Quantum Hall thermoelectrics

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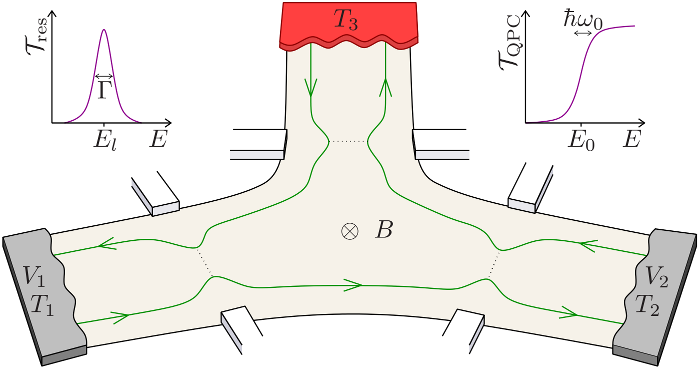
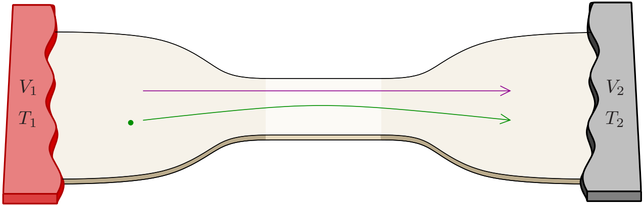


In collaboration with: Björn Sothmann (Genève)
Andrew N. Jordan (Rochester)

Palma de Mallorca, 22 April, 2015

Two terminal thermoelectrics



Charge current:

$$I_l^e = \frac{e}{h}\int dE\mathcal{T}(E)[f_l(E) - f_{\bar{l}}(E)]$$

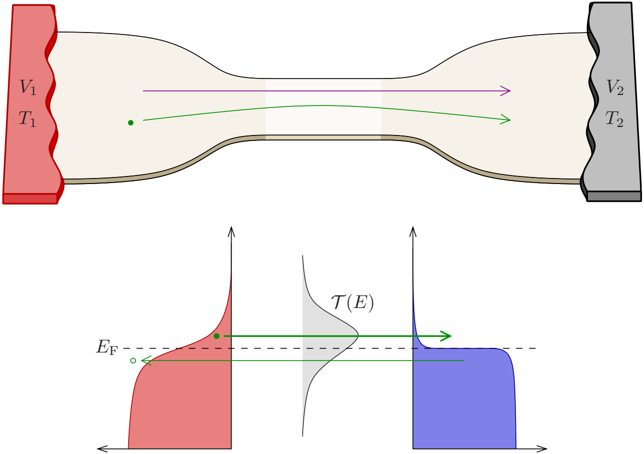
Heat current:

$$I_l^h = \frac{1}{h}\int dE(E - E_{\mathrm{F}})\mathcal{T}(E)[f_l(E) - f_{\bar{l}}(E)]$$

$$f_l(E) = \left[1 + e^{(E - eV_l)/k_{\mathrm{B}}T_l}\right]^{-1}$$

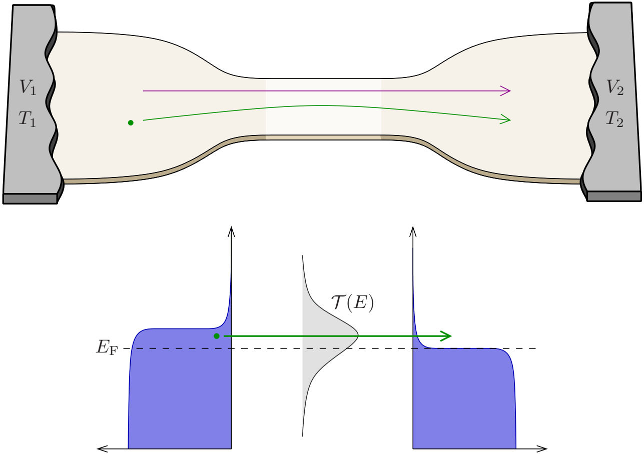
U. Sivan and Y. Imry, Phys. Rev. B **33**, 551 (1986)

Seebeck effect



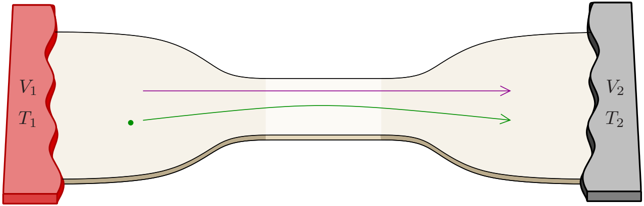
$$I_l^e = \frac{e}{2h}\int dE\,\mathcal{T}(E)(\partial_E f(E))\left[eV_l + E\frac{T_l - T}{T}\right]$$

Peltier effect



$$I_l^h = \frac{1}{2h}\int dE\, E\, \mathcal{T}(E)(\partial_E f(E))\left[eV_l + E\frac{T_l - T}{T}\right]$$

Onsager reciprocity relations



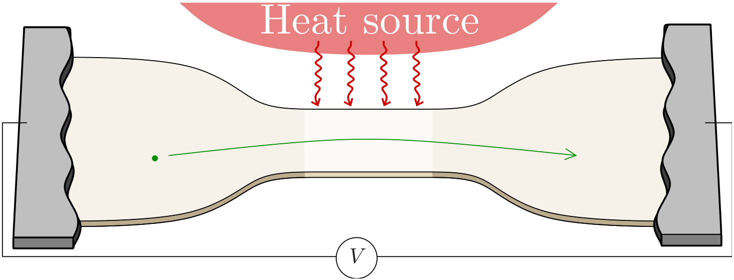
$$I_l^e = \frac{e}{2h}\sum_j \int dE[N\delta_{lj}-\mathcal{T}_{lj}(E)](-\partial_E f(E))\left[eV_j+E\frac{T_j-T}{T}\right] = \mathcal{L}_{lj}^{eV}\frac{eV_j}{k_\mathrm{B}T}+\mathcal{L}_{lj}^{eT}\frac{k_\mathrm{B}\Delta T_j}{(k_\mathrm{B}T)^2}$$

$$I_l^h = \frac{1}{2h}\sum_j \int dE\,E[N\delta_{lj}-\mathcal{T}_{lj}(E)](-\partial_E f(E))\left[eV_j+E\frac{T_j-T}{T}\right] = \mathcal{L}_{lj}^{hV}\frac{eV_j}{k_\mathrm{B}T}+\mathcal{L}_{lj}^{hT}\frac{k_\mathrm{B}\Delta T_j}{(k_\mathrm{B}T)^2}$$

$$\frac{1}{e}\mathcal{L}_{lj}^{eT} = \mathcal{L}_{jl}^{hV}$$

$$\mathcal{L}_{lj}^{hT} = \mathcal{L}_{jl}^{hT}$$

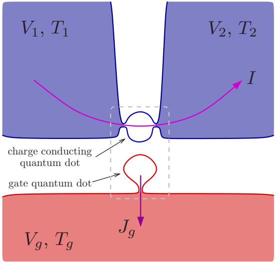
Three terminal thermoelectrics



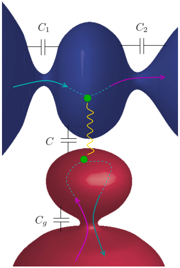
Energy harvesting demands **three terminal** devices

Separation of heat and charge currents

Three terminal thermoelectrics



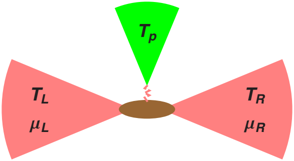
R. Sánchez and M. Büttiker, Phys. Rev. B **83**, 085428 (2011)



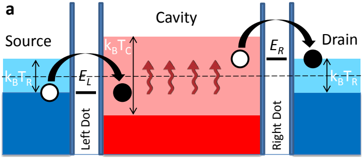
B. Sothmann, R. Sánchez, A. N. Jordan, M. Büttiker, Phys. Rev. B **85**, 205301 (2012)

Verified experimentally!

B. Roche *et al.*, Nat. Comm. **6**, 6738 (2015)
F. Hartmann *et al.*, Phys. Rev. Lett. **11**, 146805 (2015)



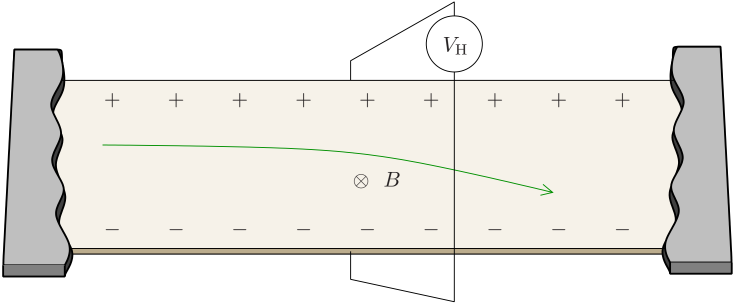
O. Entin-Wohlman, Y. Imry, A. Aharony, Phys. Rev. B **82**, 115314 (2010)



A. N. Jordan, B. Sothmann, R. Sánchez, M. Büttiker, Phys. Rev. B **87**, 075312 (2013)

Review: B. Sothmann, R. Sánchez, A. N. Jordan, Nanotechnology **26**, 032001 (2015)

(Classical) Hall effect

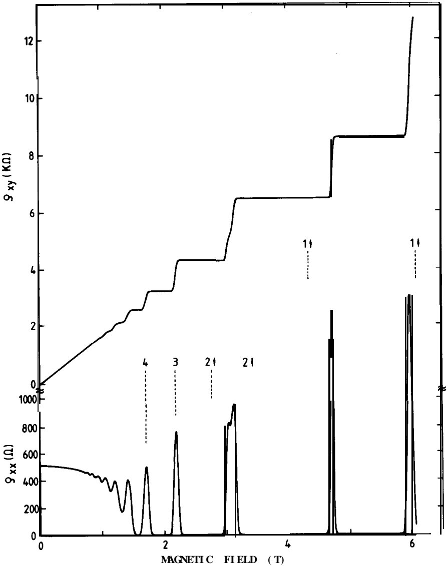


$$F = e(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

$$V_{\mathrm{H}} = -\alpha B I$$

Transverse resistance increases linearly with the magnetic field

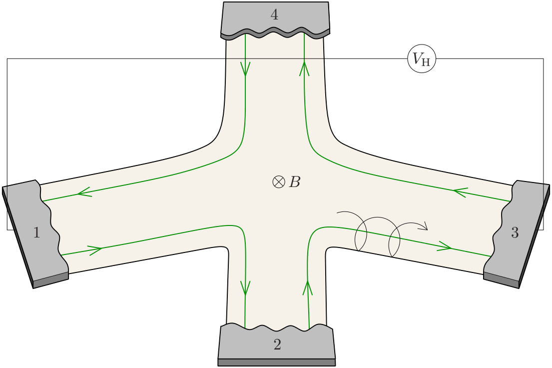
Quantum Hall effect.



$$R_{\mathrm{H}} = \frac{h}{Ne^2}$$

K. von Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. **45**, 494 (1980)
K. von Klitzing, Nobel lecture (1985)

Quantum Hall effect



$$I_l^e = \frac{e^2}{h}(V_l - V_{l-1})$$

1,3 are voltage probes

$$V_1 = V_4$$

$$V_3 = V_2$$

Charge conservation:

$$I_2 + I_4 = 0$$

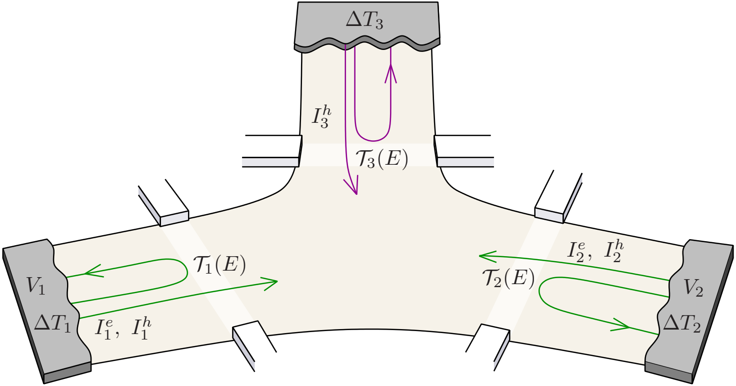
$$I_4 = \frac{e^2}{h}(V_1 - V_3)$$

$$V_{\mathrm{H}} = V_1 - V_3 = \frac{h}{e^2} I$$

Propagation without backscattering along edge states

B. I. Halperin, Phys. Rev. B **25**, 2185 (1982)
M. Büttiker, Phys. Rev. B **38**, 9375 (1988)

Back to three terminals

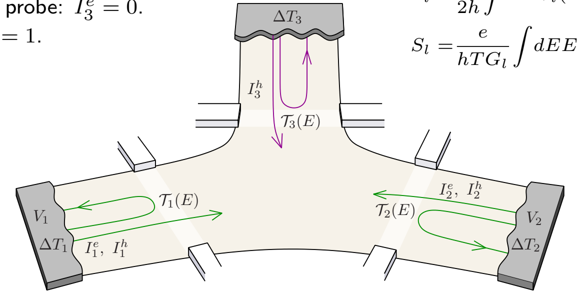


Scattering theory. Linear regime. No magnetic field

Terminal 3 is a probe: $I_3^e = 0$.
Assume $\mathcal{T}_3(E) = 1$.

$$G_l = \frac{e^2}{2h} \int dE \; \mathcal{T}_l(E) \, (-\partial_E f)$$

$$S_l = \frac{e}{hTG_l} \int dE E \; \mathcal{T}_l(E) \, (-\partial_E f)$$



Scattering theory. Linear regime. No magnetic field

Terminal 3 is a probe: $I_3^e = 0$.
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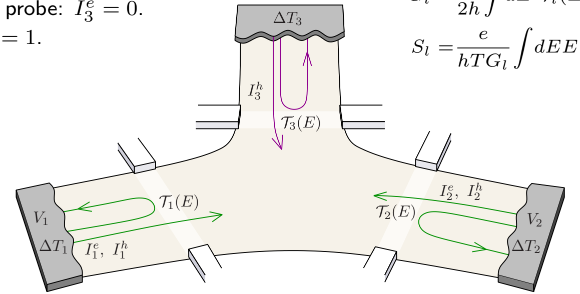
$$G_l = \frac{e^2}{2h}\int dE\ \mathcal{T}_l(E)\,(-\partial_E f)$$

$$S_l = \frac{e}{hTG_l}\int dE E\ \mathcal{T}_l(E)\,(-\partial_E f)$$



$$\begin{pmatrix} I^e \\ I_j^h \end{pmatrix} = \frac{1}{k_{\mathsf{B}}T}\begin{pmatrix} k_{\mathsf{B}}TG & \mathcal{L}_{1i}^{eT} \\ \mathcal{L}_{j1}^{hV} & \mathcal{L}_{ji}^{hT} \end{pmatrix}\begin{pmatrix} e(V_1 - V_2) \\ \Delta T_i/T \end{pmatrix}$$

$$G = \left(\frac{1}{G_1} + \frac{1}{G_2}\right)^{-1}$$

Energy harvesting:

$$\mathcal{L}_{13}^{eT} = k_{\mathsf{B}}T^2 G(S_2 - S_1)$$

Energy harvesting if we break:

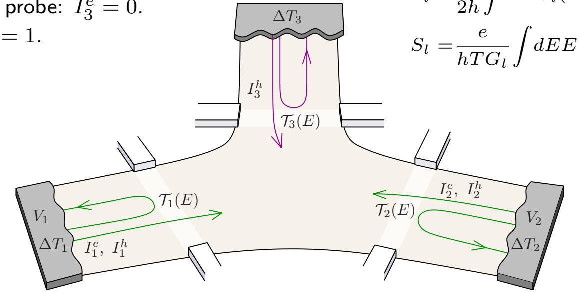
- **Left-right** symmetry
- **Particle-hole** symmetry

Scattering theory. Linear regime. No magnetic field

Terminal 3 is a probe: $I_3^e = 0$.
Assume $\mathcal{T}_3(E) = 1$.

$$G_l = \frac{e^2}{2h}\int dE\ \mathcal{T}_l(E)\,(-\partial_E f)$$

$$S_l = \frac{e}{hTG_l}\int dEE\ \mathcal{T}_l(E)\,(-\partial_E f)$$



$$\begin{pmatrix} I^e \\ I_j^h \end{pmatrix} = \frac{1}{k_{\mathrm{B}}T}\begin{pmatrix} k_{\mathrm{B}}TG & \mathcal{L}_{1i}^{eT} \\ \mathcal{L}_{j1}^{hV} & \mathcal{L}_{ji}^{hT} \end{pmatrix}\begin{pmatrix} e(V_1-V_2) \\ \Delta T_i/T \end{pmatrix}$$

$$G = \left(\frac{1}{G_1}+\frac{1}{G_2}\right)^{-1}$$

Energy harvesting:

$$\mathcal{L}_{13}^{eT} = k_{\mathrm{B}}T^2G(S_2-S_1)$$

No thermal rectification:

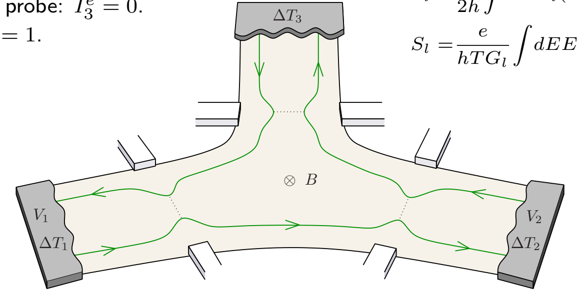
$$\mathcal{L}_{12}^{hT} = \mathcal{L}_{21}^{hT}$$

Edge states in the Quantum Hall regime

Terminal 3 is a probe: $I_3^e = 0$.
Assume $\mathcal{T}_3(E) = 1$.

$$G_l = \frac{e^2}{2h}\int dE\ \mathcal{T}_l(E)\,(-\partial_E f)$$

$$S_l = \frac{e}{hTG_l}\int dEE\ \mathcal{T}_l(E)\,(-\partial_E f)$$



$$G = \frac{1}{\Lambda}\left(\frac{1}{G_1} + \frac{1}{G_2}\right)^{-1}$$

$$\Lambda = 1 - J_1/(G_1+G_2)$$

$$J_n = \frac{e^2}{hk_{\mathsf{B}}T}\int dEE^{n-1}\mathcal{T}_1(E)\mathcal{T}_2(E)\,(-\partial_E f)\,,$$

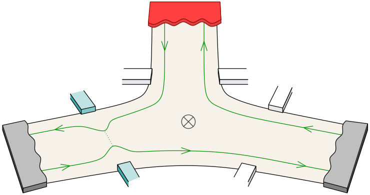
Energy harvesting:

$$\mathcal{L}_{13}^{eT} = k_{\mathsf{B}}T^2G(S_2 - S_1) + e\mathcal{X}_1$$

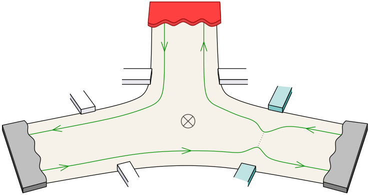
$$\mathcal{X}_l = \frac{k_{\mathsf{B}}T}{e^2}\frac{GG_l}{G_1G_2}\left(eTS_lJ_1 - J_2\right)$$

Current in symmetric configurations!

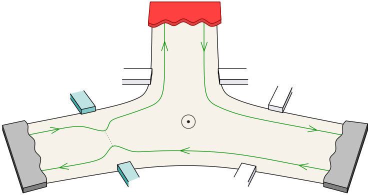
Chiral (crossed) thermopower



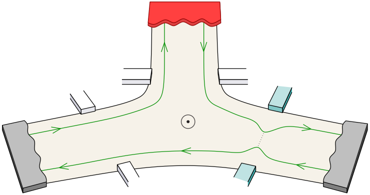
$$\left.\begin{array}{l} S_2 = 0 \\ \mathcal{X}_1 = 0 \end{array}\right\} \Rightarrow \mathcal{L}^{eT}_{13}(B) = e k_\mathsf{B} T^2 G_1 S_1$$



$$\left.\begin{array}{l} S_1 = 0 \\ \mathcal{X}_1 = -e k_\mathsf{B} T^2 G_2 S_2 \end{array}\right\} \Rightarrow \mathcal{L}^{eT}_{13}(B) = 0$$

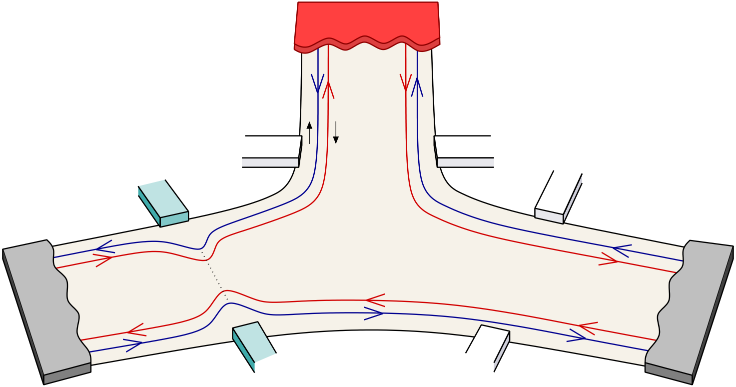


$$\left.\begin{array}{l} S_2 = 0 \\ \mathcal{X}_2 = -e k_\mathsf{B} T^2 G_1 S_1 \end{array}\right\} \Rightarrow \mathcal{L}^{eT}_{13}(-B) = 0$$



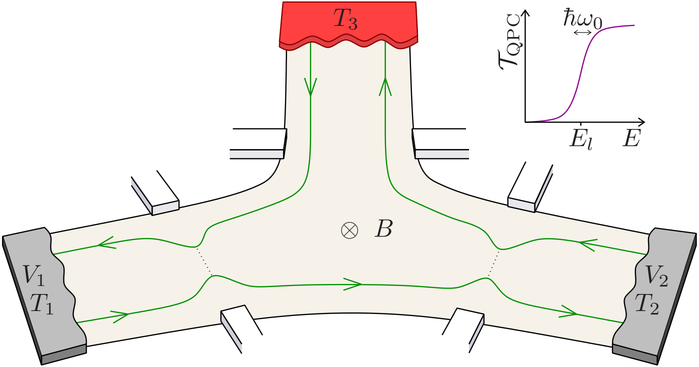
$$\left.\begin{array}{l} S_1 = 0 \\ \mathcal{X}_2 = 0 \end{array}\right\} \Rightarrow \mathcal{L}^{eT}_{13}(-B) = e k_\mathsf{B} T^2 G_2 S_2$$

Topological insulator

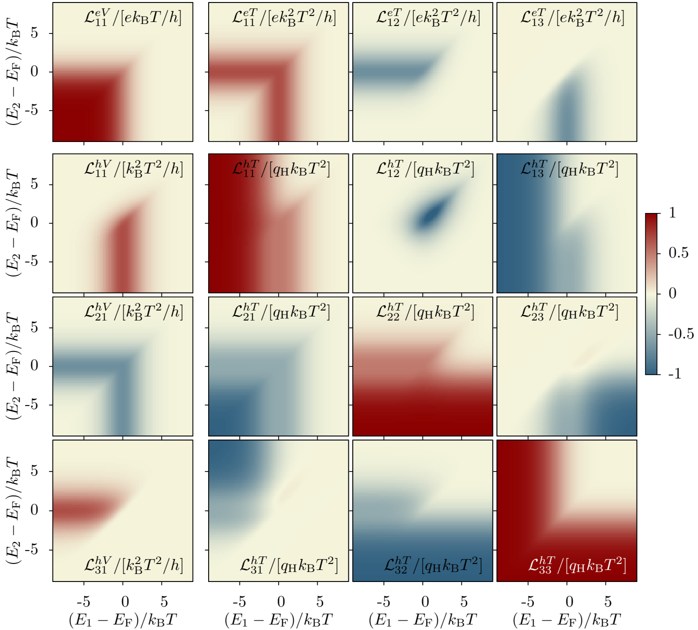


Spin polarized current controlled by the gates

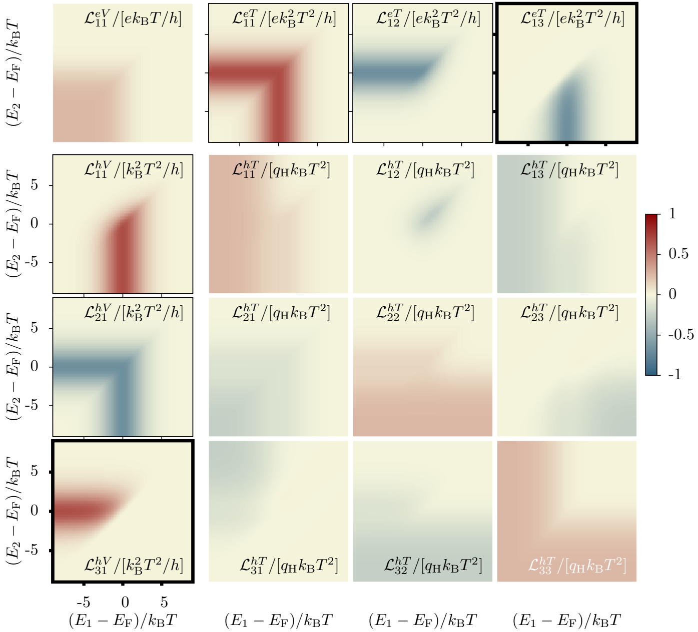
Quantum point contacts



Quantum point contacts. Onsager matrix



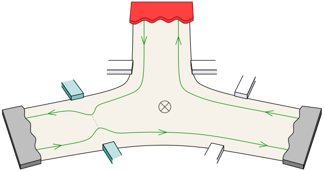
Crossed thermoelectrics



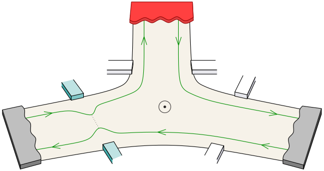
Crossed thermoelectrics

Extreme Seebeck to Peltier asymmetry!

$$\mathcal{L}_{13}^{eT}(B) = ek_{\mathsf{B}}T^2G_1S_1$$

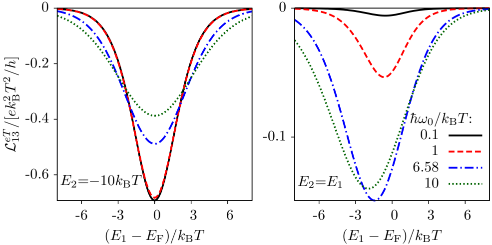


$$\mathcal{L}_{31}^{hV}(B) = \mathcal{L}_{13}^{eT}(-B) = 0$$



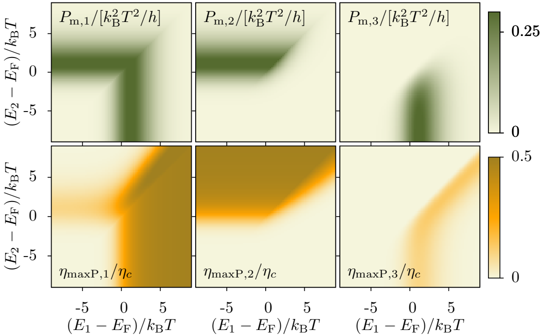
$$\frac{\mathcal{L}_{13}^{eT}(B)}{\mathcal{L}_{31}^{hV}(B)} = \infty$$

Crossed response for symmetric configurations: $\mathcal{L}_{13}^{eT} = e\mathcal{X}_1$



R. Sánchez, B. Sothmann, A.N. Jordan, Phys. Rev. Lett. **114**, 146801 (2015)

Efficiency at maximum power: $\eta_{\mathrm{maxP},l}$

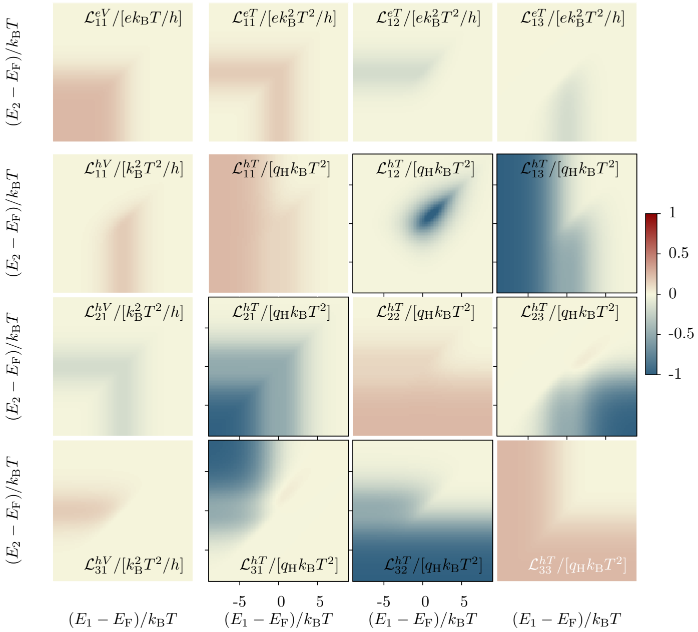


$$P_{\mathrm{m},l} = I_l^e(V_{\mathrm{m},l})V_{\mathrm{m},l}$$

$$\eta_{\mathrm{maxP},l} = \frac{P_{\mathrm{m},l}}{I_l^h(V_{\mathrm{m},l})}$$

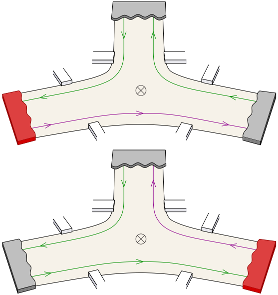
R. Sánchez, B. Sothmann, A.N. Jordan, arXiv:1503.02926

Heat rectification



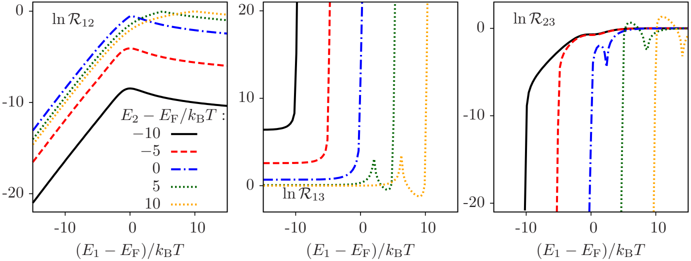
Thermal rectification. Turning heat around the bend

$$\mathcal{R}_{ij} = \frac{\mathcal{L}_{ij}^{hT}}{\mathcal{L}_{ji}^{hT}}$$



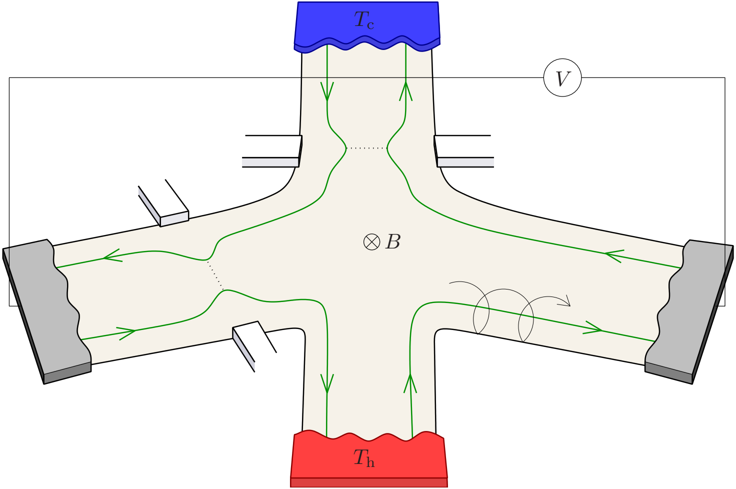
$\mathcal{R}_{ij} = 1$: No thermal rectification

$|\ln \mathcal{R}_{ij}| \gg 1$: Thermal diode



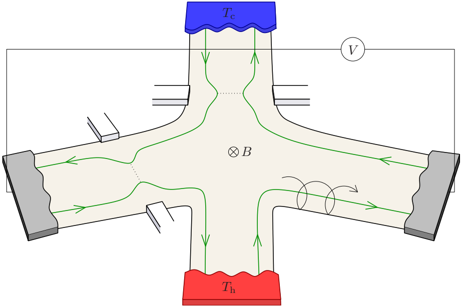
R. Sánchez, B. Sothmann, A.N. Jordan, arXiv:1503:02926

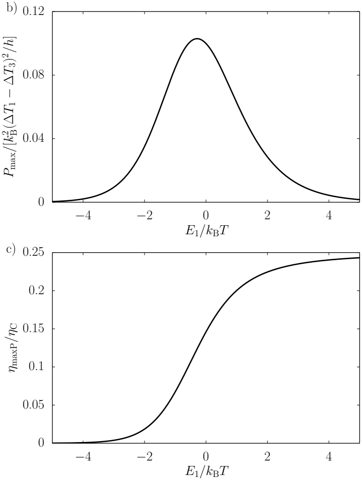
Quantum Nernst engines



Inject only heat. Measure only charge.

Quantum Nernst engines





Boundary conditions:

$I_1^h = I_3^h = 0$

$I_2^e = I_4^e = 0$

$$\mathcal{L}_{eT} \neq 0$$

$$\mathcal{L}_{hV} = 0$$

Classical Nernst engine: $\eta_\mathrm{maxP} \leq \eta_c/6$

J. Stark, K. Brander, U. Seifert, Phys. Rev. Lett. **112**, 140601 (2014)

B. Sothmann, R. Sánchez, A.N. Jordan, Europhys. Lett. **107**, 47003 (2014)

Conclusions

- Chirality detected by thermoelectric measurements
- Three terminal junctions separate heat and charge flows
- Edge states permit the manipulation of heat currents
- Extreme asymmetries of Onsager matrix
- Powerful and efficient **energy harvesting** in the crossed response
- Ideal **thermal diodes** in the longitudinal terms
- Gate control of spin polarization in topological insulators

- Heat engine based on the (quantum Hall) Nernst effect outperforms its classical version

B. Sothmann, R. Sánchez, A.N. Jordan, Europhys. Lett. **107**, 47003 (2014)
R. Sánchez, B. Sothmann, A.N. Jordan, Phys. Rev. Lett. **114**, 146801 (2015)
R. Sánchez, B. Sothmann, A.N. Jordan, arXiv:1503:02926