Landauer's principle in multipartite open quantum system dynamics

A collision model based approach

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Talk Summary

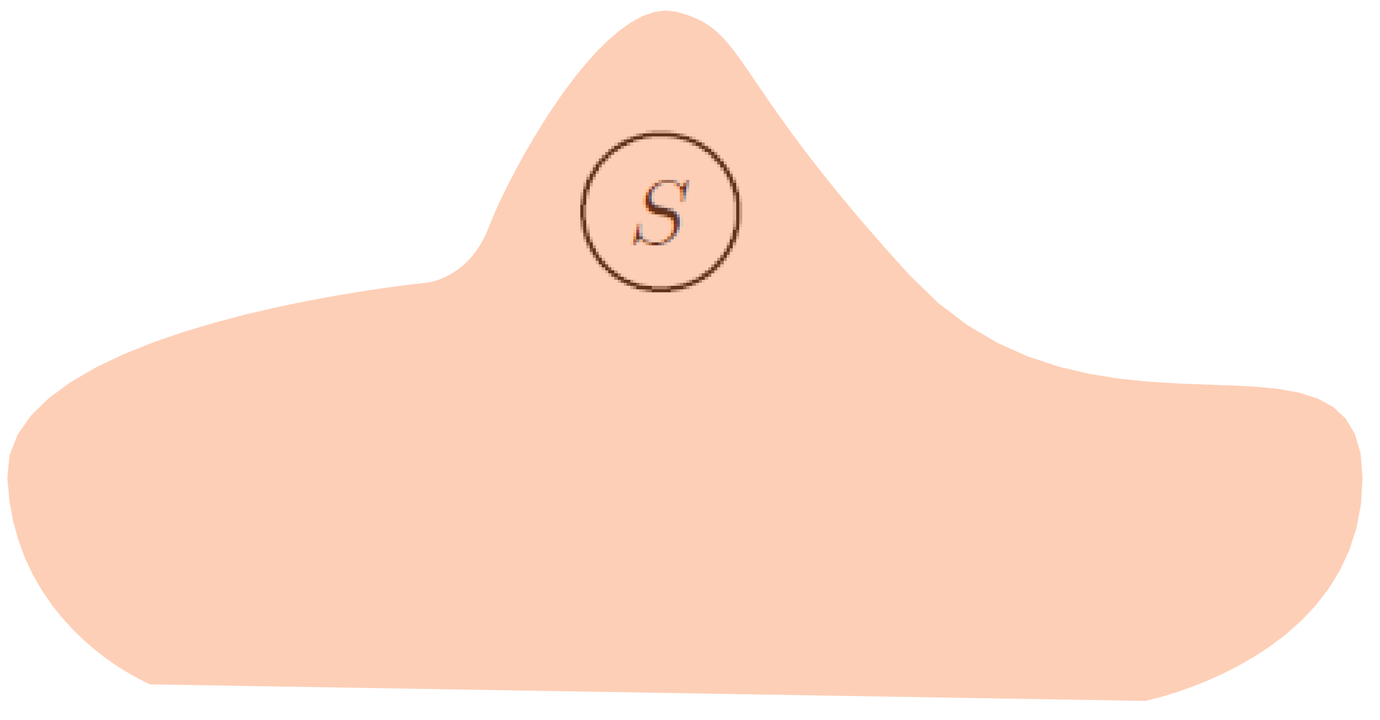
- Landauer Power
- Recycling Environment Model

  Landauer at different temperatures

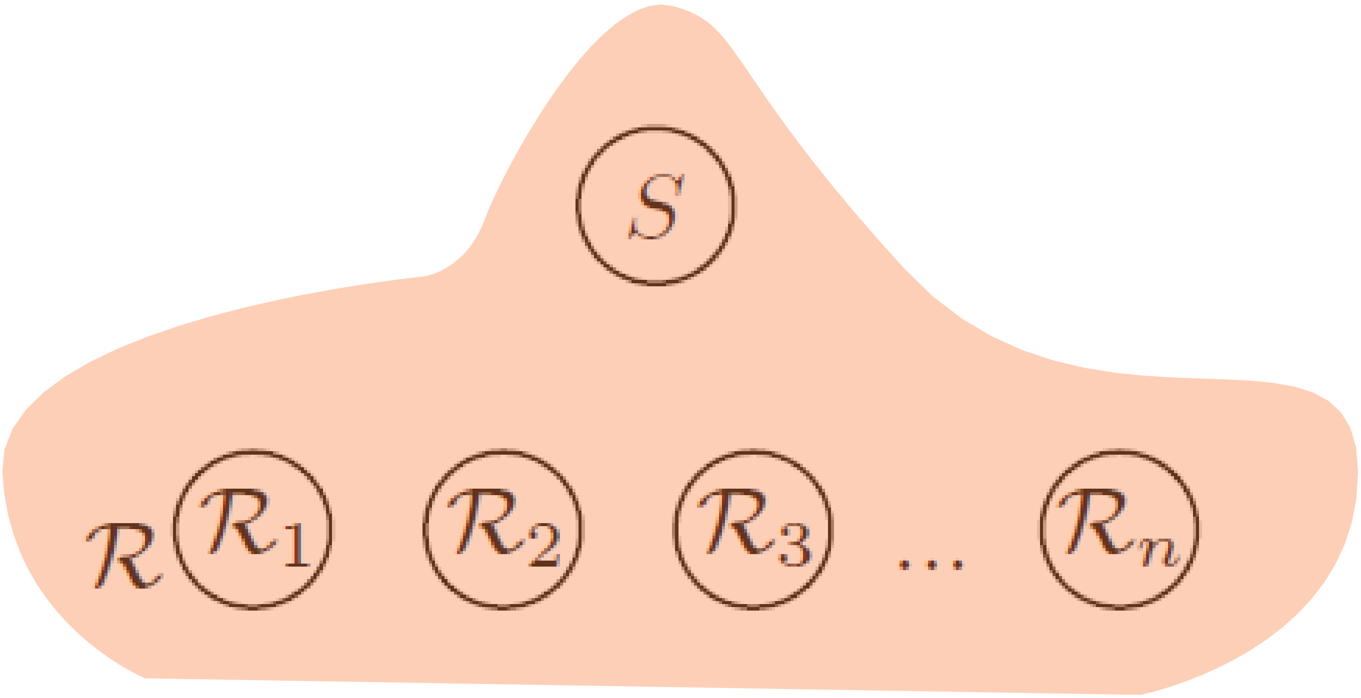
  Dependence on N
- Indirect Erasure Model

  Landauer and Non-Markovianity different at temperatures

Landauer Power



Landauer Power

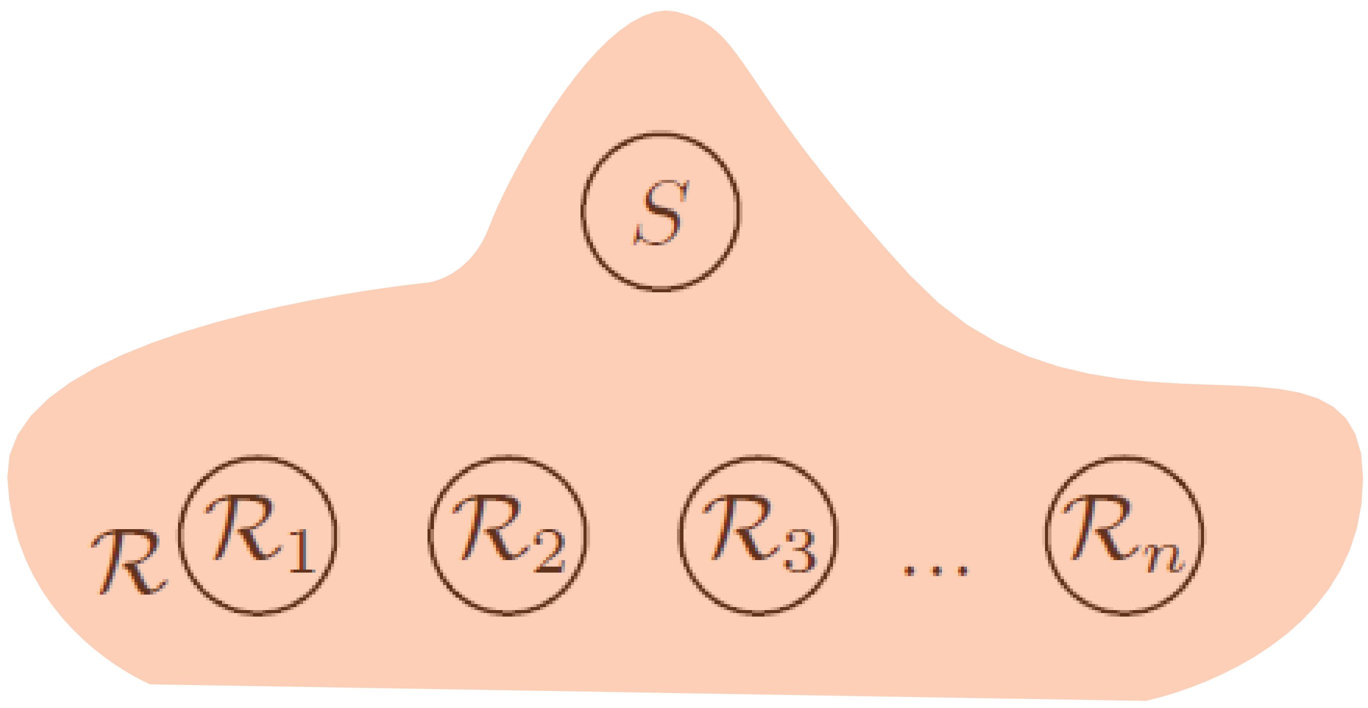


$$\eta = \frac{e^{-\beta \hat{H}_{R_n}}}{\mathrm{Tr}\left[e^{-\beta \hat{H}_{R_n}}\right]}$$

$$\hat{U} = e^{-ig\hat{V}\tau}$$

$$\hat{V} = \sum_k \hat{S}_k \otimes \hat{R}_k$$

Landauer Power



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$$\hat{U} = e^{-ig\hat{V}\tau}$$

$$\hat{V} = \sum_k \hat{S}_k \otimes \hat{R}_k$$

$$\rho_{n+1} = \mathrm{Tr}_{\mathcal{R}}\left[\hat{U}\rho_n \otimes \eta \hat{U}^\dagger\right] = \underset{\text{(CPTP)}}{\Phi\left[\rho_n\right]}$$

$$\eta_{n+1} = \mathrm{Tr}_{S}\left[\hat{U}\rho_n \otimes \eta \hat{U}^\dagger\right] = \Lambda_n\left[\eta\right]$$

$$\Delta E_{n+1} = \mathrm{Tr}\left[\hat{H}_S\left(\Phi - \mathbb{I}\right)\left[\rho_n\right]\right] \quad \Delta Q_{n+1} = \mathrm{Tr}\left[\hat{H}_{\mathcal{R}}\left(\Lambda_n - \mathbb{I}\right)\left[\eta\right]\right]$$

Assumption: $\langle\hat{\mathcal{R}}_k\rangle_\eta = \mathrm{Tr}\left[\hat{\mathcal{R}}_k\eta\right] = 0$

$$\tau << 1$$

$$\Delta\rho_{n+1} = \left(\Phi - \mathbb{I}\right)\rho_n = \mathcal{K}\rho_n$$

$$\Delta E_{n+1} = g^2\tau^2\sum_{kj}\langle\hat{\mathcal{R}}_k\hat{\mathcal{R}}_j\rangle_\eta\langle\hat{S}_k\hat{H}_S\hat{S}_j - \frac{1}{2}\left\{\hat{S}_k\hat{S}_j, \hat{H}_S\right\}\rangle_{\rho_n}$$

$$\Delta Q_{n+1} = g^2\tau^2\sum_{kj}\langle\hat{S}_k\hat{S}_j\rangle_{\rho_n}\langle\hat{\mathcal{R}}_k\hat{H}_{\mathcal{R}}\hat{\mathcal{R}}_j - \frac{1}{2}\left\{\hat{\mathcal{R}}_k\hat{\mathcal{R}}_j, \hat{H}_{\mathcal{R}}\right\}\rangle_\eta$$

$$\Delta E_{n+1} = \mathrm{Tr}\left[\hat{H}_S\left(\Phi - \mathbb{I}\right)[\rho_n]\right] \qquad \Delta Q_{n+1} = \mathrm{Tr}\left[\hat{H}_{\mathcal{R}}\left(\Lambda_n - \mathbb{I}\right)[\eta]\right]$$

Assumption: $\tau \to 0 \text{ s.t. } t = n\tau \text{ and } g^2\tau \to \gamma$

$$\dot{\rho} = \mathcal{K}_2\left[\rho\left(t\right)\right]$$

$$\dot{E} = \gamma \sum_{k,j} \langle \hat{\mathcal{R}}_k \hat{\mathcal{R}}_j \rangle_\eta \langle \hat{S}_k \hat{H}_S \hat{S}_j - \frac{1}{2}\left\{\hat{S}_k \hat{S}_j, \hat{H}_S\right\}\rangle_\rho$$

$$\dot{Q} = \gamma \sum_{k,j} \langle \hat{S}_k \hat{S}_j \rangle_\rho \langle \hat{\mathcal{R}}_k \hat{H}_{\mathcal{R}} \hat{\mathcal{R}}_j - \frac{1}{2}\left\{\hat{\mathcal{R}}_k \hat{\mathcal{R}}_j, \hat{H}_{\mathcal{R}}\right\}\rangle_\eta$$

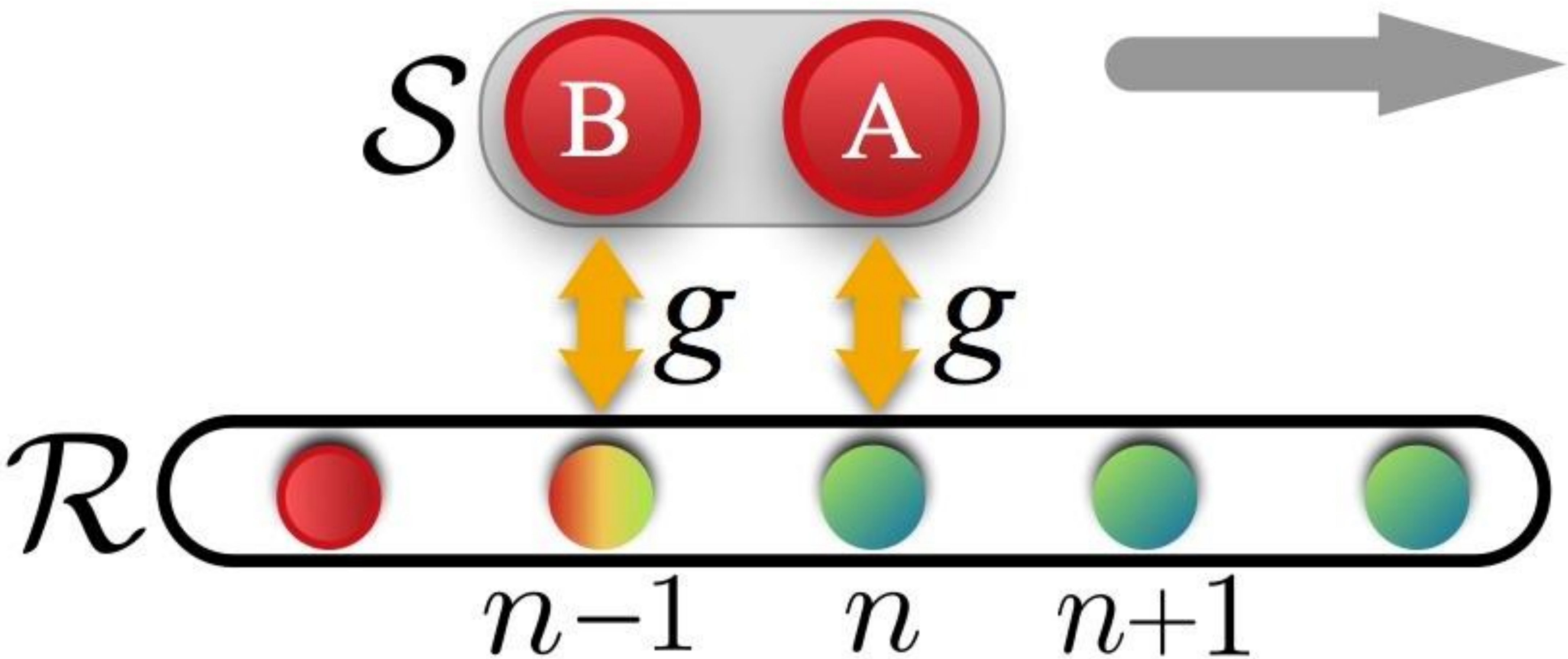
Assumption: $\left[\hat{U}, \left(\hat{H}_S + \hat{H}_{\mathcal{R}}\right)\right] = 0$

$$\rho^{\mathrm{eq}} = \frac{e^{-\beta \hat{H}_S}}{\mathrm{Tr}\left[e^{-\beta \hat{H}_S}\right]} \qquad \dot{Q} = -\dot{E}$$

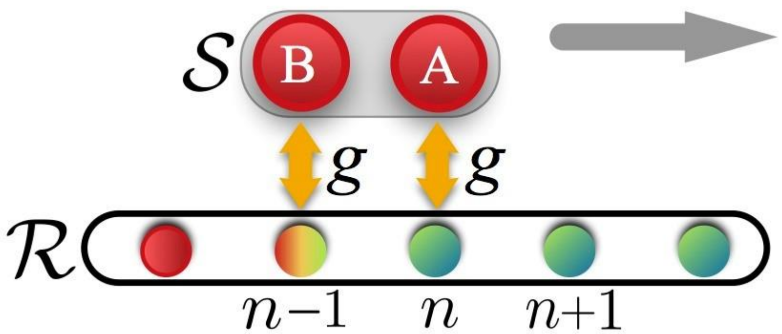
$$\dot{S}\left(\rho \mid \rho^{\mathrm{eq}}\right) = \mathrm{Tr}\left[\dot{\rho}\left(\ln\rho - \ln\rho^{\mathrm{eq}}\right)\right] = -\dot{S}\left(\rho\right) + \beta\dot{E}$$

$$\beta\dot{Q}(t) \geq \dot{\tilde{S}}(\rho)$$

Recycling Environment Model



Recycling Environment Model



$$\dot{\rho} = \sum_{X=A,B} \mathcal{L}_X[\rho] + \mathcal{D}_{AB}[\rho]$$

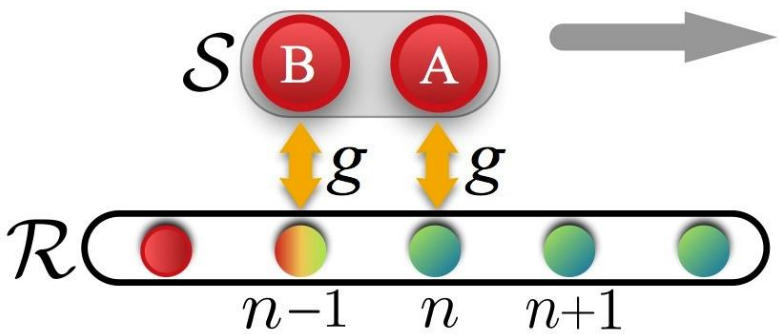
$$\mathcal{L}_X[\rho] = \gamma \sum_{kj} \langle \hat{R}_k \hat{R}_j \rangle_\eta \left(\hat{S}_{Xj} \rho \hat{S}_{xk} - \frac{1}{2} \left\{ \hat{S}_{Xk} \hat{S}_{Xj}, \rho \right\} \right)$$

$$\mathcal{D}_{AB}[\rho] = \gamma \sum_{kj} \langle \hat{R}_j \hat{R}_k \rangle_\eta \left[\hat{S}_{Ak} \rho, \hat{S}_{Bj} \right] + \langle \hat{R}_k \hat{R}_j \rangle_\eta \left[\hat{S}_{Bj} \rho, \hat{S}_{Ak} \right]$$

$$\xi = \tanh(\beta\omega/2)$$

$$\hat{S}^X = \left\{\sigma_x^X, \sigma_y^X\right\} \qquad \hat{R} = \left\{\sigma_x^R, \sigma_y^R\right\}$$

$$\eta = \begin{pmatrix} \frac{1-\xi}{2} & 0 \\ 0 & \frac{1+\xi}{2} \end{pmatrix}$$



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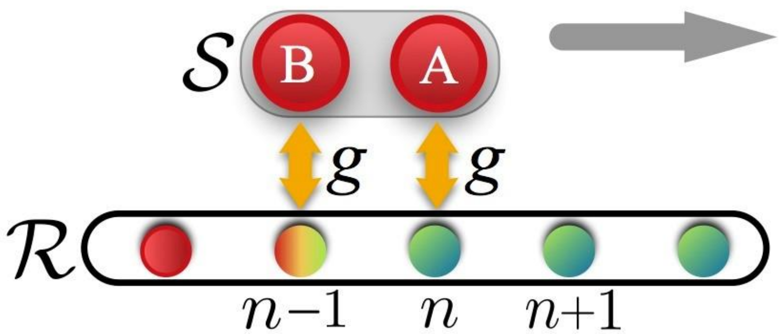


$$\dot{\rho} = \sum_{X=A,B} \mathcal{L}_X[\rho] + \mathcal{D}_{AB}[\rho]$$

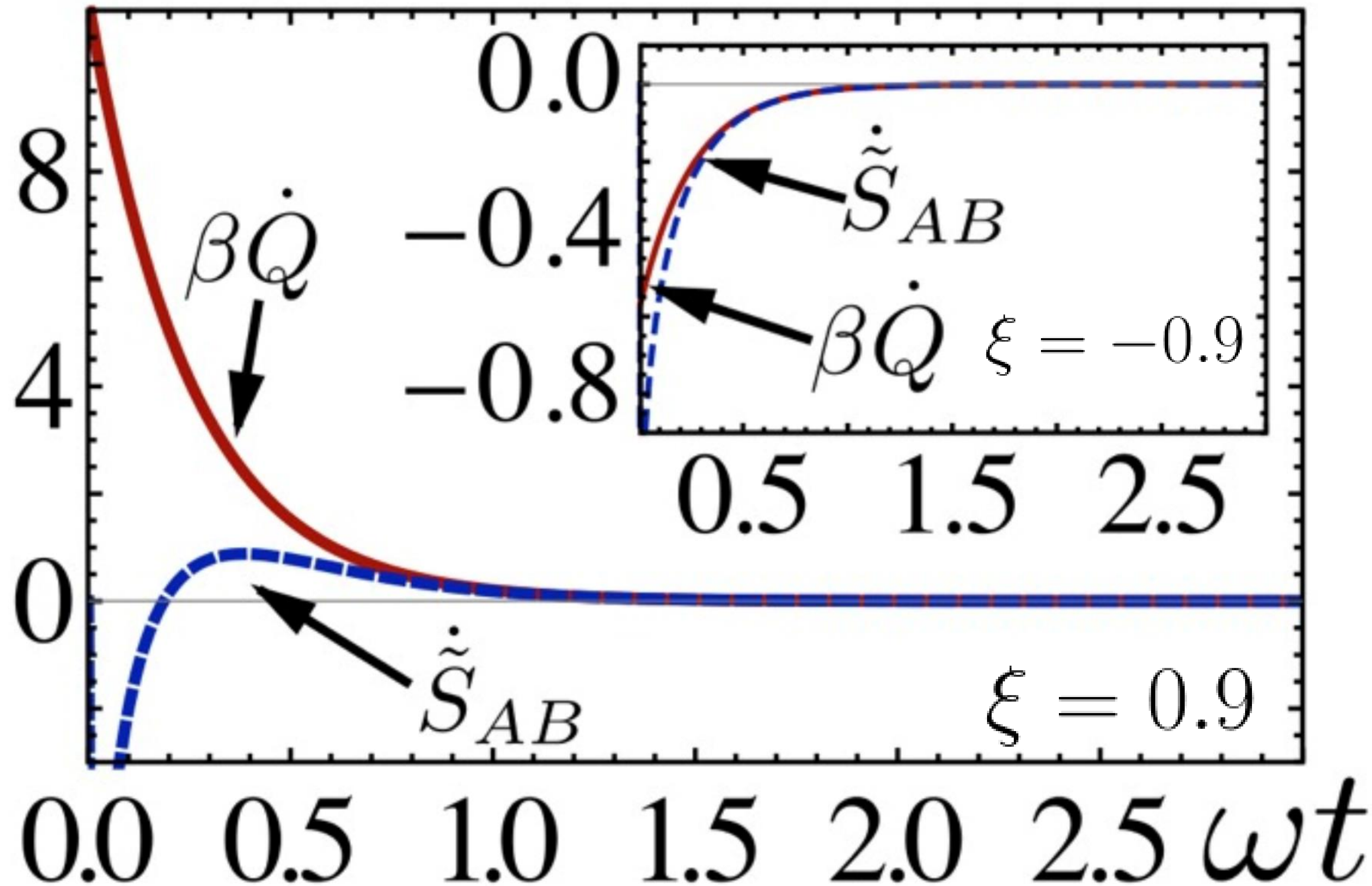
$$\mathcal{L}_X[\rho] = \Gamma^+ L[\sigma^-_{S_x}](\rho) + \Gamma^- L[\sigma^+_{S_x}](\rho)] \qquad \Gamma^\pm = 2\gamma(1\pm\xi)$$

$$\mathcal{D}_{AB}[\rho] = \Gamma^+ \left(\sigma^-_{S_A}[\rho, \sigma^+_{S_B}] - [\rho, \sigma^-_{S_B}]\sigma^+_{S_A}\right) + \Gamma^- \left(\sigma^+_{S_A}[\rho, \sigma^-_{S_B}] - [\rho, \sigma^+_{S_B}]\sigma^-_{S_A}\right)$$

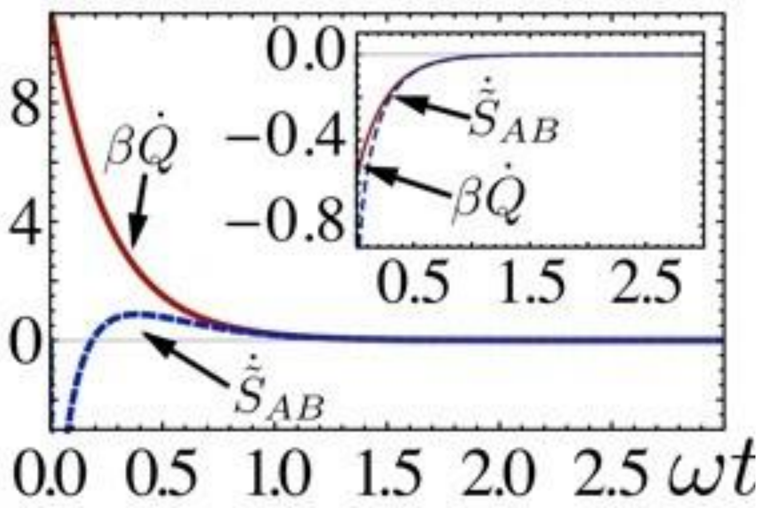
$$\dot{\rho}_{S_A} = \Gamma^+ L[\sigma^-_{S_A}]\rho_{S_A} + \Gamma^- L[\sigma^+_{S_A}]\rho_{S_A}$$

$$\dot{\rho}_{S_B} = \Gamma^+ L[\sigma^-_{S_B}]\rho_{S_B} + \Gamma^- L[\sigma^+_{S_B}]\rho_{S_B}$$
$$- i\frac{\Gamma^+ - \Gamma^-}{2}\langle[\sigma^x_{S_B}, \sigma^y_{S_A}\rho] - [\sigma^y_{S_B}, \sigma^x_{S_B}\rho]\rangle_{S_A}$$

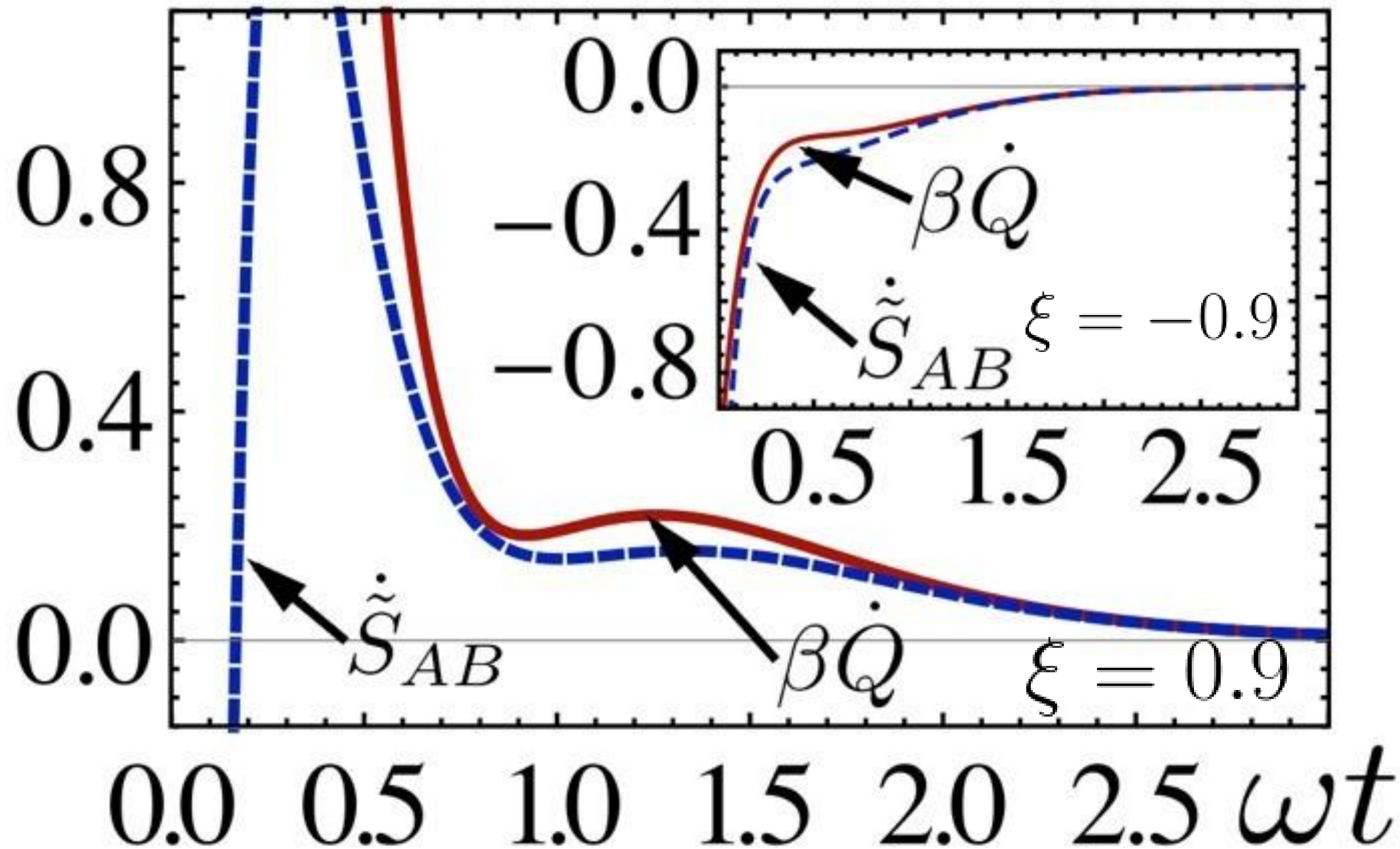
Single Qubit:



$$\dot{\rho}_{S_A} = \Gamma^+ L[\sigma^-_{S_A}]\rho_{S_A} + \Gamma^- L[\sigma^+_{S_A}]\rho_{S_A}$$

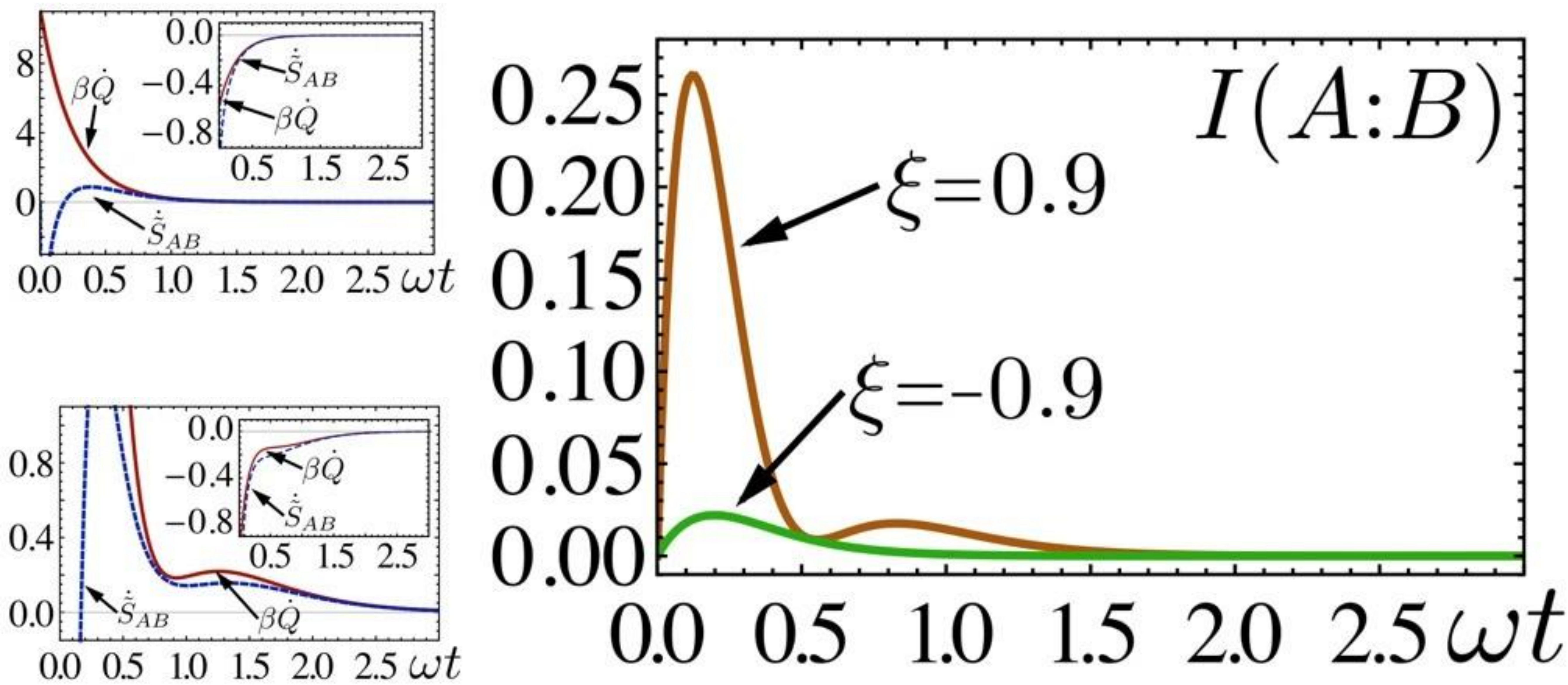


Two Qubits:



$$\dot{\rho}_{S_A} = \Gamma^{+} L[\sigma^{-}_{S_A}]\rho_{S_A} + \Gamma^{-} L[\sigma^{+}_{S_A}]\rho_{S_A}$$

$$\dot{\rho}_{S_B} = \Gamma^{+} L[\sigma^{-}_{S_B}]\rho_{S_B} + \Gamma^{-} L[\sigma^{+}_{S_B}]\rho_{S_B} \\ - i\frac{\Gamma^{+} - \Gamma^{-}}{2}\langle[\sigma^{x}_{S_B}, \sigma^{y}_{S_A}\rho] - [\sigma^{y}_{S_B}, \sigma^{x}_{S_B}\rho]\rangle_{S_A}$$



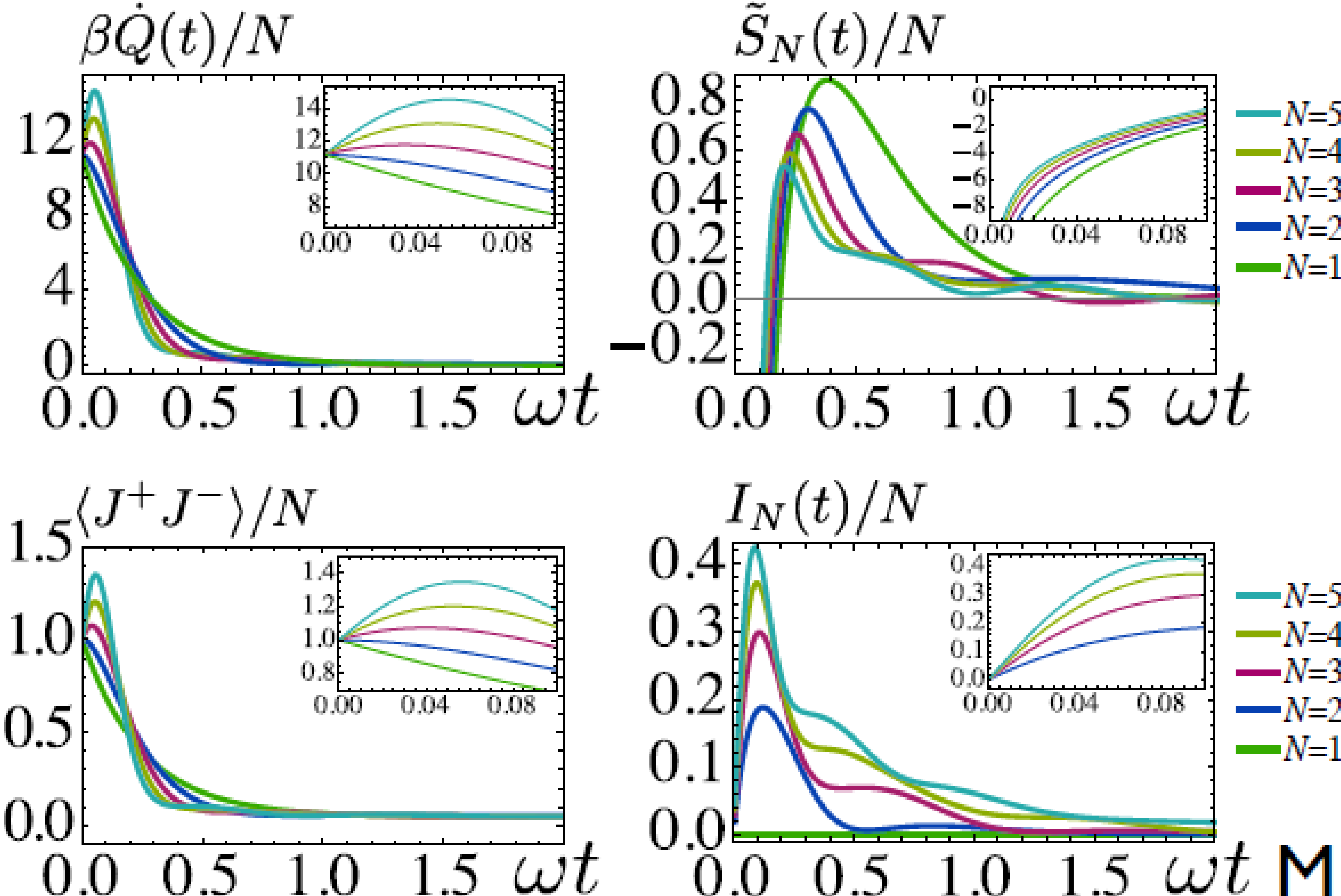
$$\dot{\rho}_{S_A} = \Gamma^{+} L[\sigma_{S_A}^{-}]\rho_{S_A} + \Gamma^{-} L[\sigma_{S_A}^{+}]\rho_{S_A}$$

$$\dot{\rho}_{S_B} = \Gamma^{+} L[\sigma_{S_B}^{-}]\rho_{S_B} + \Gamma^{-} L[\sigma_{S_B}^{+}]\rho_{S_B} - i\frac{\Gamma^{+} - \Gamma^{-}}{2}\langle[\sigma_{S_B}^{x}, \sigma_{S_A}^{y}\rho] - [\sigma_{S_B}^{y}, \sigma_{S_B}^{x}\rho]\rangle_{S_A}$$

Dependence on N

Heat flux

Entropy variation

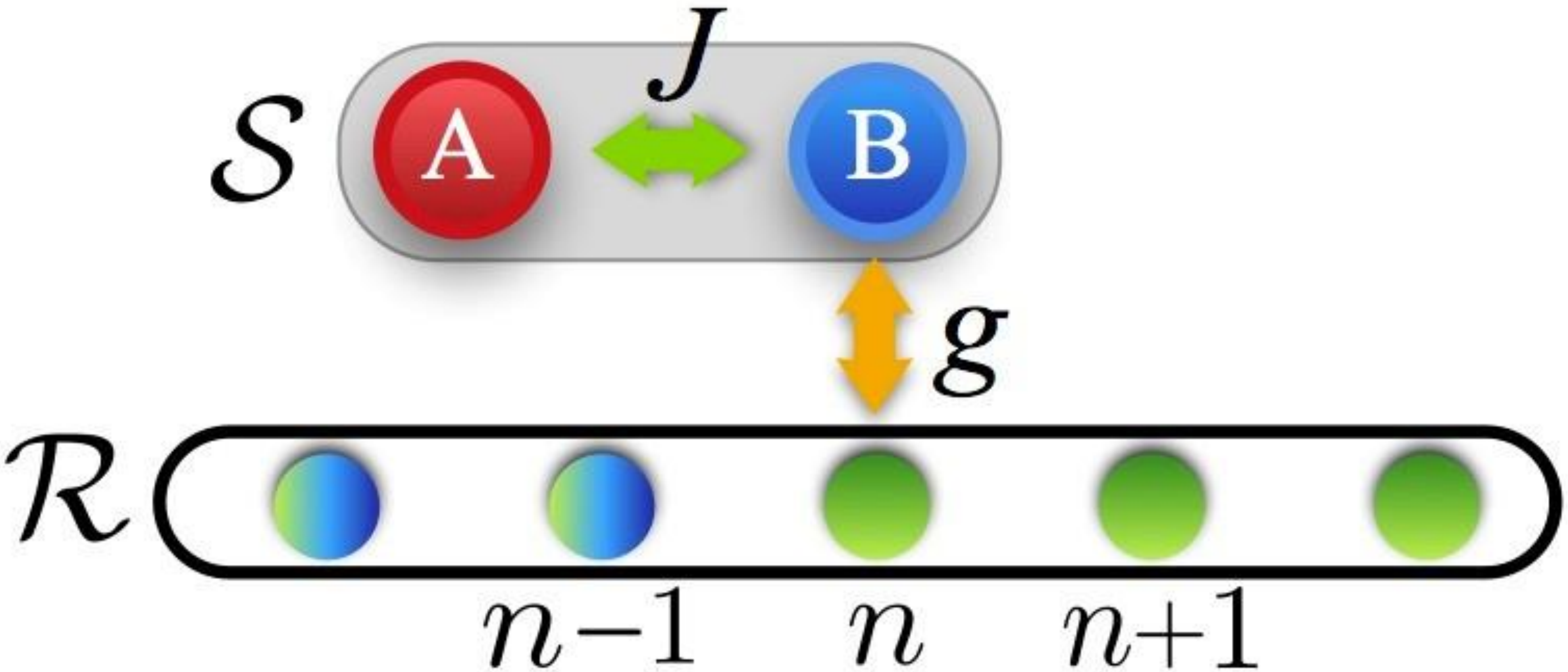


Radiation rate

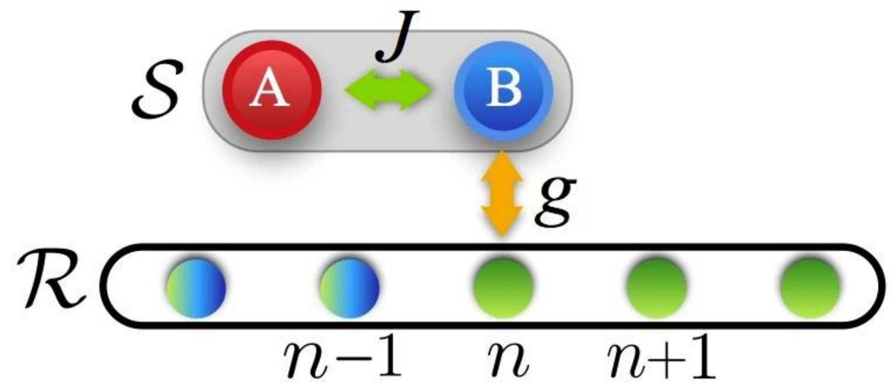
Mutual information

$\xi = 0.9$ $\gamma/\omega = 1$

Indirect Erasure Model

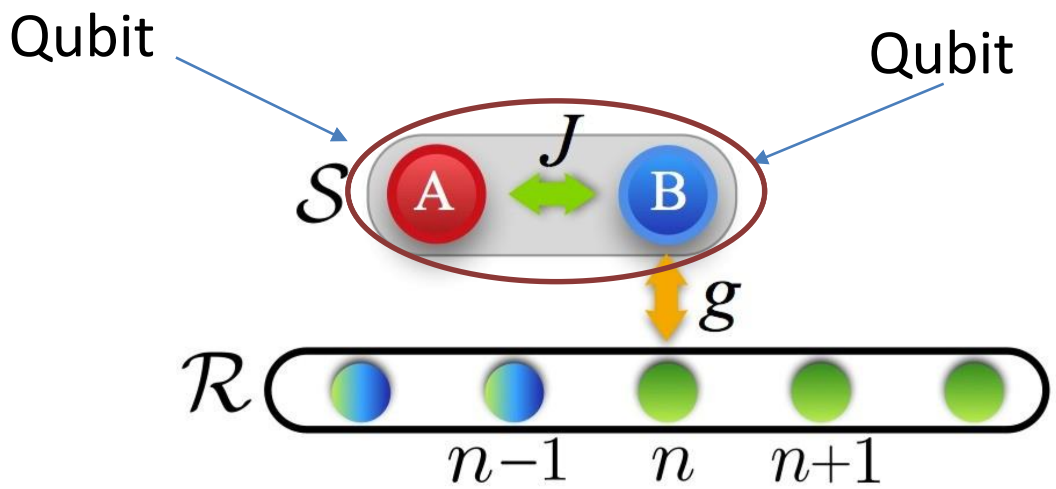


Indirect Erasure Model



$$\dot{\rho} = -[J\hat{H}_{AB}, \rho] + \gamma \sum_{kj} \langle \hat{R}_k \hat{R}_j \rangle_\eta \left(\hat{S}_{Bj} \rho \hat{S}_{Bk} - \frac{1}{2} \left\{ \hat{S}_{Bk} \hat{S}_{Bj}, \rho \right\} \right)$$

$$\hat{S}^X = \left\{\sigma_x^X, \sigma_y^X\right\} \qquad \hat{U} = e^{-iJ\hat{V}t} \qquad \hat{V} = \sum_{XY} \hat{S}^X \otimes \hat{S}^Y$$



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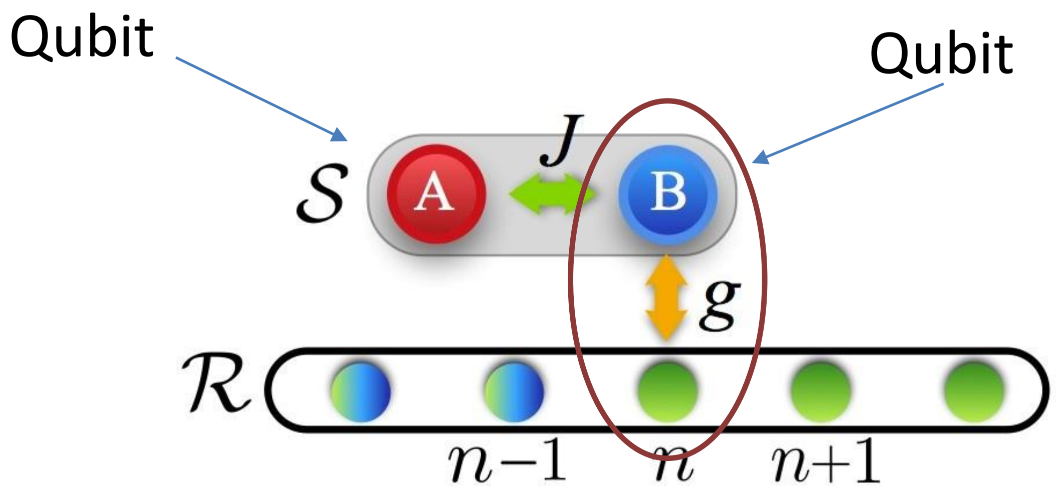
$$\hat{S}^B = \left\{\sigma_x^B, \sigma_y^B\right\} \qquad \hat{R} = \left\{\sigma_x^R, \sigma_y^R\right\}$$

$$\xi = \tanh(\beta\omega/2)$$

$$\eta = \begin{pmatrix} \frac{1-\xi}{2} & 0 \\ 0 & \frac{1+\xi}{2} \end{pmatrix}$$



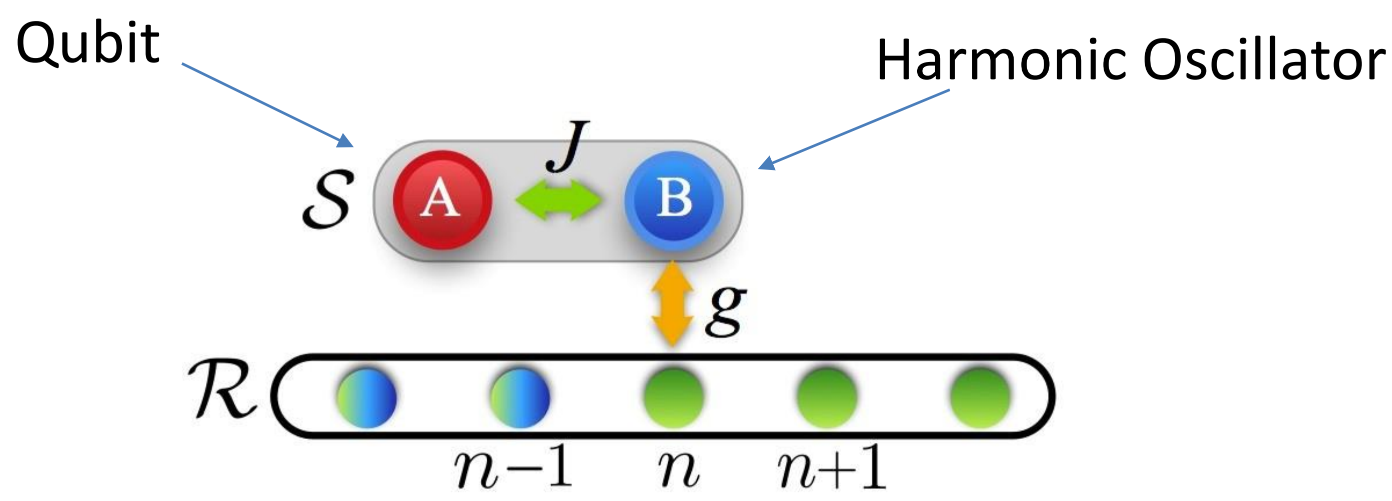
$$\hat{U} = e^{-ig\hat{V}\tau}$$

$$\hat{V} = \sum_k \hat{S}_k \otimes \hat{R}_k$$

$$\dot{\rho} = -[J\hat{H}_{AB}, \rho] + \gamma \sum_{kj} \langle \hat{R}_k \hat{R}_j \rangle_\eta \left(\hat{S}_{Bj} \rho \hat{S}_{Bk} - \frac{1}{2} \left\{ \hat{S}_{Bk} \hat{S}_{Bj}, \rho \right\} \right)$$

$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma(1-\xi) L[\sigma^+](\rho) + \gamma(1+\xi) L[\hat{\sigma}^-](\rho))$$

Indirect Erasure Model



$$\hat{q} = (\hat{b} + \hat{b}^\dagger)/\sqrt{2}$$

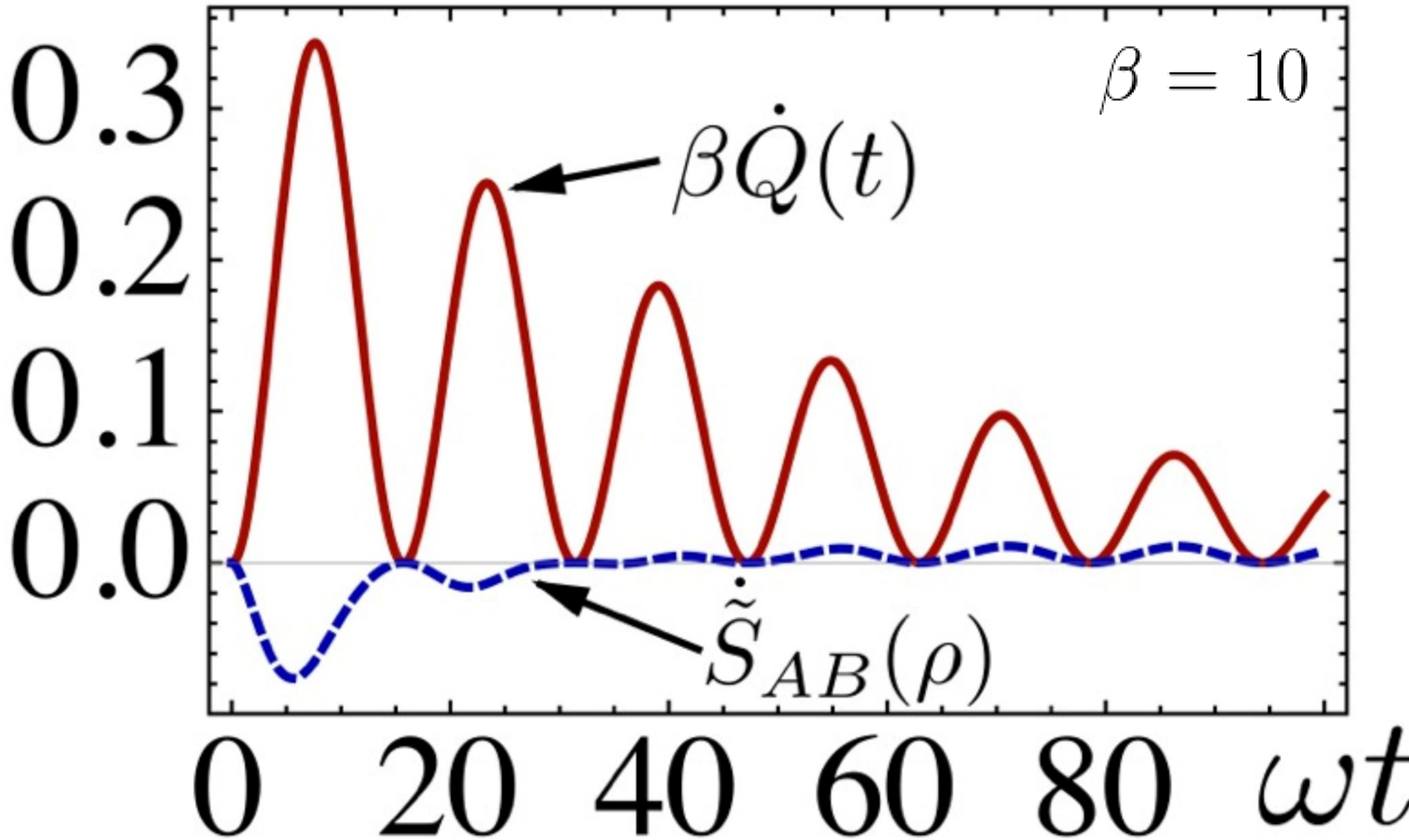
$$\hat{p} = i(\hat{b}^\dagger - \hat{b})/\sqrt{2}$$

$$\hat{H}_{AB} = \omega(\hat{q}^2 + \hat{p}^2 + \hat{\sigma}^z_{S_A}/2) + J(\hat{q}\hat{\sigma}^x_{S_A} + \hat{p}\hat{\sigma}^y_{S_A})$$

Jaynes-Cummings

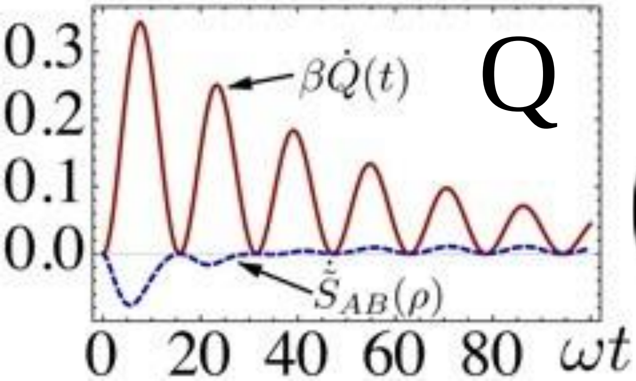
$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma_g(1-\xi)L[\hat{b}^\dagger](\rho) + \gamma_g(1+\xi)L[\hat{b}](\rho)$$

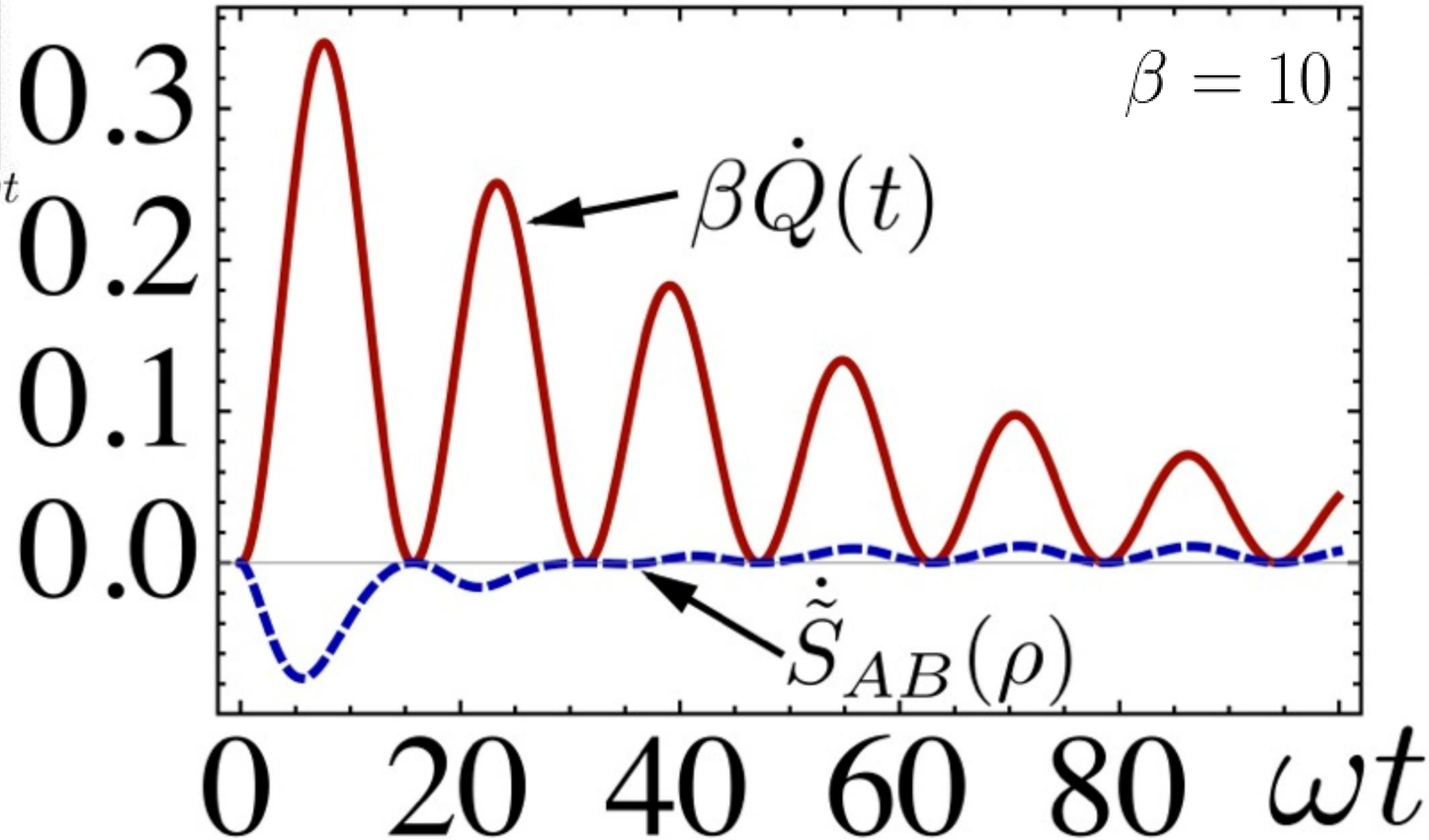
Qubit-Qubit



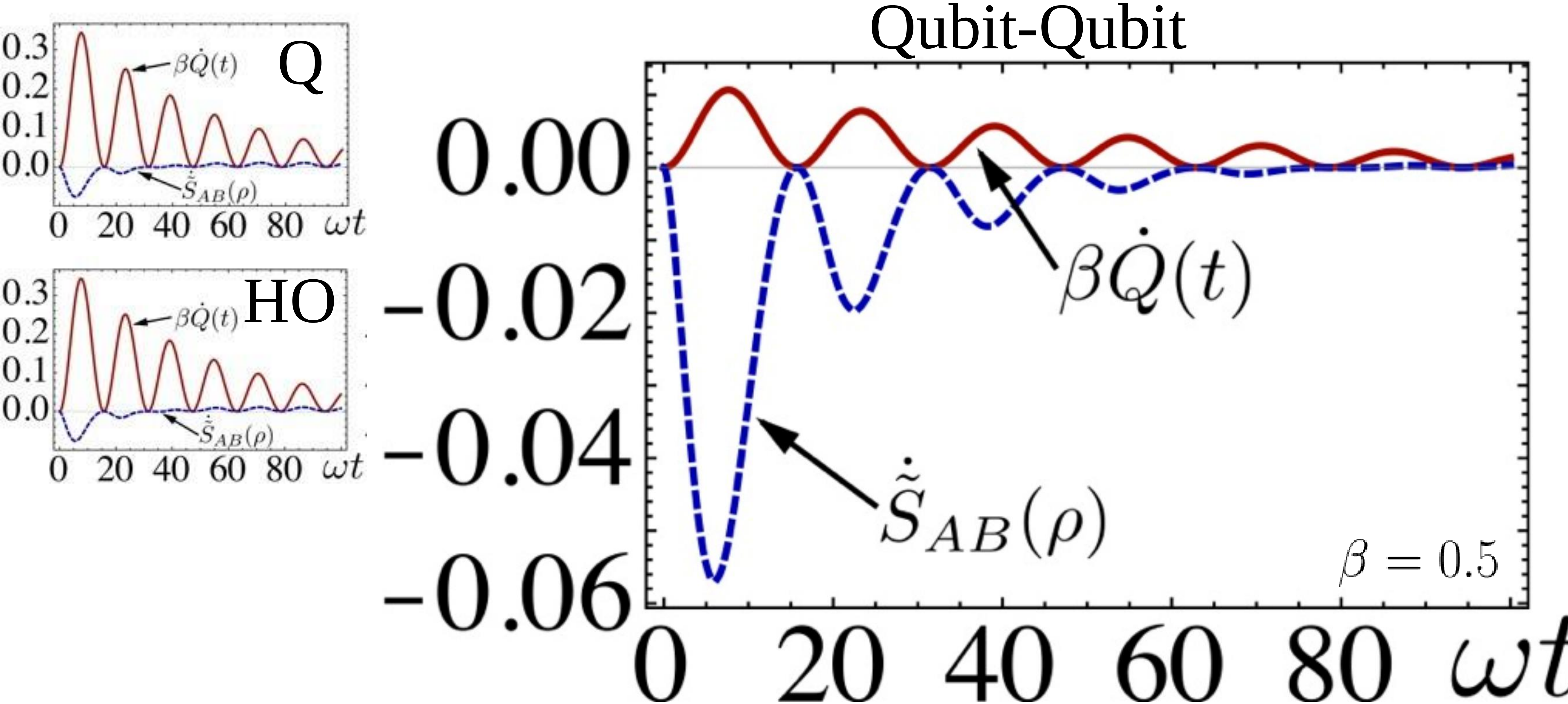
$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma(1-\xi)L[\sigma^{+}](\rho) + \gamma(1+\xi)L[\hat{\sigma}^{-}](\rho))$$

Qubit-Harmonic Oscillator



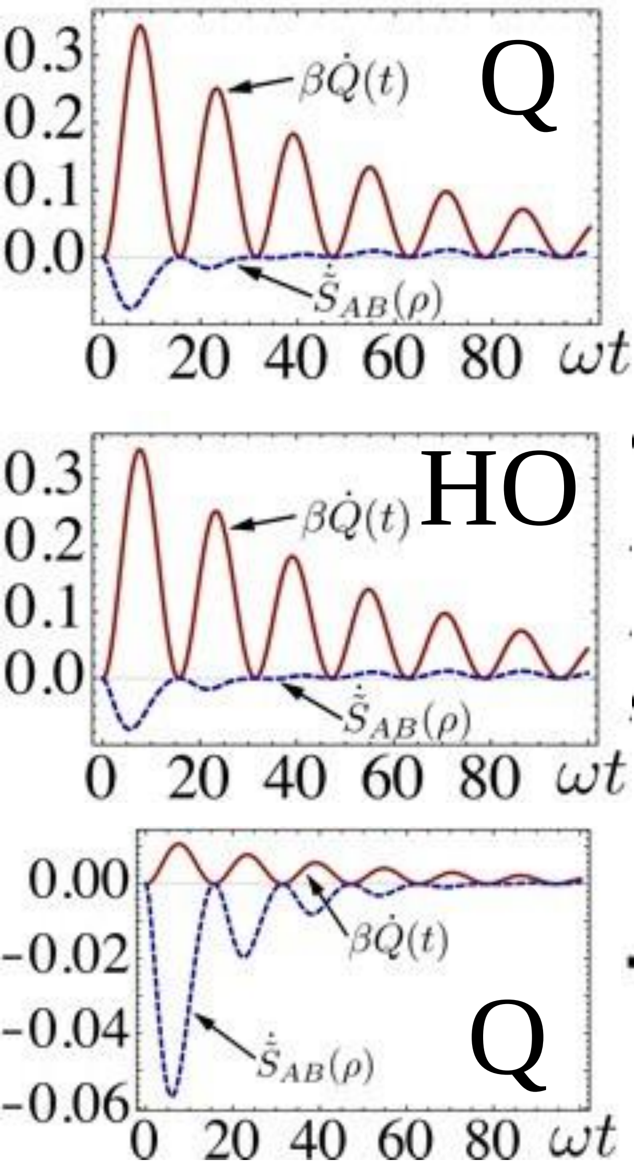


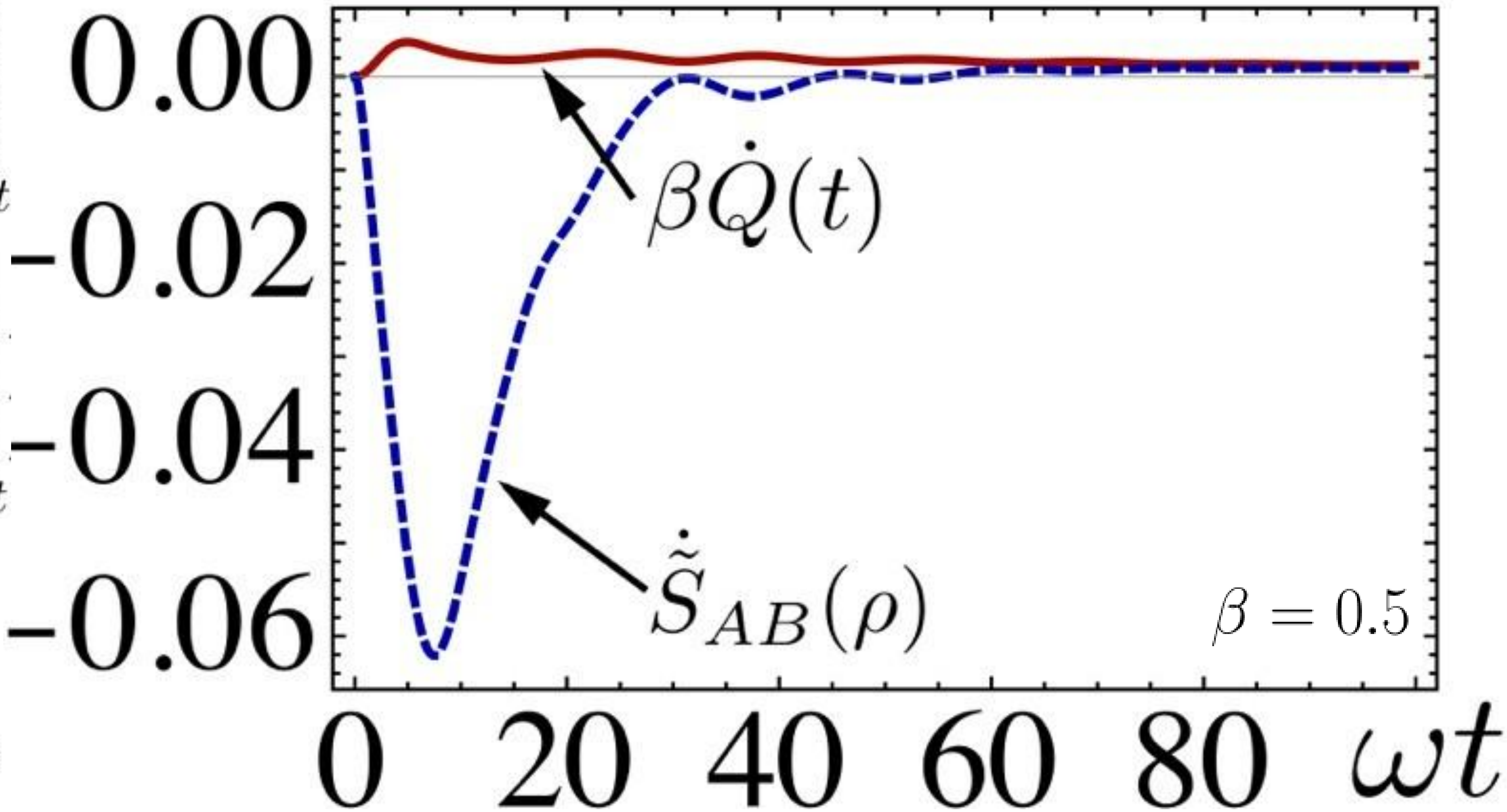
$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma_g(1-\xi)L[\hat{b}^\dagger](\rho) + \gamma_g(1+\xi)L[\hat{b}](\rho)$$



$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma(1-\xi)L[\sigma^{+}](\rho) + \gamma(1+\xi)L[\hat{\sigma}^{-}](\rho))$$

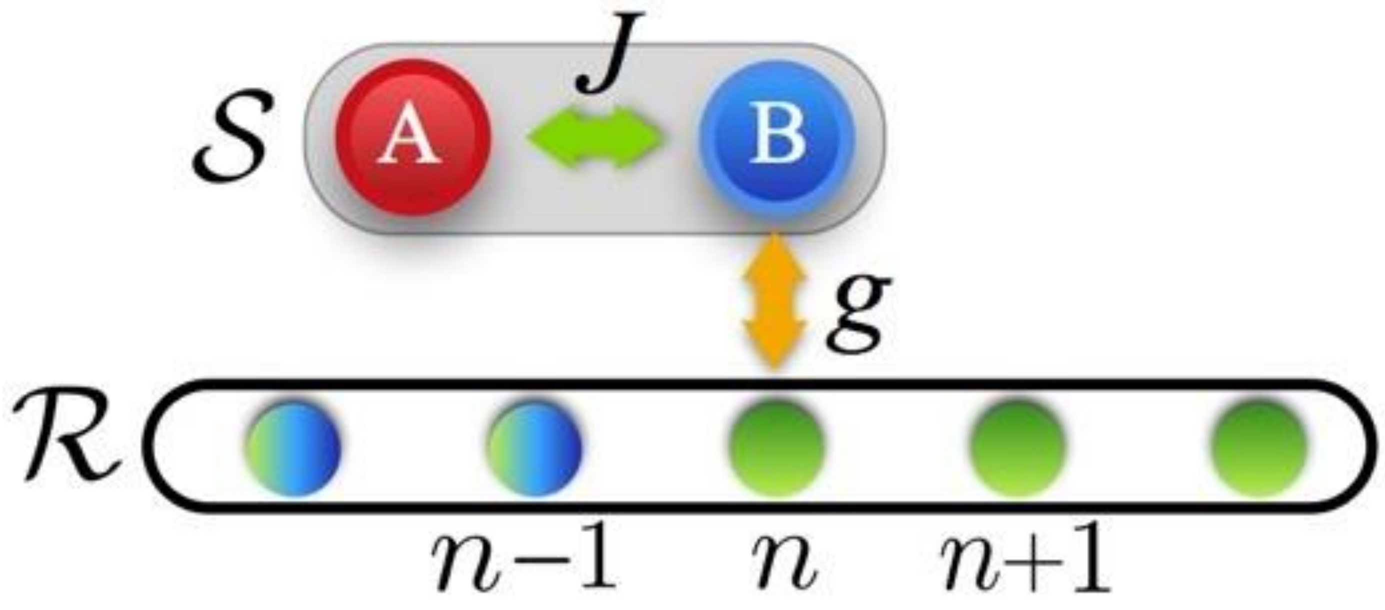
Qubit-Harmonic Oscillator



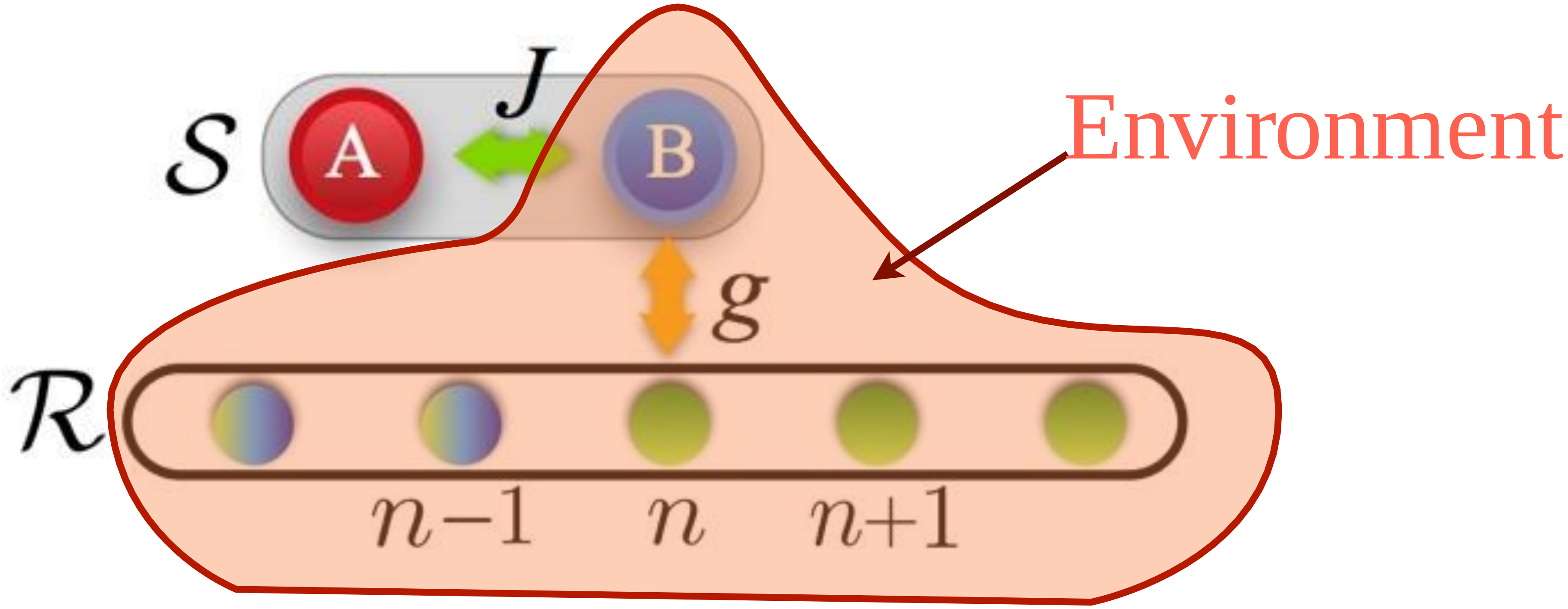


$$\dot{\rho} = -i[\hat{H}_{AB}, \rho] + \gamma_g(1-\xi)L[\hat{b}^{\dagger}](\rho) + \gamma_g(1+\xi)L[\hat{b}](\rho)$$

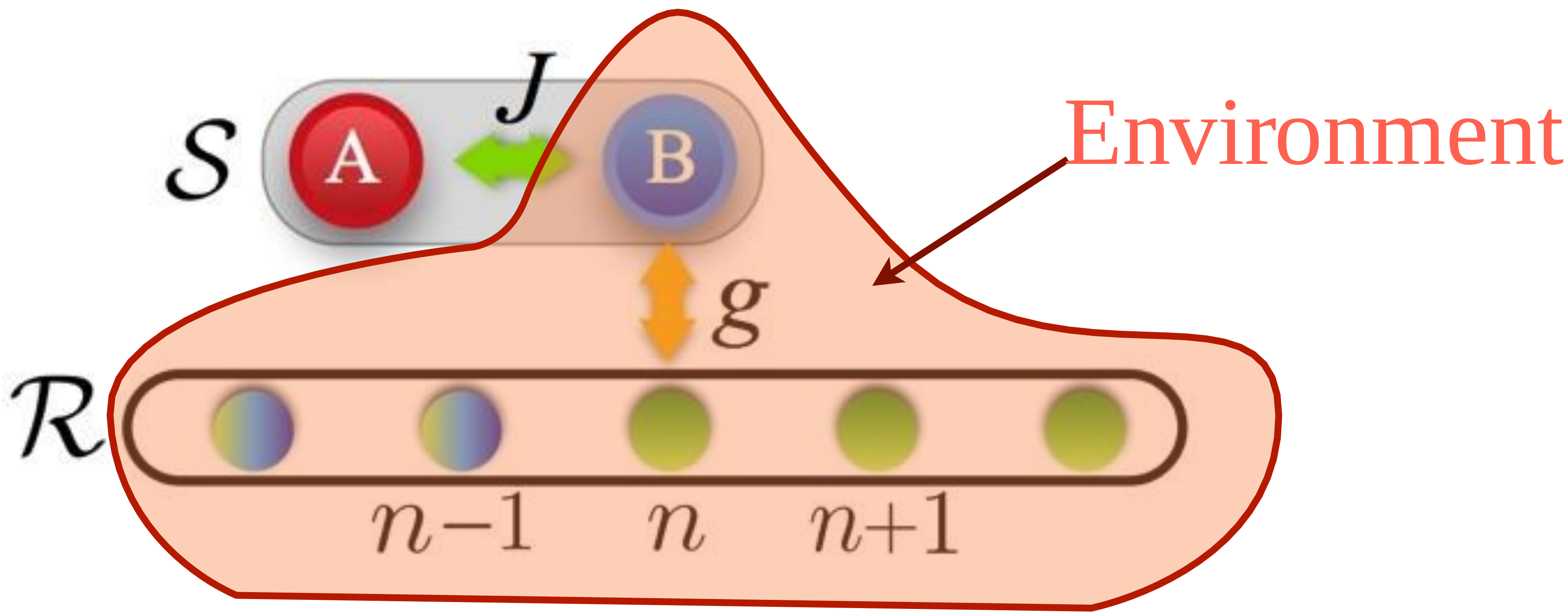
Non-Markovianity in Q-HO model

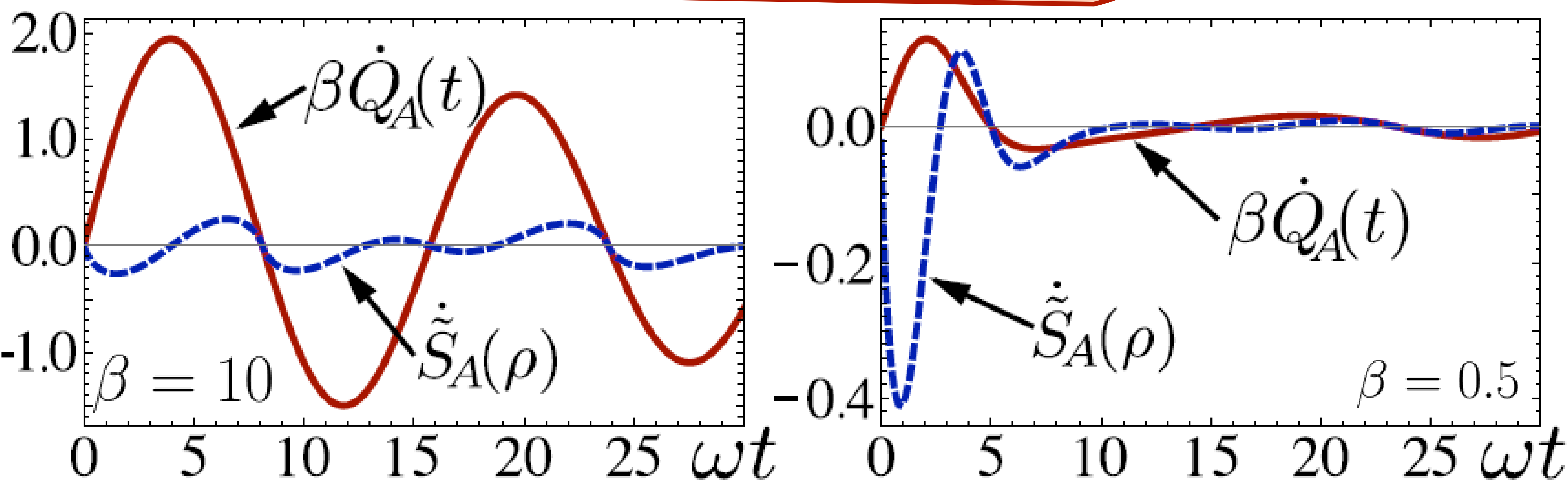


Non-Markovianity in Q-HO model



Non-Markovianity in Q-HO model





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