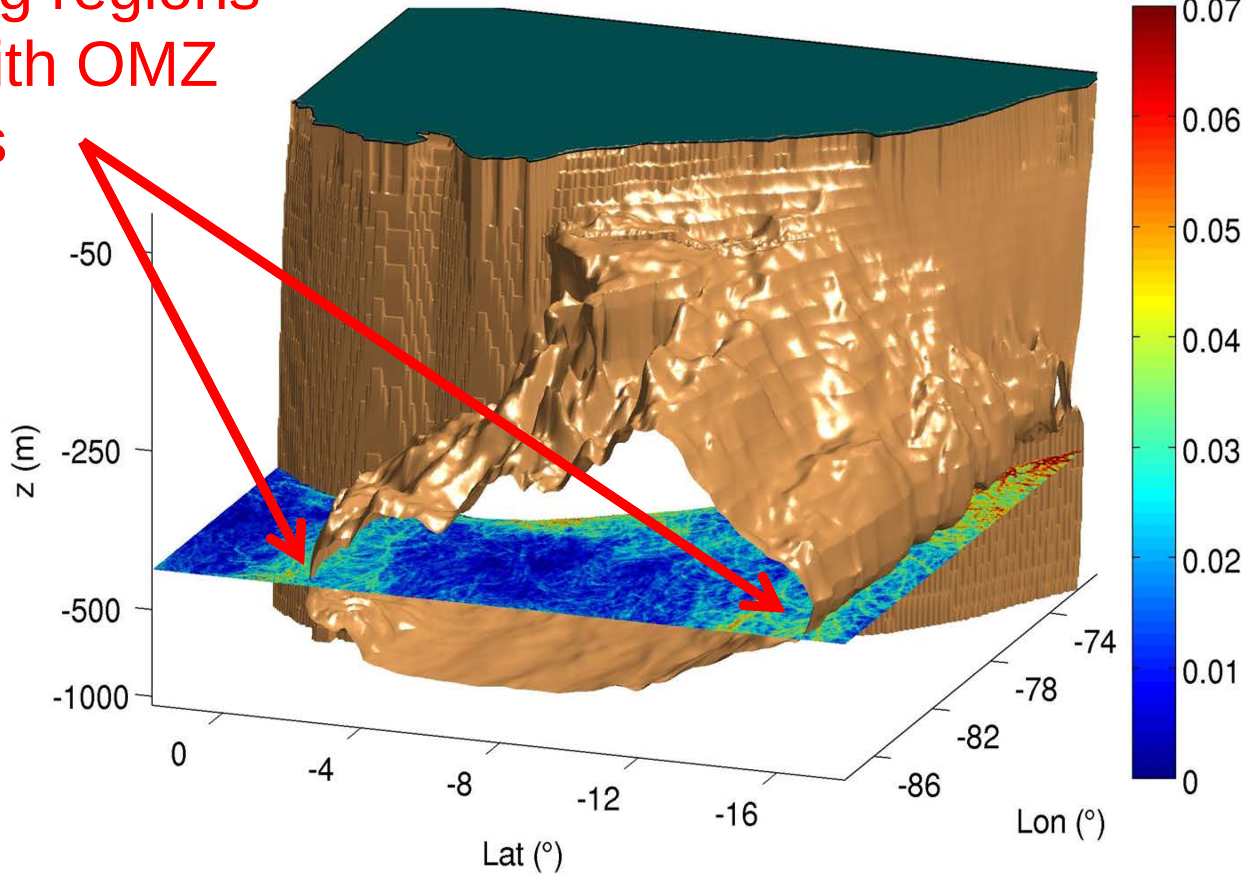




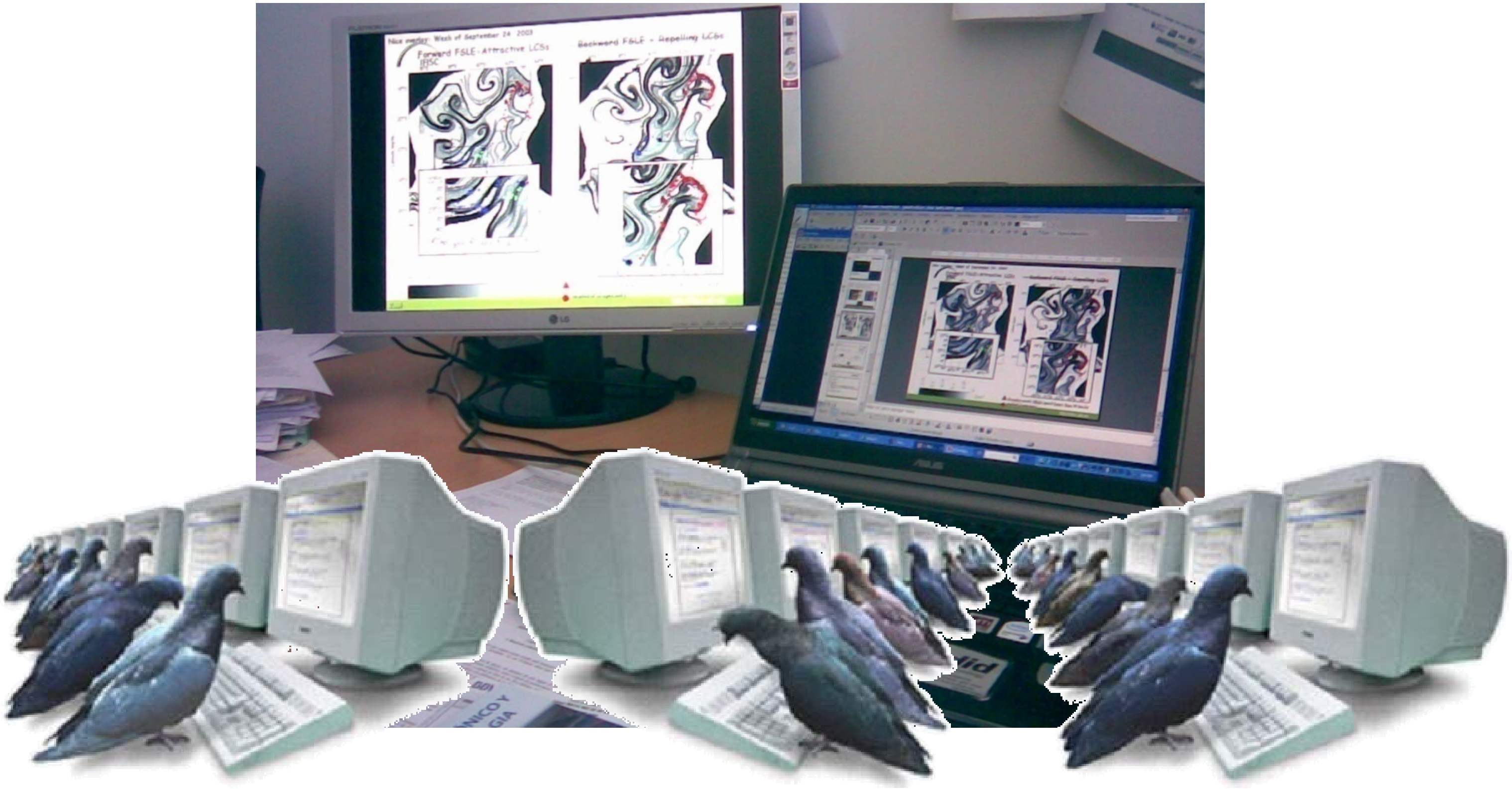
Backward FSLE (day^{-1})
Particles released in horizontal planes and integrated in 3D
δ_0=4 km ; δ_f=100 km

Mean Oxigen Minimum Zone boundary at 20 µM

High stirring regions coincide with OMZ boundaries



Do birds know about Lyapunov exponents?



Tew Kai, Rossi, Sudre, Weimerskirch, Lopez, Hernandez-Garcia, Marsac, Garçon,
PNAS 106, 8245 (2009)

Satellite transmitter and altimeter
(total weight : 1 to 3% mass of adults, max 45g)

8 birds (from Europa Island community) fitted with satellite transmitter and altimeter.

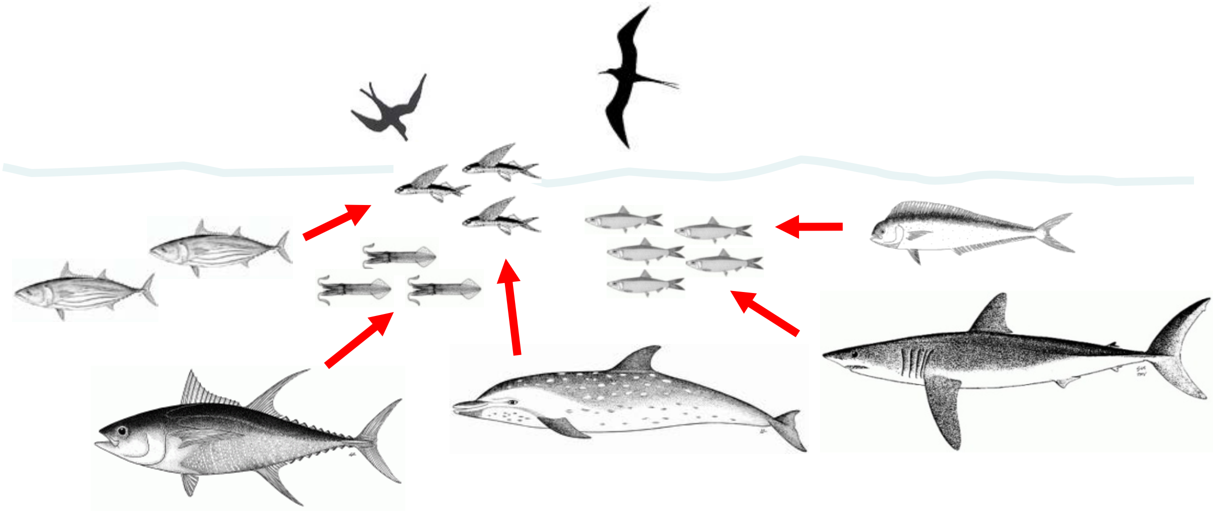
Followed for their foraging trips from August 18 to September 30, 2003.

1600 Argos from 50 trips positions, distributed into 17 long trips (> 614 km) and 33 short trips.

(Weimerskirch et al., 2004

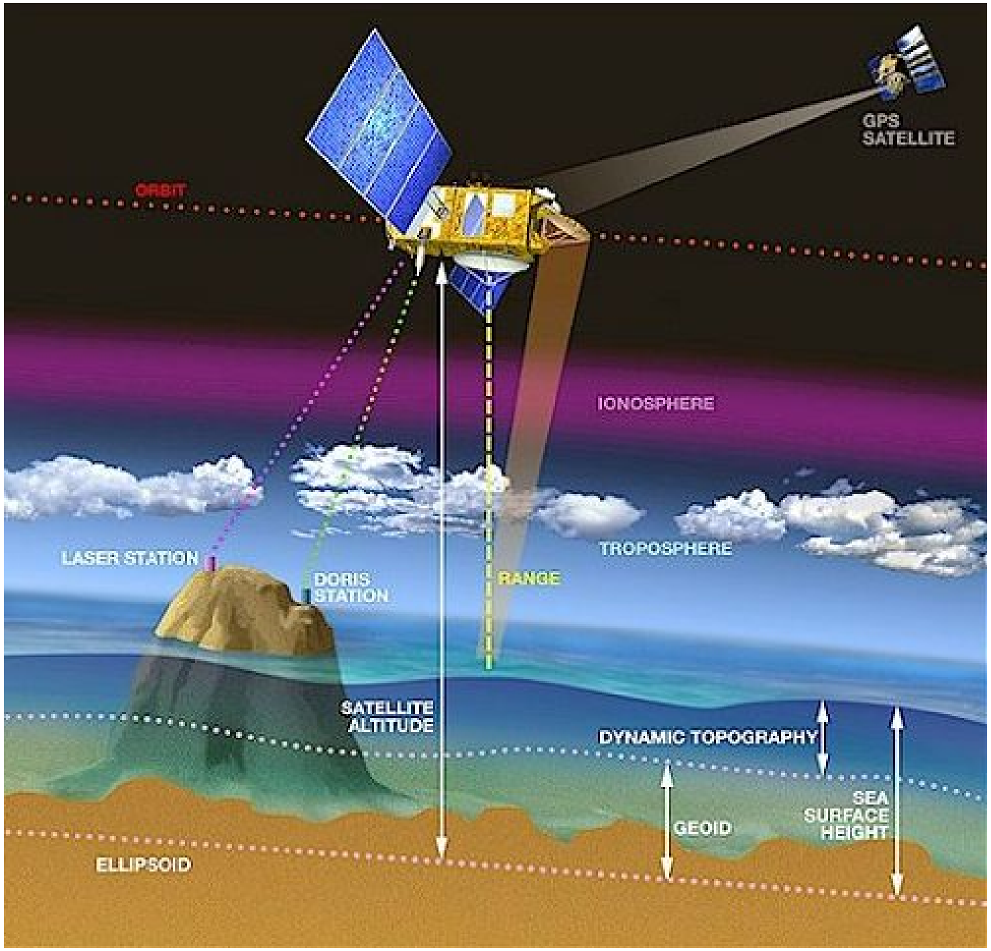
Great frigatebird (*fregata minor*):

- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**



Weimerskirch et al. 2004; Jaquemet et al. 2005

SATELLITE ALTIMETRY FROM TOPEX/POSEIDON, ERS-2, JASON, ENVISAT, …



Dynamic Topography (DT)=
Sea Surface Heigh (SSH) – Geoid (G)

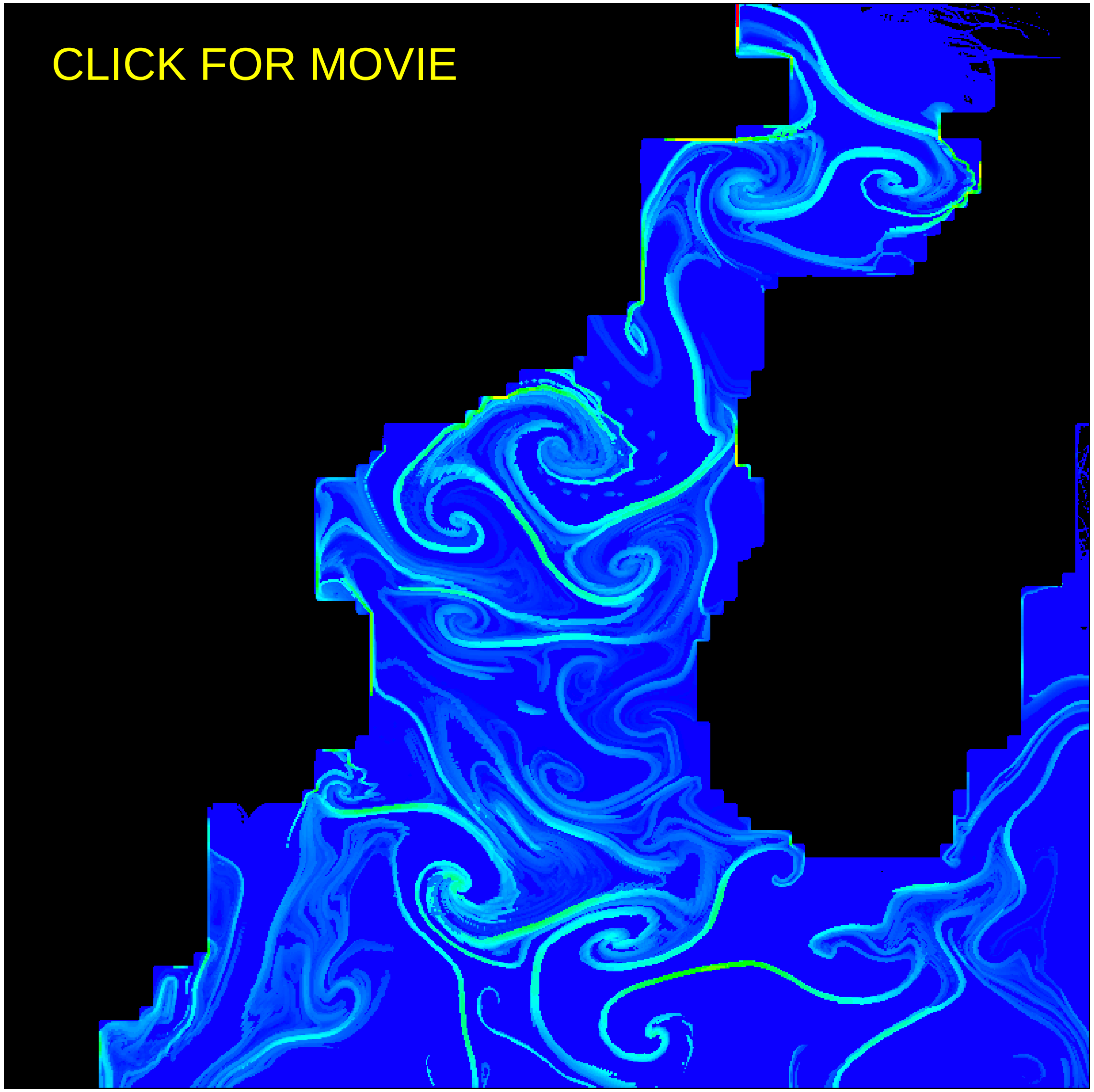
SSH ≈ 3 cm
G ≈ meters …

Sea Level Anomalies (SLA) =
SSH - $\langle SSH \rangle_t$ = DT - $\langle DT \rangle_t$

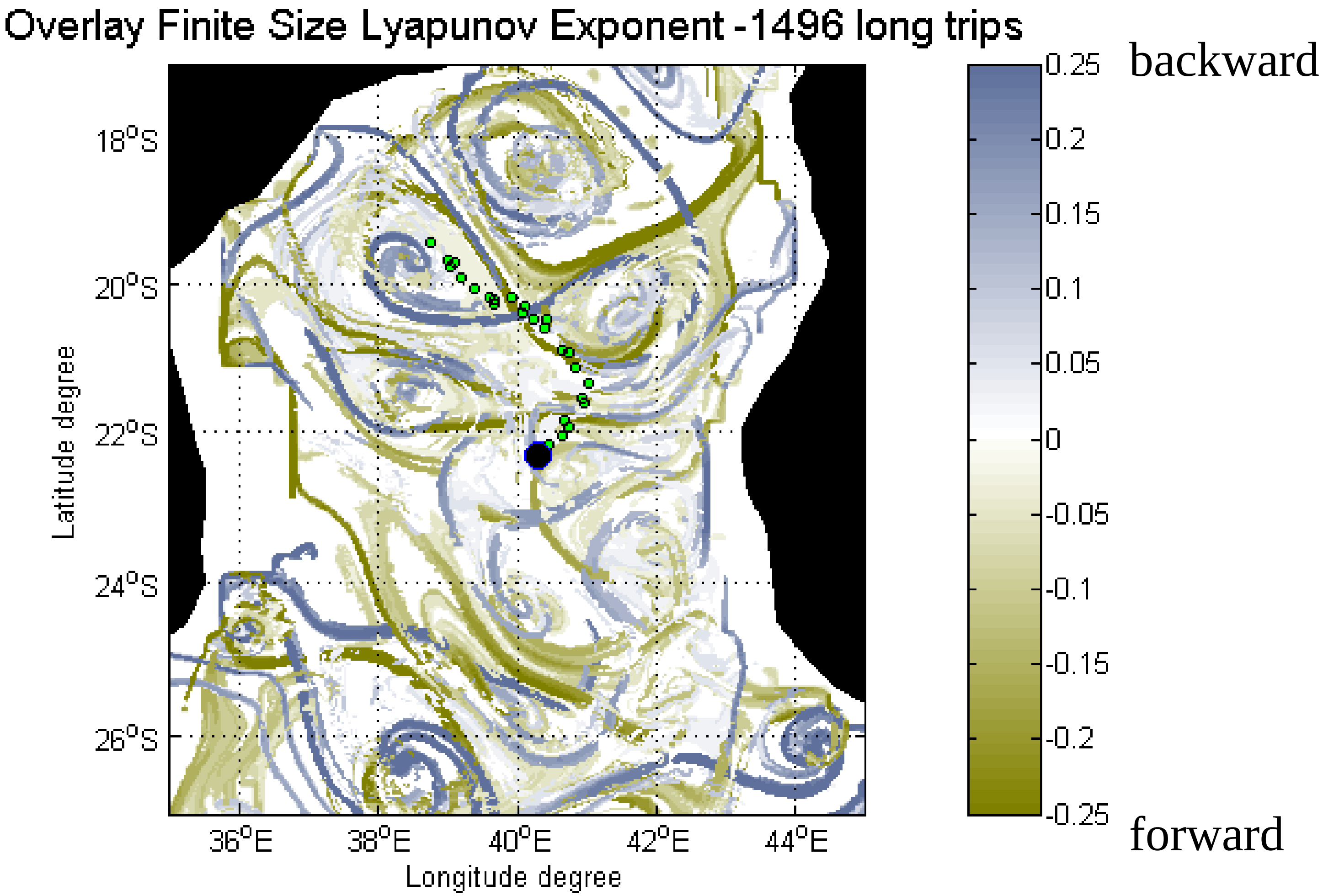
Dynamic topography determines, via the Colioris force, the velocity field (at large scales, geostrophic approximation)

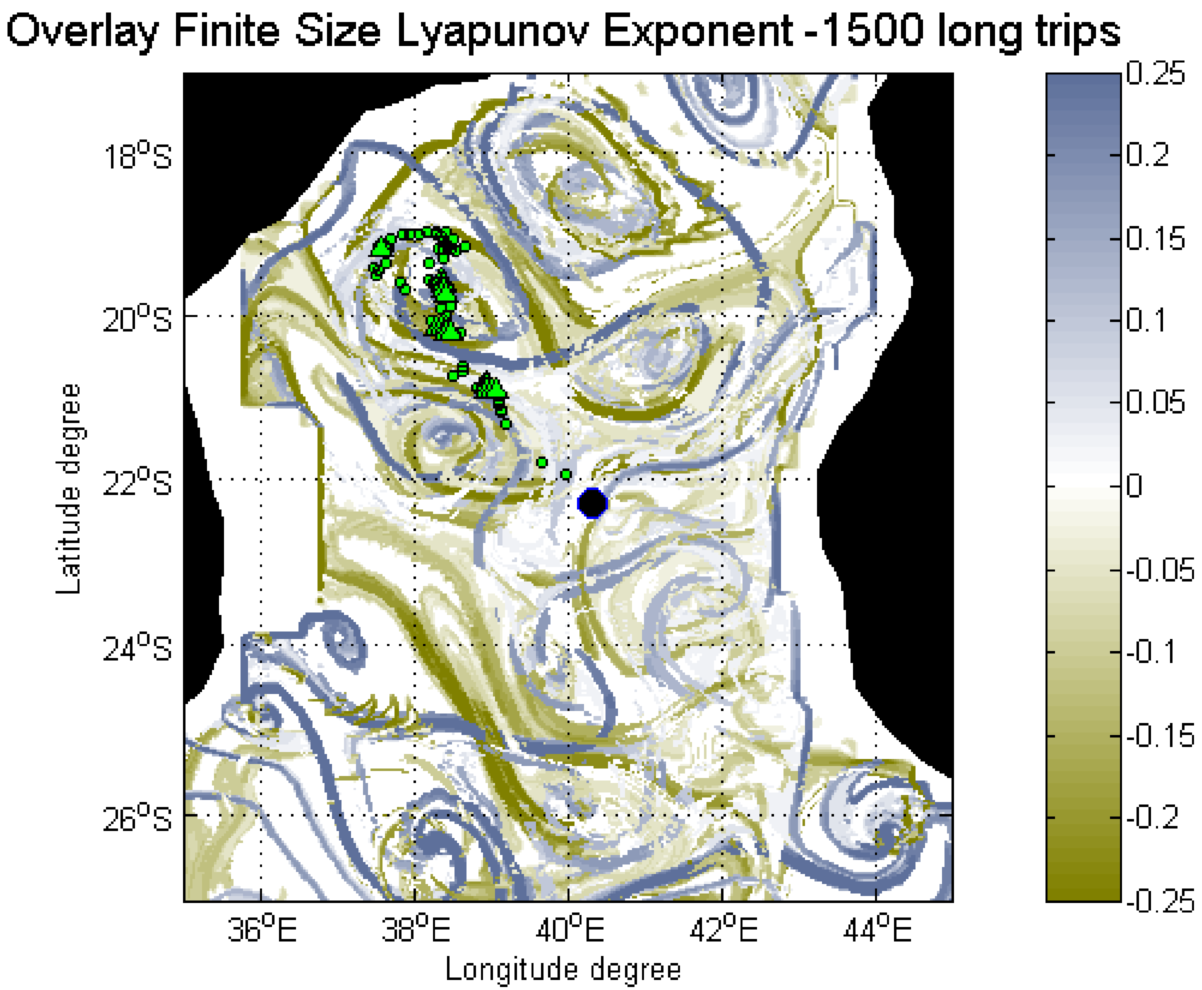
Ageostrophic components Can be estimated from scatterometer data
(Surface roughness → wind → Eckman component)

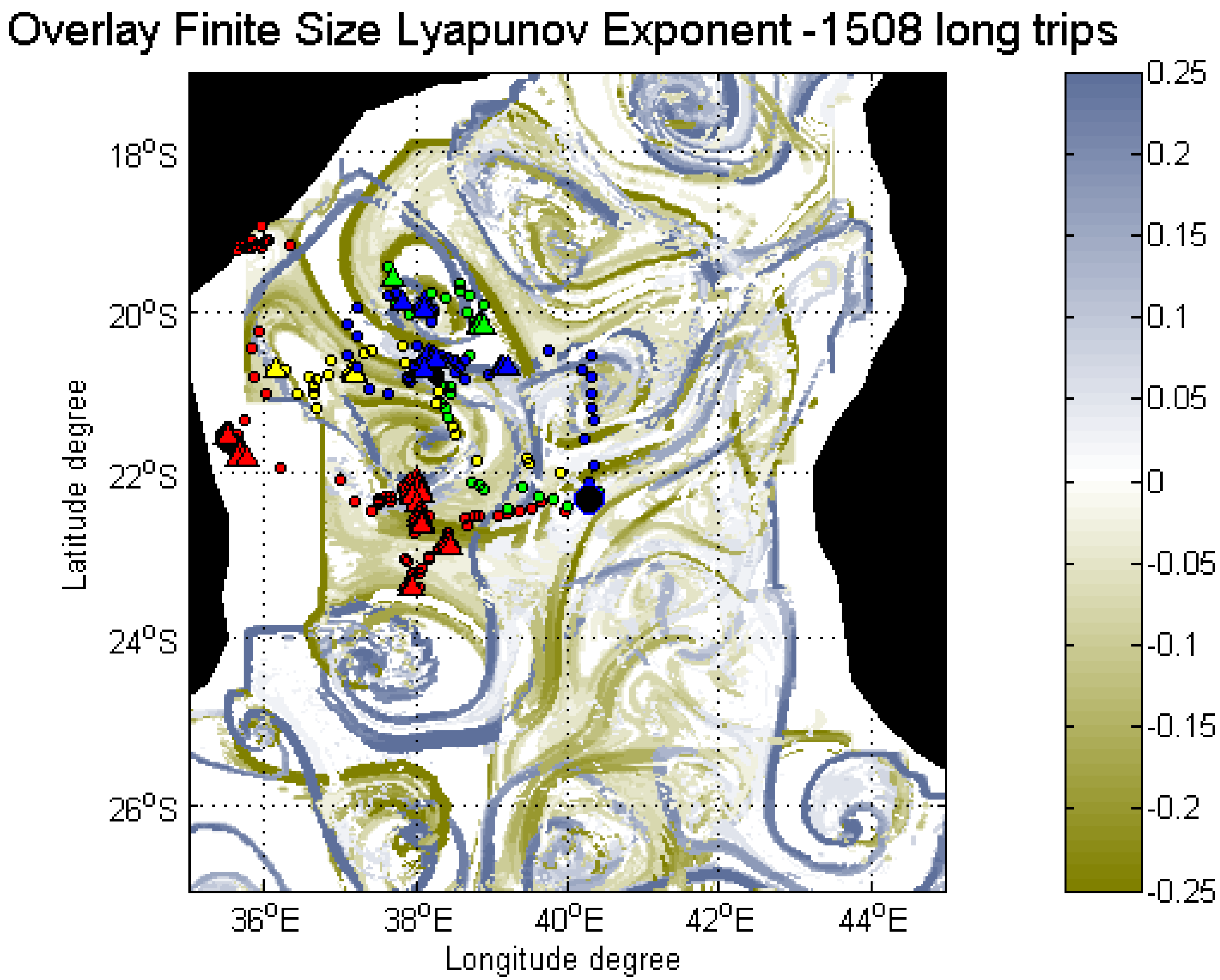
Backwards FSLE

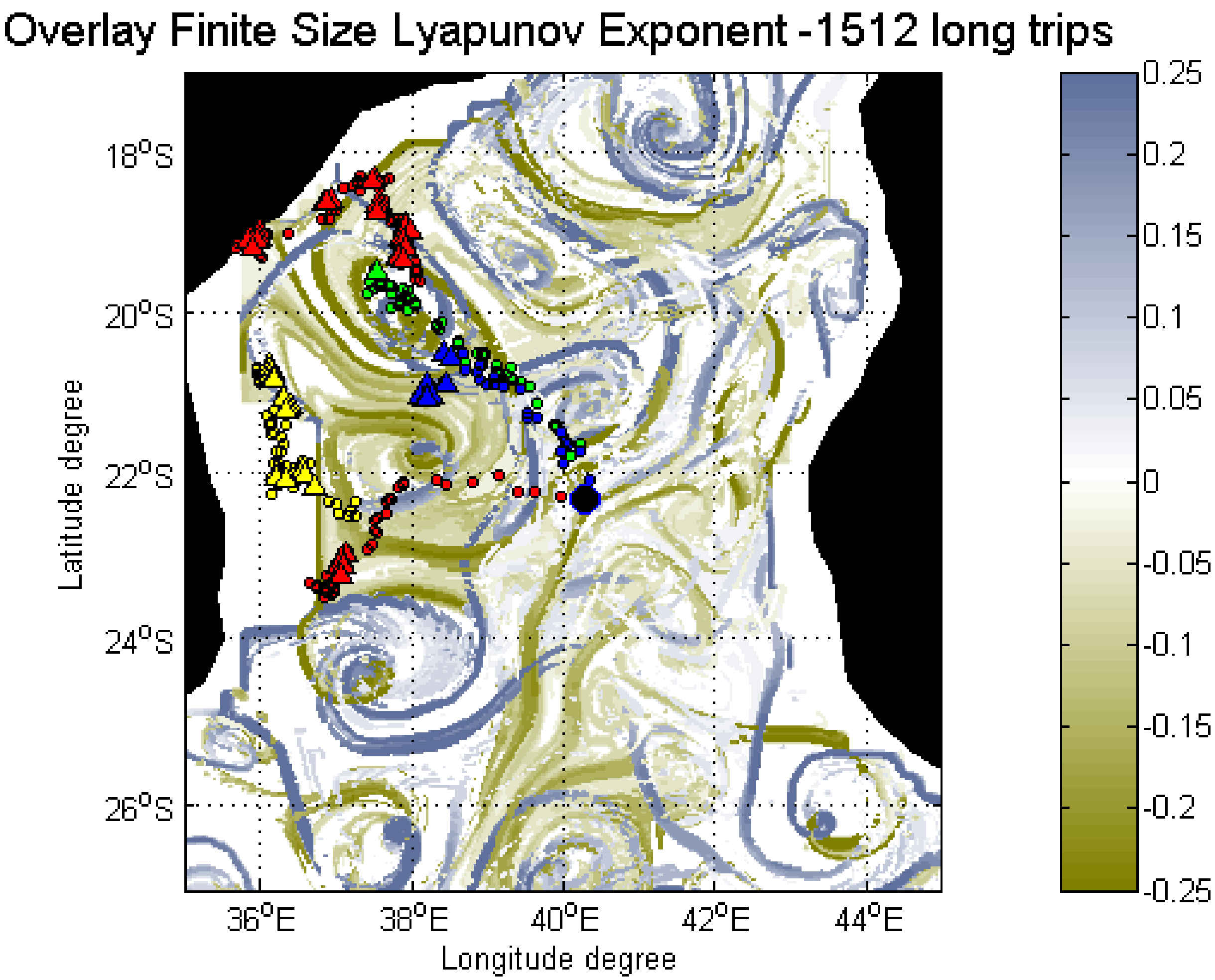


August 18 - September 30, 2003.

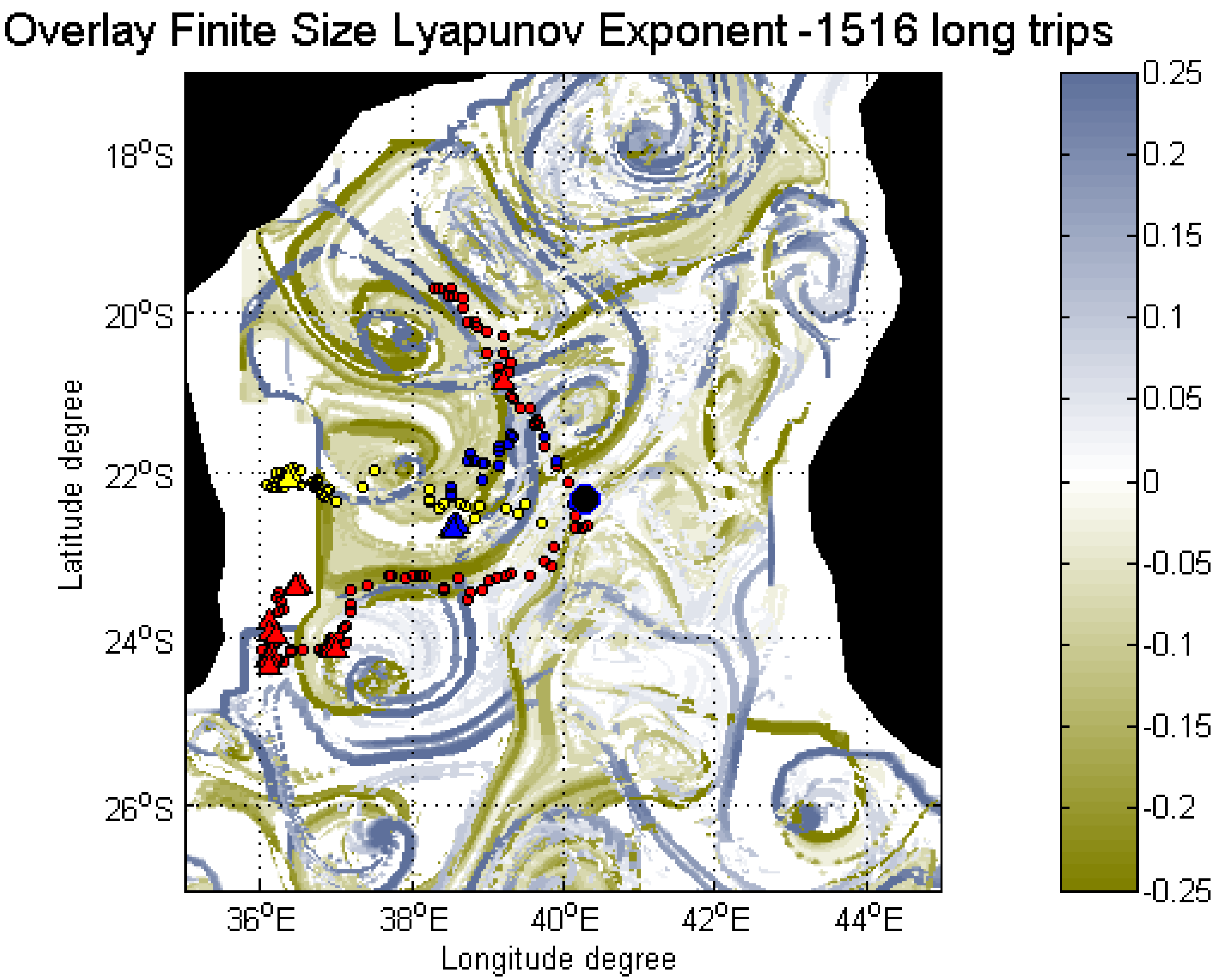

Overlay Finite Size Lyapunov Exponent -1496 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
backward
forward

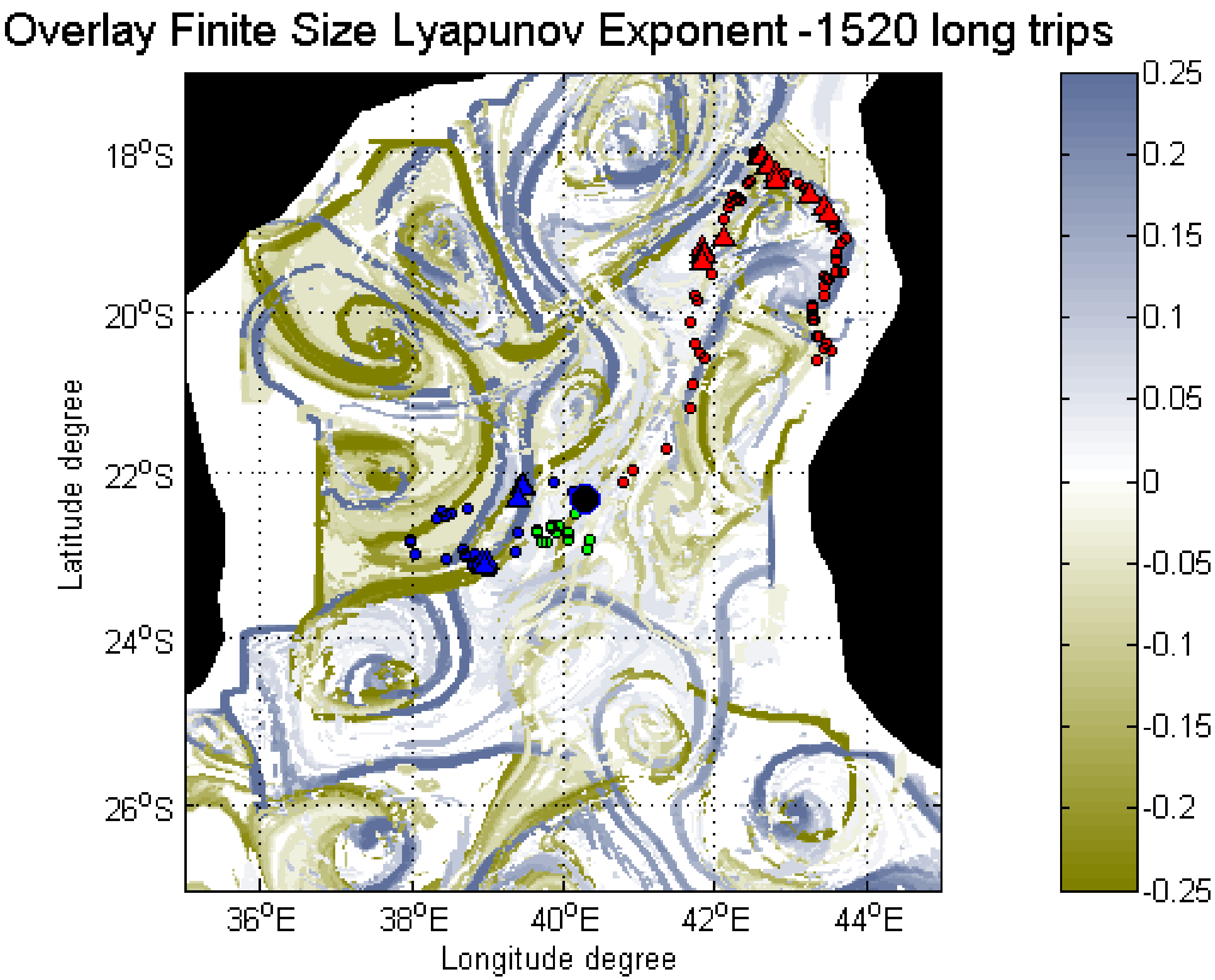

Overlay Finite Size Lyapunov Exponent -1500 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

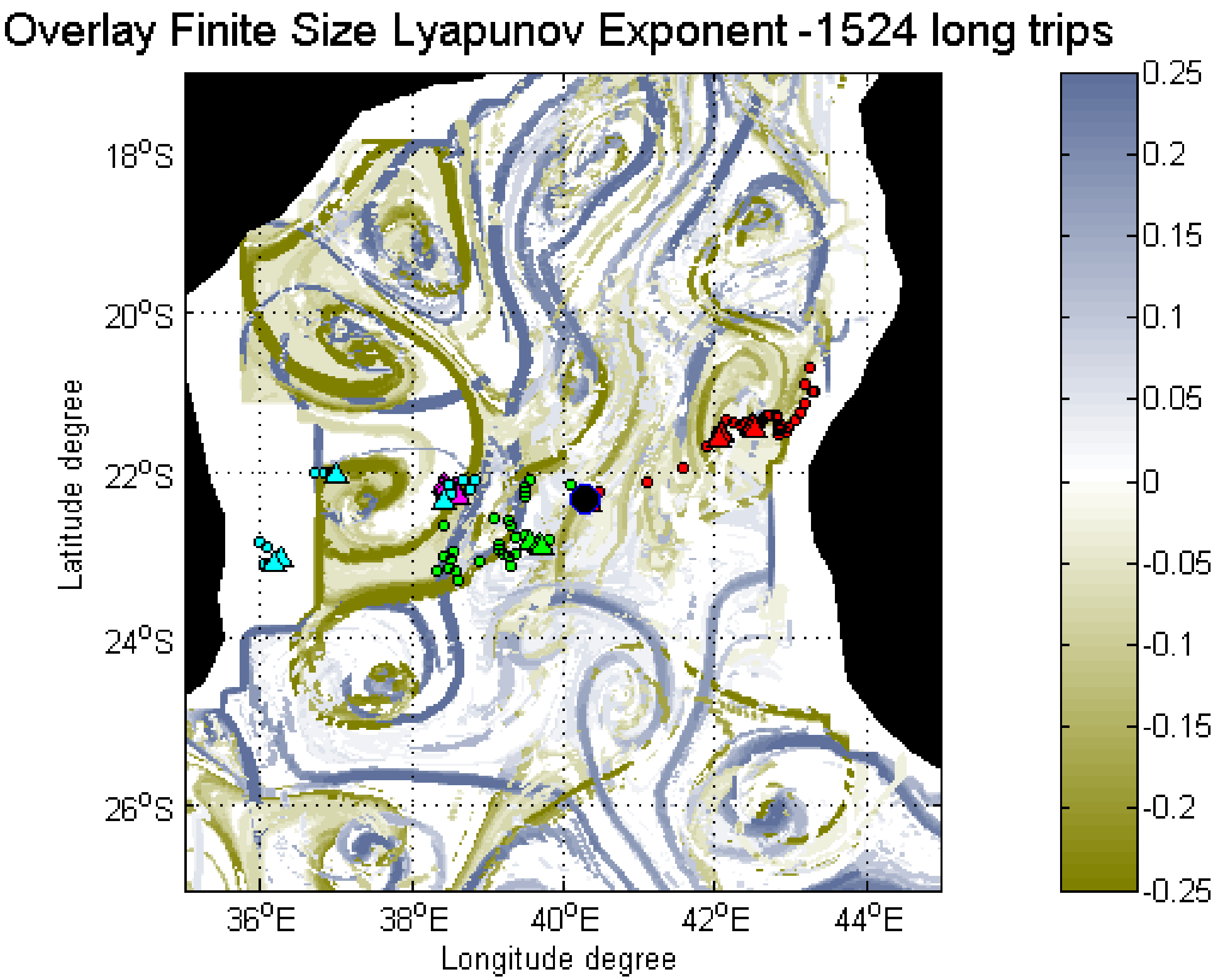

Overlay Finite Size Lyapunov Exponent -1508 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25



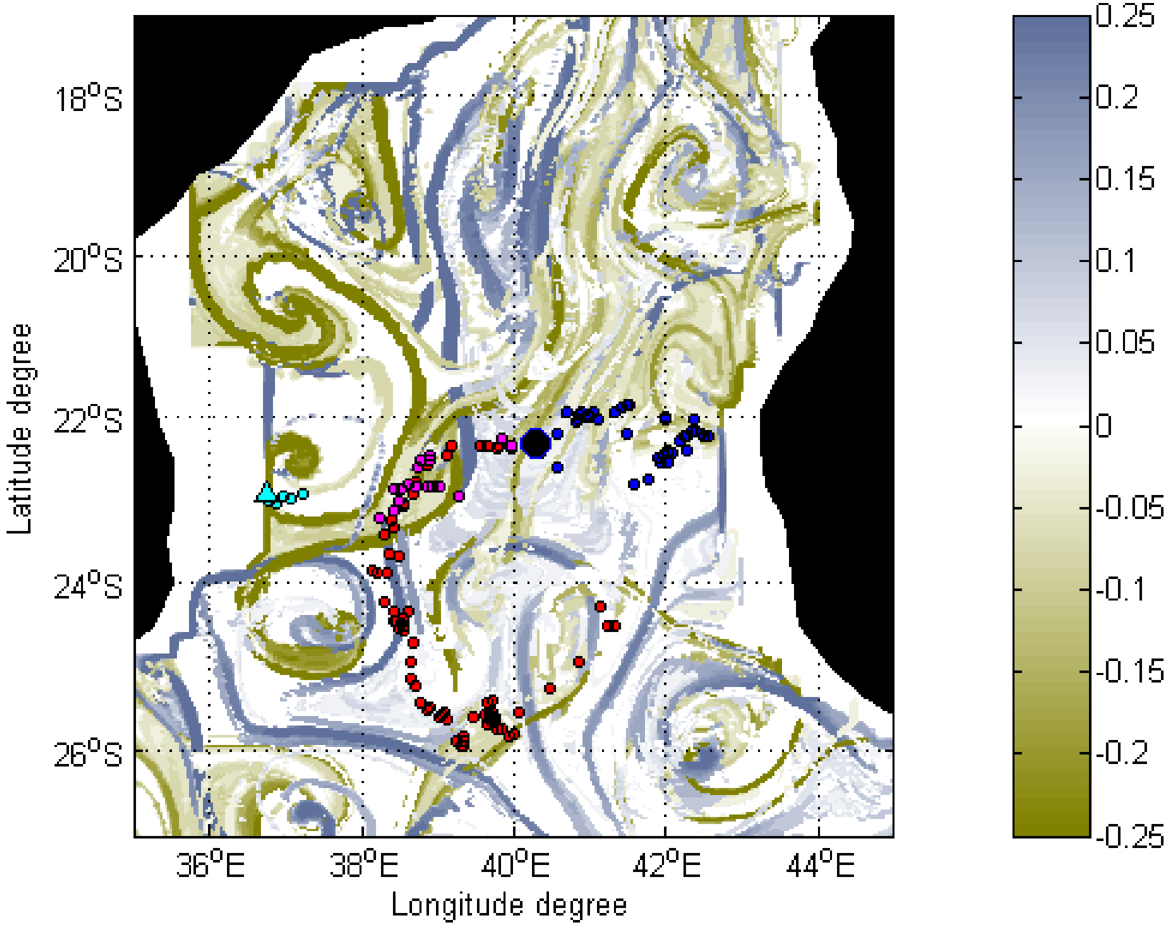
Overlay Finite Size Lyapunov Exponent -1512 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

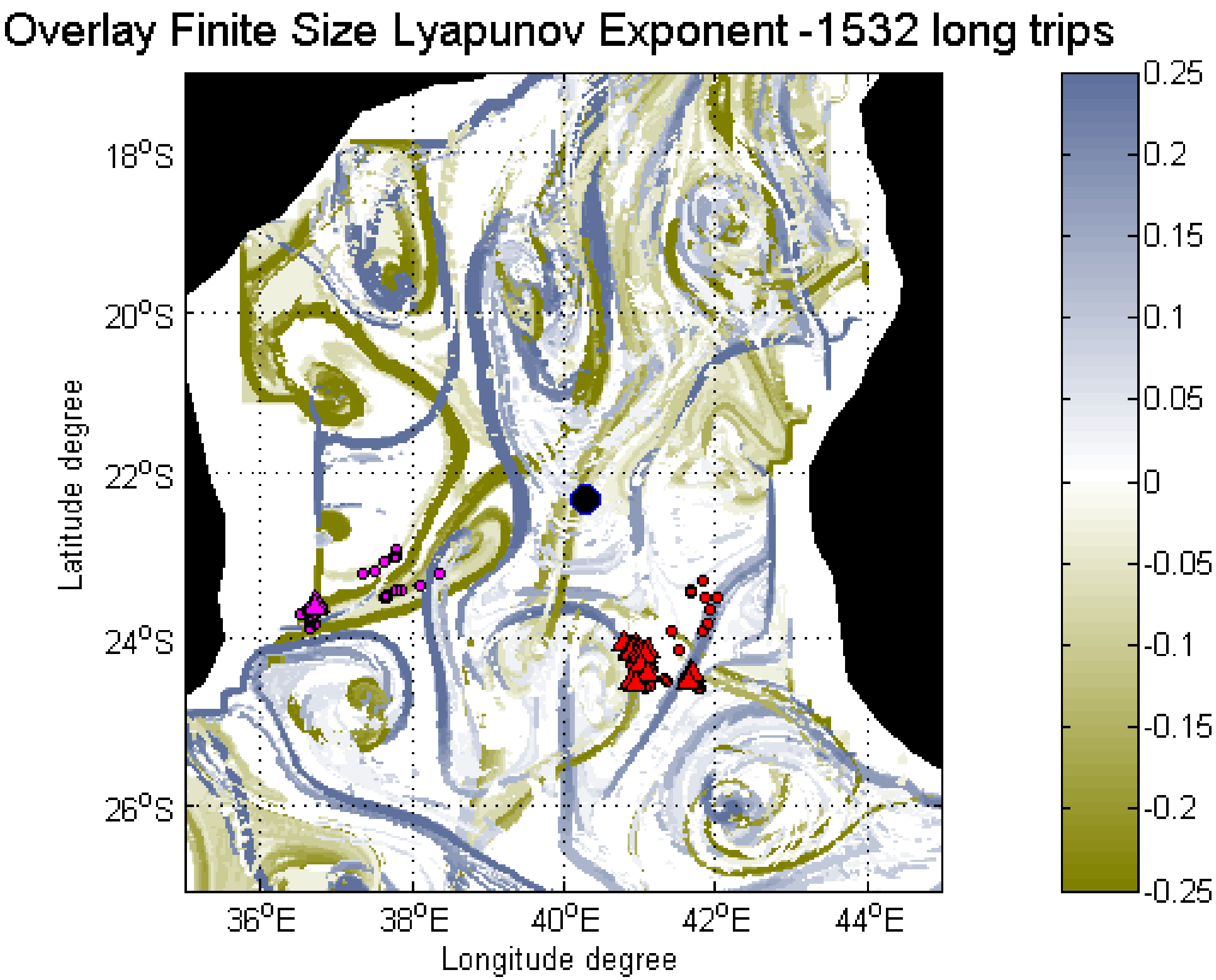

Overlay Finite Size Lyapunov Exponent -1516 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

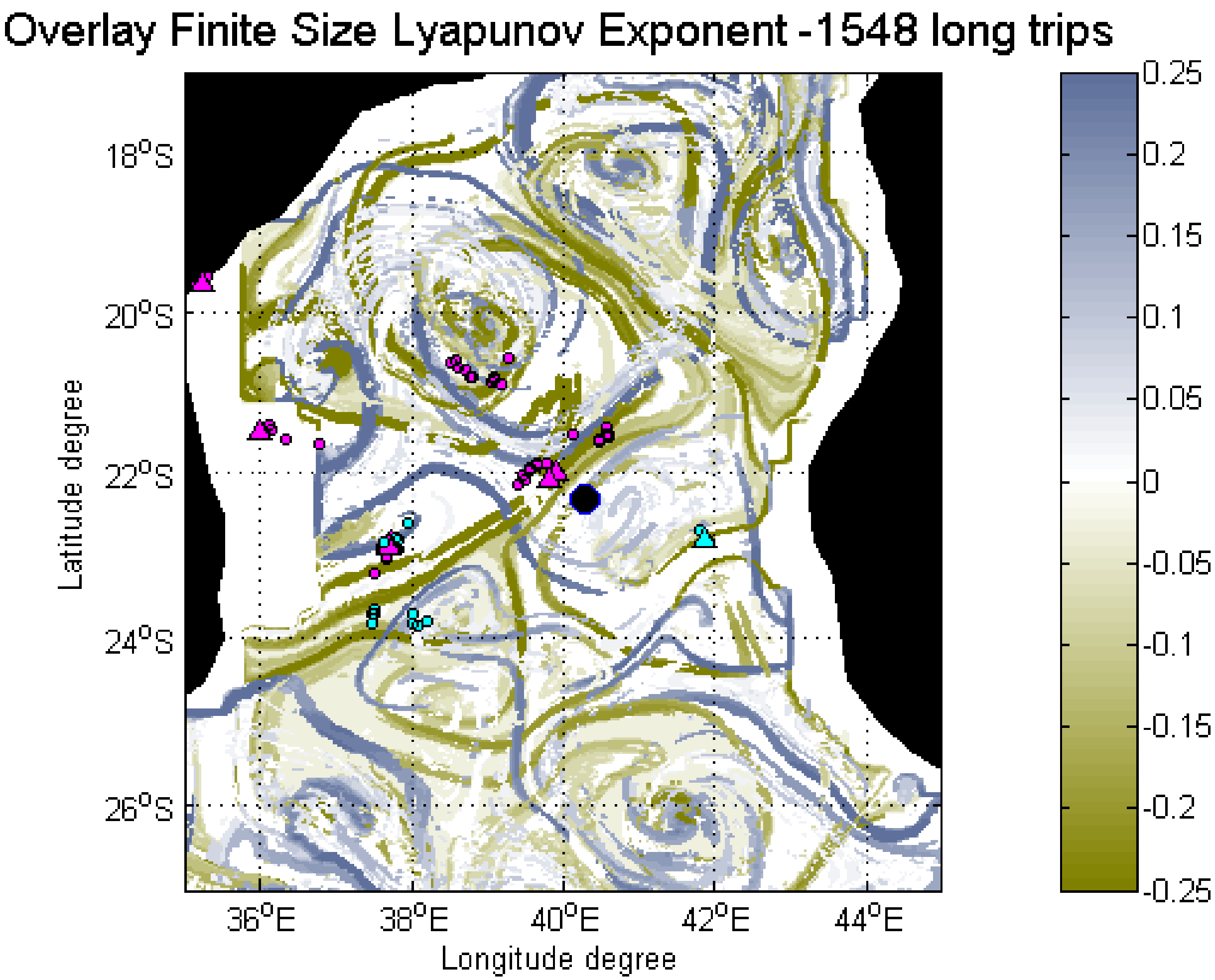

Overlay Finite Size Lyapunov Exponent -1520 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

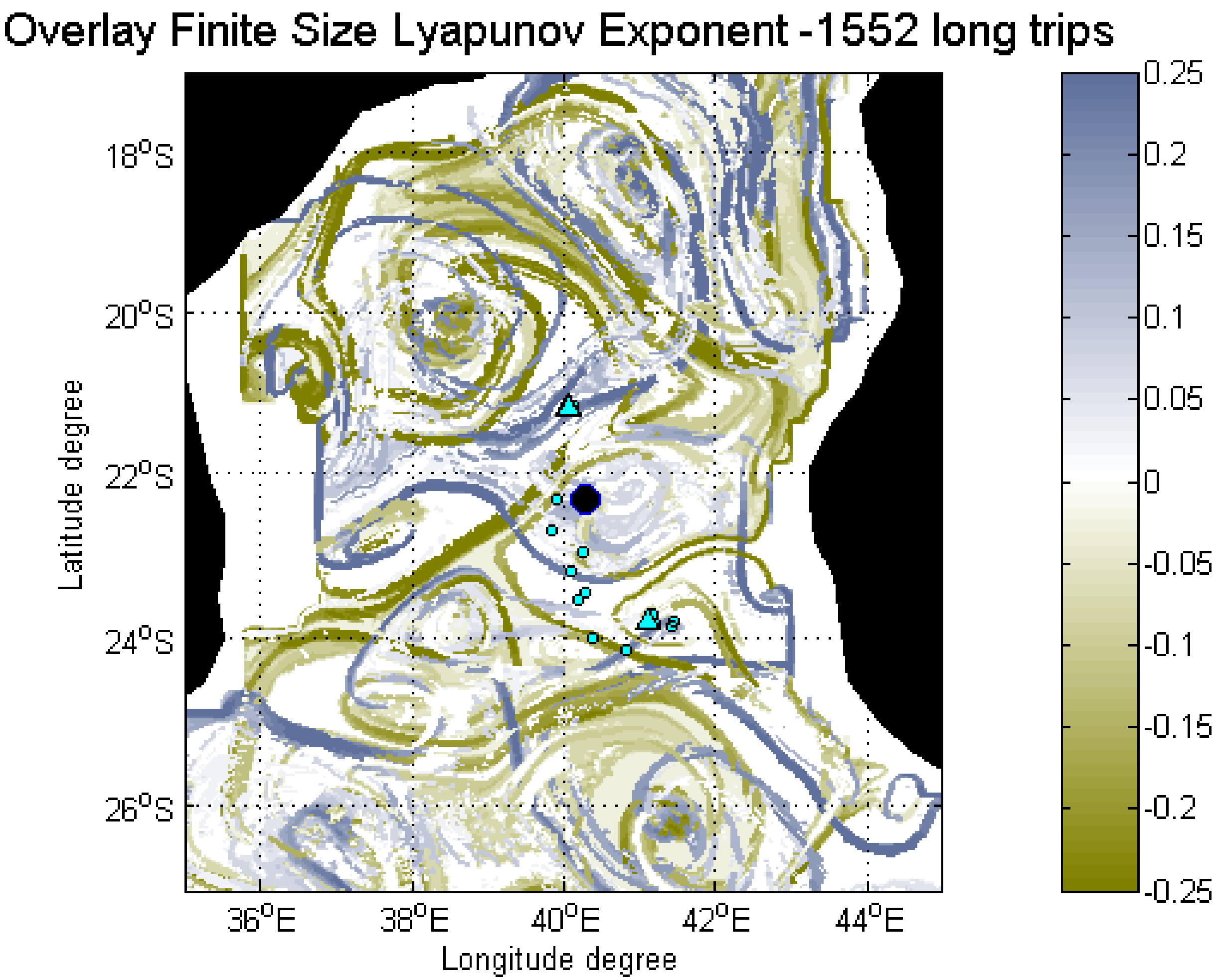

Overlay Finite Size Lyapunov Exponent -1524 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Overlay Finite Size Lyapunov Exponent -1528 long trips

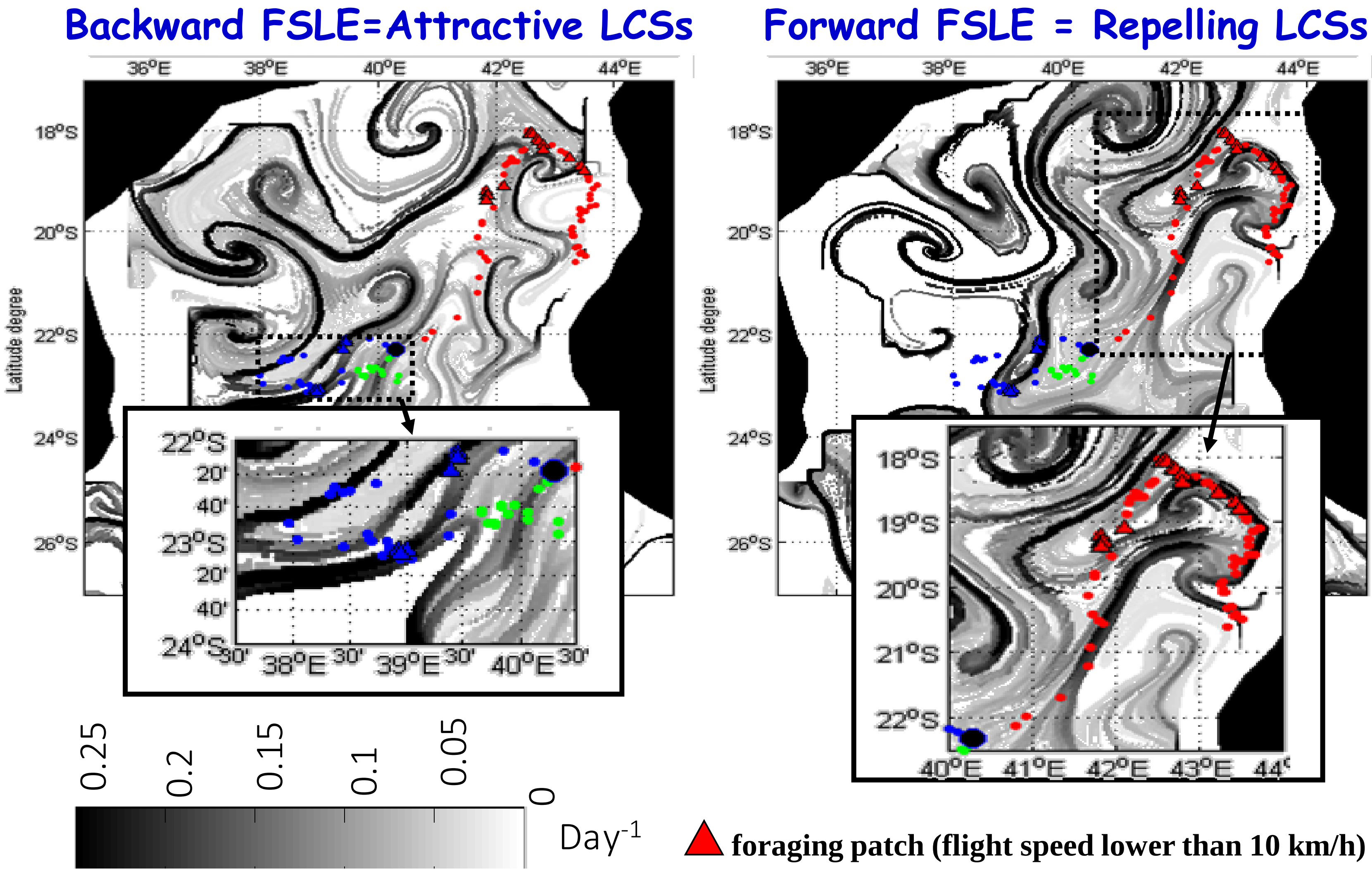



Overlay Finite Size Lyapunov Exponent -1532 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1548 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1552 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Week of September 24, 2003

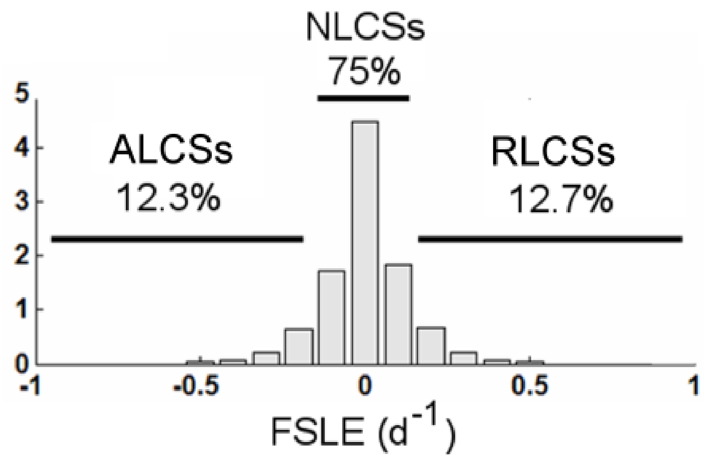


▲ **foraging patch (flight speed lower than 10 km/h)**

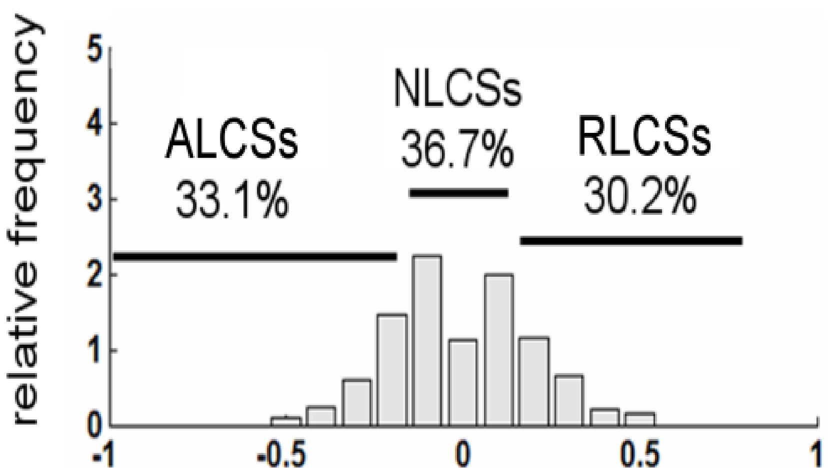
● **seabird trajectory**

Histograms of FSLE values

On the whole area



On the birds positions



ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day^{-1}
RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day^{-1}
NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird's positions are on them