The role of quantum correlations in measurement-based feedback cooling

P. Liuzzo-Scorpo[1], L. A. Correa[2], R. Schmidt[1], D. Girolami[3] and G. Adesso[1]

[1]School of Mathematical Sciences, The University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom

[2]Department de Fìsica, Universitàt Autonòma de Barcelona E08193 Bellaterra, Spain

[3]Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU,United Kingdom

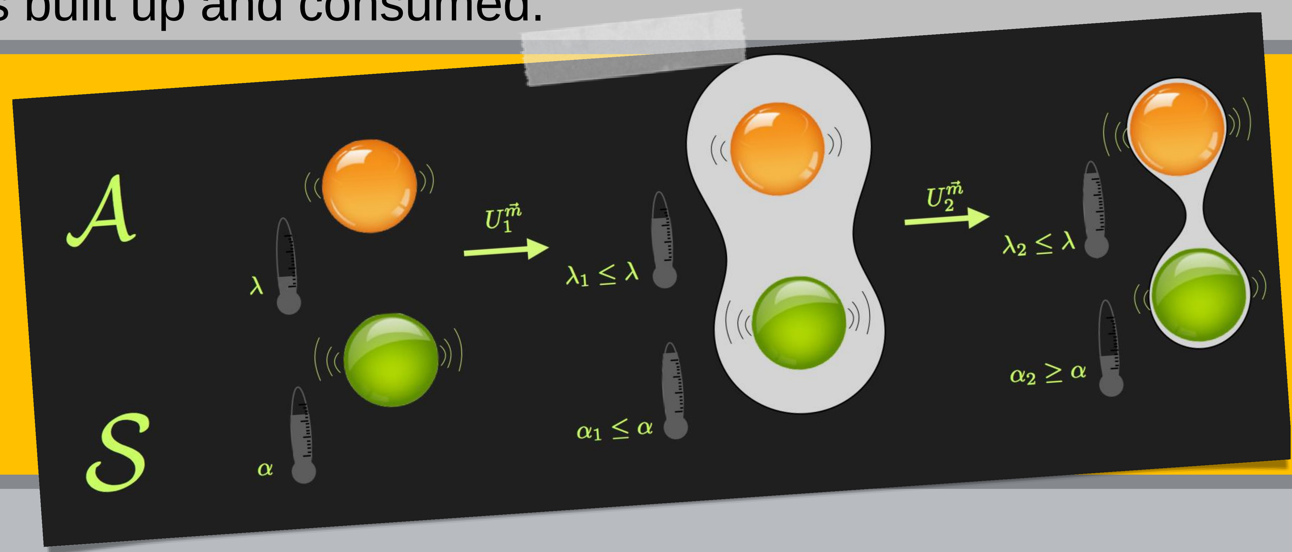
Abstract

The study of a measurement-based feedback protocol applied to a, initially uncorrelated, system consisting of two qubits (identified as principal system and auxiliary respectively) [1] led us to investigate the relation between correlations and efficiency of the feedback protocol. In particular we studied the nature of correlations at each step of the protocol, i.e. the amount of classical and quantum correlations built up and consumed.

The protocol

$$r_0 \xrightarrow[U_1^{\vec{m}}]{} r_1 = U_1^{\vec{m}} r_0 U_1^{\vec{m}\dagger} \xrightarrow[U_2^{\vec{m}}]{} r_2 = U_2^{\vec{m}} r_1 U_2^{\vec{m}\dagger}$$

First Step Second Step



Initial State

We consider a composite system, r_0 of two qubits: the principal system, ρ_0, which we aim to cool, and the auxiliary χ_0. We consider an initial product of thermal states

$$r_0 = \rho_0 \otimes \chi_0 \qquad \text{where} \qquad \rho_0 = \frac{1-\alpha\sigma_z}{2}\,; \qquad \chi_0 = \frac{1-\lambda\sigma_z}{2},$$

with α and λ mixing parameters of the system and the auxiliary, respectively such that $0 \le \alpha \le \lambda \le 1$.

First Step: Building up correlations

In order to build up the correlations between the two qubits we choose the first unitary to be

$$U_1^{\vec{m}} = e^{-i\frac{\theta}{2}\sigma_{\vec{m}} \otimes \sigma_y},$$

Where $\sigma_{\vec{m}} = \vec{m}\cdot\vec{\sigma}$ and $\vec{m} = (m_x, m_y, m_z)$ s.t. $|\vec{m}| = 1$.

Second Step: Feedback

To the composite system is now applied the unitary

$$U_2^{\vec{m}} = U_+^{\vec{m}} \otimes |+\rangle\langle+| + U_-^{\vec{m}} \otimes |-\rangle\langle-|$$

where, {|+⟩, |−⟩} are the auxiliary's σ_x eigenstates, and

$$U_\pm^{\vec{m}} = e^{\mp i\phi(\vec{m})\frac{-m_y\sigma_x + m_x\sigma_y}{2\sqrt{1-m_z^2}}}.$$

Efficiency

A measure of efficiency was proposed in [1]

$$\varepsilon = -\frac{kT\Delta H_{\mathcal{S}}}{Q_{reset}},$$

where $\Delta H_{\mathcal{S}}$ is the entropy reduction of the principal system, Q_{reset} is the heat dumped into a thermal reservoir by the auxiliary at the end of the feedback process (reset). In [1] it is assumed that this reset operation is "optimally" performed, by this meaning that it is saturated the second law-like inequality

$$k\Delta H - Q/T \ge 0\,, \qquad \text{leading to}$$

$$\varepsilon_{HJ} = -\frac{\Delta H_{\mathcal{S}}}{\Delta H_{\mathcal{A}}} = \frac{|\Delta H_{\mathcal{S}}|}{|\Delta H_{\mathcal{S}}| + \mathcal{I}(r_2)}.$$

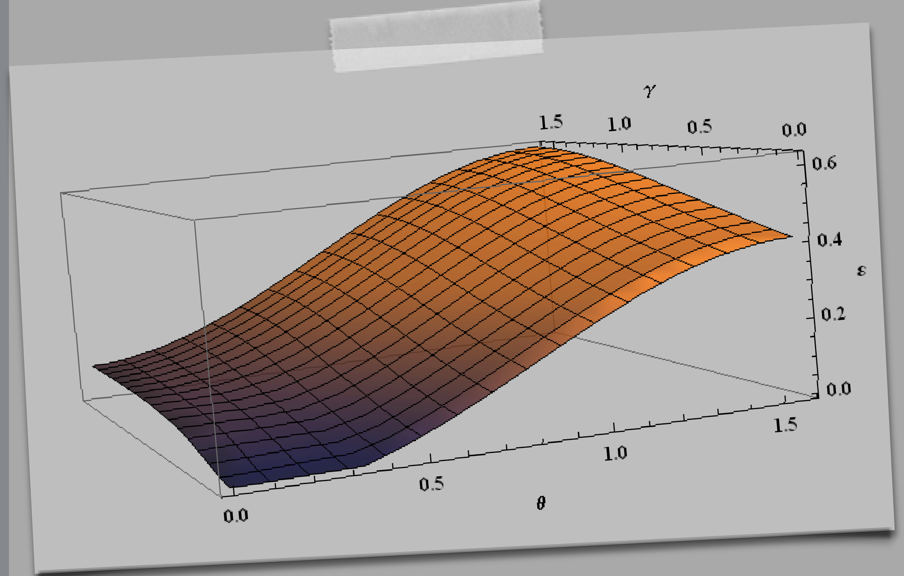
This efficiency is upper bounded by 1 (it is equal to 1 if there are no residual correlations at the end of the feedback step) which justifies its name. However, since we can think at the auxiliary's reset as a Landauer's process, as was proved in [2], the (Landauer's) bound holds with equality if and only if $\Delta H = \Delta Q = 0$, i.e. any reset is performed.

For this reason, we have adopted the same definition of efficiency but we have explicitly computed the heat transferred by the auxiliary to the reservoir during the reset operation, i.e

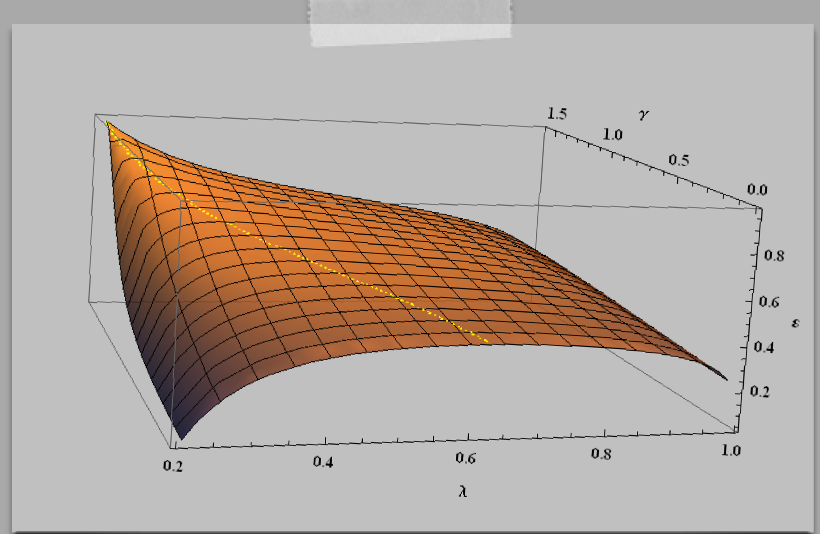
$$Q_{reset} = \mathrm{Tr}\left\{\hat{H}(r_2 - \rho_2 \otimes \chi_0)\right\} > k\Delta H_{\mathcal{A}}$$

$$\text{where} \qquad \hat{H} = \frac{\omega_{\mathcal{S}}}{2}\sigma_z^{\mathcal{S}} \otimes \mathbb{I}_{\mathcal{A}} + \frac{\omega_{\mathcal{A}}}{2}\mathbb{I}^{\mathcal{S}} \otimes \sigma_z^{\mathcal{A}}$$

$$\omega_{\mathcal{S}} = T\log\frac{1+\alpha}{1-\alpha} \qquad \omega_{\mathcal{A}} = T\log\frac{1+\lambda}{1-\lambda}$$



Efficiency, for $\alpha = 0.2$ and $\lambda = 0.6$, as a function of the parameters θ and γ (which parametrizes $\vec{m} = (\sin\gamma, 0, \cos\gamma)$). It is worth noticing that the efficiency increases monotonically varying $\vec{m}$ from $\hat{z}$ to $\hat{x}$ for every value of θ.

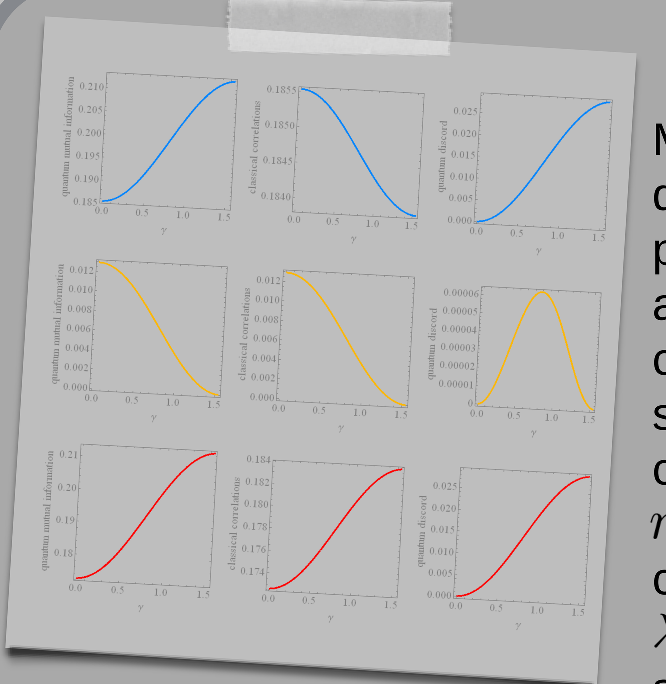


Efficiency, for $\alpha = 0.2$ and $\theta = \pi/2$, as a function of the parameters λ and γ (which parametrizes $\vec{m} = (\sin\gamma, 0, \cos\gamma)$). The yellow dotted line has equation

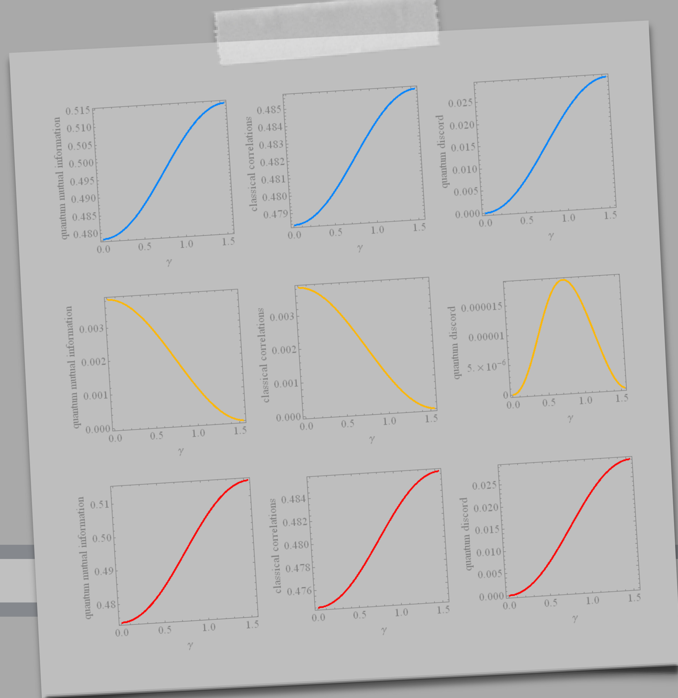
$$\partial\varepsilon/\partial\lambda = 0$$

As stated above we see that the efficiency reaches the value ~ 1 when $\lambda \sim \alpha$ i.e. there is no cooling of the principal system.

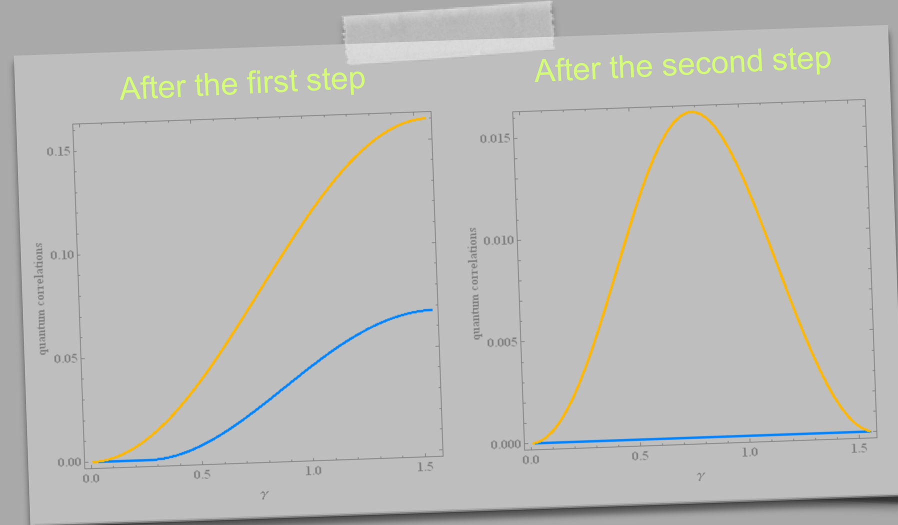
Correlations: θ = π/2



Mutual information, classical correlations and quantum discord for $\alpha = 0.2$, $\lambda = 0.6$ (left panel), $\lambda = 0.9$ (right panel), and $\theta = \pi/2$, after the first step (blue curves), after the second step (yellow curves) and their consumption (red curves). We notice that after the first step the amount of quantum correlations does not change with λ and it is monotonically growing varying $\vec{m}$ from $\hat{z}$ to $\hat{x}$. On the other hand, the behaviour of the classical correlations is different in the two cases: for $\lambda = 0.6$ the classical correlations built up during the first step decreases with γ reaching their minimum value when $\vec{m} = \hat{x}$ while, for $\lambda = 0.9$, we have that the minimum amount of classical correlations built up is when $\vec{m} = \hat{z}$. We also observe that the protocol, for every value of λ, is better in both acquiring purely quantum information (evaluated as quantum discord) and making use of it. The residual total correlations after the feedback (responsible of a lower efficiency [1]) decrease monotonically varying $\vec{m}$ from $\hat{z}$ to $\hat{x}$.



Nature of Correlations



We were also interested in quantifying entanglement, in particular we wanted to use an entanglement measure which we could compare with a measure of total quantum correlations (yellow lines).To this aim we studied the 2-tangle τ_2 (blue line), i.e. the square of the concurrence C defined by [3]. This entanglement measure can be compared with the interferometric power $\mathcal{P}^A(r)$, a computable measure of discord-type quantum correlations [4].

These measures of entanglement and total quantum correlations are such that, $\mathcal{P}^A(r) \ge \tau_2(r)$ where equality holds for pure states.

[Parameters: $\theta = \pi/2$, $\alpha = 0.4$, $\lambda = 0.8$.]

References

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http://quantumcorrelations.weebly.com/

pmxpl2@nottingham.ac.uk