Correlations and work

Martí Perarnau Llobet

Institut de Ciències Fotòniques

2nd COST Quantum Thermodynamics Conference
Palma de Mallorca, April 2015.

Marcus Huber
(UAB & ICFO)

Karen V. Hovhannisyan
(ICFO)

Paul Skrzypczyk
(ICFO)

Antonio Acín
(ICFO)

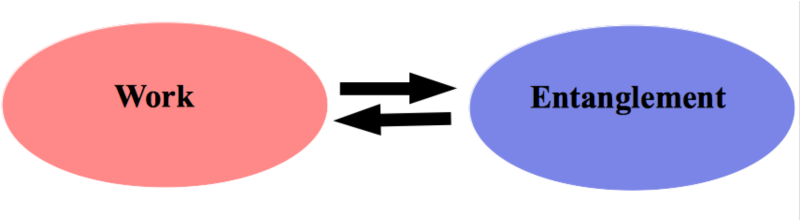
Claude Klöckl
(UAB)

Nicolas Brunner
(University of Geneva)

David Edward Bruschi
(University of Jerusalem)

Nicolai Friis
(Innsbruck)

Interconversion between two resources: entanglement (quantum information theory) and work (thermodynamics).



I. Correlations from work.

M. Huber, M. P.L., K. Hovhannisyan, P. Skrzypczyk, C. Klöckl, N. Brunner, and A. Acín, arXiv:1404.2169.

D. Bruschi, M. P.-L., N. Friis, K. Hovhannisyan, M. Huber, arXiv:1409.4647.

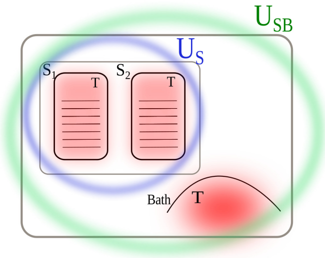
Scenario

- The initial state is in thermal equilibrium and uncorrelated.
- We can perform any operation to correlate the state, but
  - the initial temperature is fixed, and
  - every operation has an energy cost.

Scenario

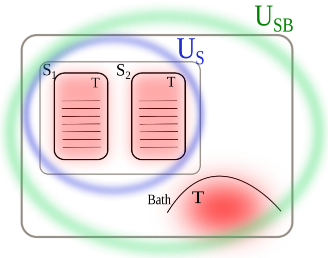
- The initial state is in thermal equilibrium and uncorrelated.
- We can perform any operation to correlate the state, but
  - the initial temperature is fixed, and
  - every operation has an energy cost.
- How does the initial temperature limits the amount of achievable correlations?
- What is the minimal cost of creating (quantum) correlations? How complex is the optimal process?

Scenario



- Hamiltonian: $H = H_{S_1} + H_{S_2} + H_B$
- Initial state: $\tau = exp\{-\beta H\}/\mathcal{Z} = \tau_\beta^{(S_1)} \otimes \tau_\beta^{(S_2)} \otimes \tau_\beta^{(B)}.$
- Evolution: $U\tau U^\dagger$, with $[U, H] \neq 0$

Scenario



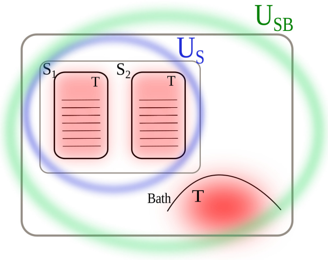
Every U_{SB} defines a cyclic process:

State	Hamiltonian	Time
τ	H	$t = 0$
$U(t)\tau U^\dagger(t)$	$H + V(t)$	$0 < t < \tau$
$U\tau U^\dagger$	H	$t = \tau$

Average work cost,

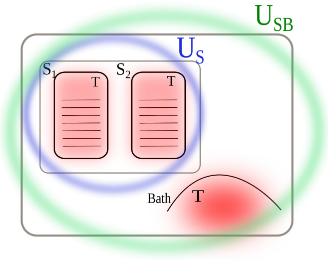
$$W = \mathrm{Tr}(HU\tau U^\dagger) - \mathrm{Tr}(H\tau) \geq 0.$$

Scenario



- Hamiltonian: $H = H_{S_1} + H_{S_2} + H_B$
- Initial state: $\tau = \frac{exp\{-\beta H\}}{\mathcal{Z}}$
- Evolution: $U\tau U^{\dagger}$, with $[U_{SB}, H] \neq 0$
- Work cost: $W = \mathrm{Tr}(HU\tau U^{\dagger}) - \mathrm{Tr}(H\tau)$

Scenario



- Hamiltonian: $H = H_{S_1} + H_{S_2} + H_B$
- Initial state: $\tau = \frac{exp\{-\beta H\}}{\mathcal{Z}}$
- Evolution: $U \tau U^{\dagger}$, with $[U_{SB}, H] \neq 0$
- Work cost: $W = \mathrm{Tr}(HU\tau U^{\dagger}) - \mathrm{Tr}(H\tau)$

Questions

- How does the initial temperature limit the achievable correlations?
- What is the minimal energy cost of correlating thermal states?

Limitations arising from the temperature

We consider,

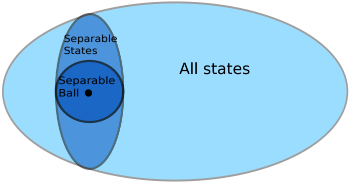
- closed systems: $U = U_S$.
- the system is a collection of N qubits: $\tau_\beta^{(S)} = \tau_\beta \otimes ... \otimes \tau_\beta$.

Limitations arising from the temperature

We consider,

- closed systems: $U = U_S$.
- the system is a collection of N qubits: $\tau_\beta^{(S)} = \tau_\beta \otimes \ldots \otimes \tau_\beta$.

If $\rho = U \tau_\beta \otimes \ldots \otimes \tau_\beta U^\dagger$, where τ is a thermal state, what is the maximal temperature that allows for the generation of entanglement?



Entanglement

Consider a $N-$partite system,

- Entanglement,

$$\rho \text{ is } \text{ entangled} \Longleftrightarrow \rho \neq \sum_i p_i \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_N$$

- Genuine multipartite entanglement (GME),

$$\rho \text{ is } \text{ GME} \Longleftrightarrow \rho \neq \sum_i p_i \rho_{1\ldots k} \otimes \rho_{k+1,\ldots,N}, \qquad k < N$$

Limitations arising from the temperature

- Maximal temperature for two qubits,[1] $K_{\rm B}\, T_{\rm max} \approx 1.19\epsilon$.

[1] S. Ishizaka and T. Hiroshima, Phys. Rev. A **62**, 22310 (2000); F. Verstraete et al, Phys. Rev. A 64, 012316 (2001)

Limitations arising from the temperature

- Maximal temperature for two qubits,[1] $K_{\mathrm{B}} T_{\max} \approx 1.19\epsilon$.
- This temperature can be increased by considering more copies. An intuitive explanation comes from algorithmic cooling:

$$\tau_\beta^{\otimes N} \xrightarrow{U} |0\rangle\langle 0|^{\otimes l} \gamma^{\otimes N-l}$$

[1] S. Ishizaka and T. Hiroshima, Phys. Rev. A **62**, 22310 (2000); F. Verstraete et al, Phys. Rev. A 64, 012316 (2001)

Limitations arising from the temperature

- Maximal temperature for two qubits,[1] $K_{\mathrm{B}} T_{\max} \approx 1.19\epsilon$.
- This temperature can be increased by considering more copies. An intuitive explanation comes from algorithmic cooling:

$$\tau_\beta^{\otimes N} \xrightarrow{U} |0\rangle\langle 0|^{\otimes I} \gamma^{\otimes N-I}$$

- How does $T_{\max}$ depend on N?

[1] S. Ishizaka and T. Hiroshima, Phys. Rev. A **62**, 22310 (2000); F. Verstraete et al, Phys. Rev. A 64, 012316 (2001)

Maximal temperature allowing for entanglement generation

Explicit protocols

The optimal protocol in the 2-qubit case consists of:

I. Permutation.

II. Rotation to a maximally entangled state.

Maximal temperature allowing for entanglement generation

Explicit protocols

The optimal protocol in the 2-qubit case consists of:

I. Permutation.

II. Rotation to a maximally entangled state.

Based on this ansatz, we developed protocols for N-qubit systems, leading to:

- Entanglement in a single bipartition: $T_{max} \approx N/2 \ln(1+\sqrt{2})$.

Maximal temperature allowing for entanglement generation

Explicit protocols

The optimal protocol in the 2-qubit case consists of:

I. Permutation.

II. Rotation to a maximally entangled state.

Based on this ansatz, we developed protocols for N-qubit systems, leading to:

- Entanglement in a single bipartition: $T_{max} \approx N/2 \ln(1+\sqrt{2})$.
- Entanglement in all bipartitions: $T_{max} \approx N/\ln 3$.

Maximal temperature allowing for entanglement generation

Explicit protocols

The optimal protocol in the 2-qubit case consists of:

I. Permutation.
II. Rotation to a maximally entangled state.

Based on this ansatz, we developed protocols for N-qubit systems, leading to:

- Entanglement in a single bipartition: $T_{max} \approx N/2 \ln(1+\sqrt{2})$.
- Entanglement in all bipartitions: $T_{max} \approx N/\ln 3$.
- GHZ-like Genuine Multipartite Entanglement: $T_{max} \approx \epsilon/k_B 2 \ln 2$.

Maximal temperature allowing for entanglement generation

Explicit protocols

The optimal protocol in the 2-qubit case consists of:

I. Permutation.

II. Rotation to a maximally entangled state.

Based on this ansatz, we developed protocols for N-qubit systems, leading to:

- Entanglement in a single bipartition: $T_{max} \approx N/2\ln(1+\sqrt{2})$.
- Entanglement in all bipartitions: $T_{max} \approx N/\ln 3$.
- GHZ-like Genuine Multipartite Entanglement: $T_{max} \approx \epsilon/k_B 2\ln 2$.
- W-like Genuine Multipartite Entanglement: $T_{max} \propto N/\ln(N)$.

Maximal temperature allowing for entanglement generation

Explicit protocols

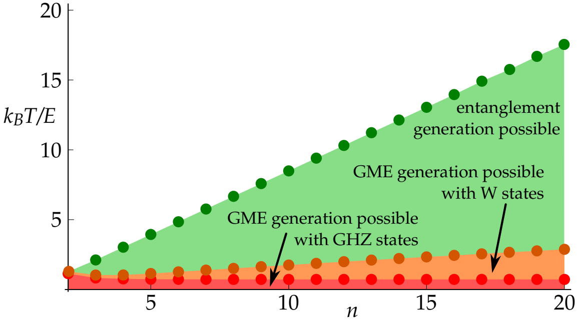
- Entanglement in a single bipartition: $T_{max} \approx N/2\ln(1+\sqrt{2})$.
- Entanglement in all bipartitions: $T_{max} \approx N/\ln 3$.
- GHZ-like Genuine Multipartite Entanglement: $T_{max} \approx \epsilon/k_B 2\ln 2$.
- W-like Genuine Multipartite Entanglement: $T_{max} \propto N/\ln(N)$.

Upper bounds on T_{max}

- Using the ball of separable states,[2] $T_{max} \propto e^N$.
- Using the form of the spectrum,[3] $T_{max} \propto N$.

[2] L. Gurvits and H. Barnum, Phys. Rev. A 68, 042312 (2003).
[3] N. Johnston, Phys. Rev. A 88, 062330 (2013)

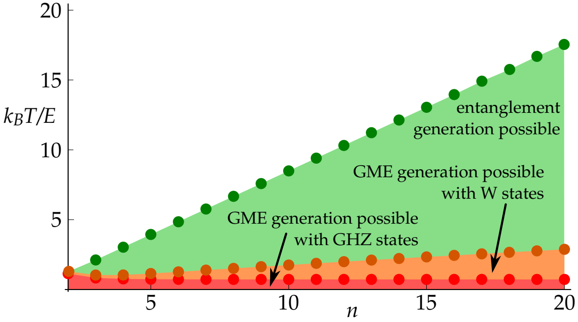
Maximal temperature allowing for entanglement generation



[4] W. Dür and J. I. Cirac, Phys. Rev. A **61**, 032341 (2000).

[5] T.M. Yu, K.R. Brown, and I.L. Chuang, Phys. Rev. A **71**, 032341 (2005).

Maximal temperature allowing for entanglement generation



Comparison with previous results. Assuming an initial (experimentally) achievable polarization in a NMR setting,

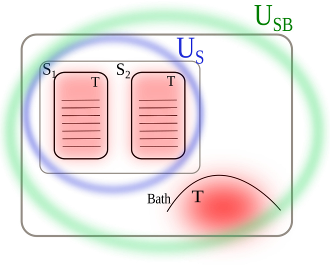
- Dür- Cirac: $N \geq 50000$ qubits [4]
- Yu-Brown-Chuang: $N \geq 22305$ qubits [5]
- Our protocols: $N \geq 5964$ qubits.

---

[4] W. Dür and J. I. Cirac, Phys. Rev. A **61**, 032341 (2000).

[5] T.M. Yu, K.R. Brown, and I.L. Chuang, Phys. Rev. A **71**, 032341 (2005).

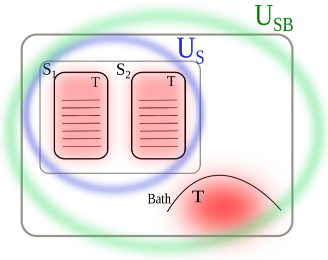
Maximal temperature: ancillary bath



- If the bath is sufficiently large, there exists $U^{\star}_{SB}$ s.t.

$$U^{\star}_{SB}\tau^{(S)}_{\beta} \otimes \tau^{(B)}_{\beta} U^{\star\dagger}_{SB} \approx |GS\rangle\langle GS| \otimes \tau^{(B)}_{\beta'}$$

Maximal temperature: ancillary bath

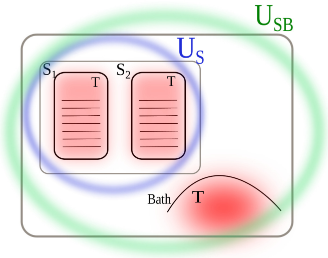


- If the bath is sufficiently large, there exists $U^{\star}_{SB}$ s.t.

$$U^{\star}_{SB}\tau^{(S)}_{\beta}\otimes\tau^{(B)}_{\beta}U^{\star\dagger}_{SB}\approx|GS\rangle\langle GS|\otimes\tau^{(B)}_{\beta'}$$

- If an arbitrary amount of work is available, a (fundamental) limiting temperature only exists if the system is closed.

Maximal temperature: ancillary bath



- If the bath is sufficiently large, there exists $U^{\star}_{SB}$ s.t.

$$U^{\star}_{SB}\tau^{(S)}_{\beta} \otimes \tau^{(B)}_{\beta} U^{\star\dagger}_{SB} \approx |GS\rangle\langle GS| \otimes \tau^{(B)}_{\beta'}$$

- If an arbitrary amount of work is available, a (fundamental) limiting temperature only exists if the system is closed.
- Nevertheless, how complex is $U^{\star}_{SB}$? what is the energy cost?

Energy Cost of Creating Correlations

If the available work is limited, $W \leq \Delta E$, how much correlations can we create? In particular, we consider

- Total correlations,

$$\mathcal{I}_{S_1 S_2} = S(\rho_{S_1}) + S(\rho_{S_2}) - S(\rho_S)$$

- Quantum correlations (entanglement),

$$E_{oF}(\rho) = \frac{1}{2} \inf_{\mathcal{D}(\rho)} \left(\sum_i p_i \mathcal{I}(|\psi_i\rangle\langle\psi_i|) \right)$$

where $\mathcal{D}(\rho) = \{p_i, |\psi_i\rangle | \sum_i p_i |\psi_i\rangle\langle\psi_i| = \rho\}$.

Energy Cost of Creating Correlations

Work cost

$$W = \mathrm{Tr}\left(H U_{SB} \tau U_{SB}^{\dagger}\right) - \mathrm{Tr}(H\tau)$$

Energy Cost of Creating Correlations

Work cost

$$W = \mathrm{Tr}\left(HU_{SB}\tau U_{SB}^{\dagger}\right) - \mathrm{Tr}(H\tau)$$

Using (i) conservation of entropy, and (ii) the form of the initial state, [6]

$$\beta W = I_{S_1 S_2} + S(\gamma_{S_1}||\tau_{S_1}) + S(\gamma_{S_2}||\tau_{S_2}) + I_{SB} + S(\gamma_B||\tau_B)$$

where γ is the final state.

[6] Esposito et al. New J. Phys. 12, 013013 (2010); D. Reeb et al, New J. Phys. 16, 103011 (2014).

Energy Cost of Creating Correlations

Work cost

$$W = \mathrm{Tr}\left(H U_{SB} \tau U_{SB}^{\dagger}\right) - \mathrm{Tr}(H\tau)$$

Using (i) conservation of entropy, and (ii) the form of the initial state, [6]

$$\beta W = I_{S_1 S_2} + S(\gamma_{S_1} || \tau_{S_1}) + S(\gamma_{S_2} || \tau_{S_2}) + I_{SB} + S(\gamma_B || \tau_B)$$

where γ is the final state.

Therefore,

$$\beta W \geq I_{S_1 S_2}$$

[6] Esposito et al. New J. Phys. 12, 013013 (2010); D. Reeb et al, New J. Phys. 16, 103011 (2014).

Energy Cost of Creating Correlations

$$\beta W = I_{S_1 S_2} + S(\gamma_{S_1} || \tau_{S_1}) + S(\gamma_{S_2} || \tau_{S_2}) + I_{SB} + S(\gamma_B || \tau_B)$$

For

- weak coupling, then $I_{SB} \approx 0$
- large bath, $S(\gamma_B || \tau_B) \approx 0$

so that $W = \Delta F_S^{\mathrm{non-eq.}}$.

Recall that,

$$\gamma_{S_1} = \mathrm{Tr}_{B,S_2}\left(U_{SB} \tau U_{SB}^\dagger\right)$$

A simple protocol achieving $\beta W = I_{S_1 S_2}$

Step 1: Cooling

$\tau_S(\beta) \to \tau_S(\beta')$
with an energy cost: $W_I = F(\tau_S(\beta)) - F(\tau_S(\beta'))$

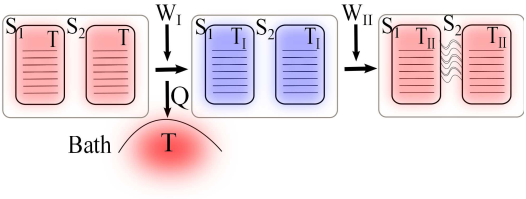
A simple protocol achieving $\beta W = I_{S_1 S_2}$

Step 1: Cooling

$\tau_S(\beta) \to \tau_S(\beta')$
with an energy cost: $W_I = F(\tau_S(\beta)) - F(\tau_S(\beta'))$

Step 2: Correlating

Isolate the system from the bath and apply a transformation U such that:
$\mathrm{Tr}_{S_1}\left(U\tau_S(\beta')U^\dagger\right) = \tau_{S_2}(\beta)$



Energy cost of entanglement

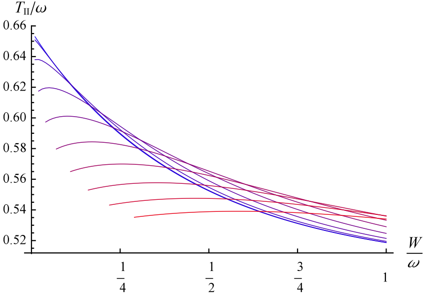
How does the protocol change if we want to generate entanglement instead?

Energy cost of entanglement

How does the protocol change if we want to generate entanglement instead?

Entanglement in two bosonic modes

Final local temperature of the state for the optimal protocol,



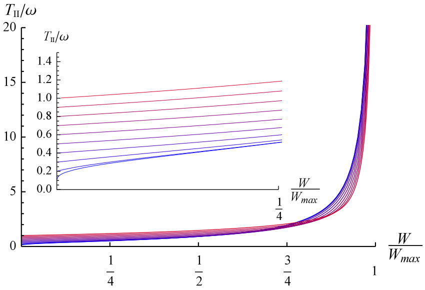
For maximising entanglement generation, it is beneficial to move the local states out of equilibrium.

Energy cost of entanglement

How does the protocol change if we want to generate entanglement instead?

Entanglement in two fermonic modes

Final local temperature of the state for the optimal protocol,



II. Work from correlations.

M. P.-L., K. Hovhannisyan, M. Huber, P. Skrzypczyk, N. Brunner, and A. Acín, arXiv:1407.7765

Scenario

- Initial state: a *correlated* state whose local states are thermal,

$$\mathrm{Tr}_{i\neq j}\left(\rho_S\right) = \frac{e^{-\beta H_S}}{\mathcal{Z}}$$

  but ρ_S is not a Gibbs state.

- In absence of correlations, the extractable work is exactly zero. How much work can we extract from the correlations?

Scenario

- Initial state: a *correlated* state whose local states are thermal,

$$\mathrm{Tr}_{i\neq j}(\rho_S) = \frac{e^{-\beta H_S}}{\mathcal{Z}}$$

  but ρ_S is not a Gibbs state.

- In absence of correlations, the extractable work is exactly zero. How much work can we extract from the correlations?
- Example: a microcanonical state.[6]

[6] A. E. Allahverdyan and K. V. Hovhannisyan, EPL **95** 60004 (2011).

Extractable work from correlations.

- Maximal work extraction in a unitary transformation:

$$W = \mathrm{Tr}(\mathrm{H}(\rho - \rho_{\mathrm{passive}}))$$

with $\rho_{\mathrm{passive}} = \sum_i \lambda_i |E_i\rangle\langle E_i|$, for $\lambda_{i+1} \leq \lambda_i$, $E_{i+1} \geq E_i$.

Extractable work from correlations.

- Maximal work extraction in a unitary transformation:

$$W = \mathrm{Tr}(\mathrm{H}(\rho - \rho_{\mathrm{passive}}))$$

with $\rho_{\mathrm{passive}} = \sum_i \lambda_i |E_i\rangle\langle E_i|$, for $\lambda_{i+1} \leq \lambda_i$, $E_{i+1} \geq E_i$.

- Given the restriction of local thermal marginals, the optimal ρ is,

$$\rho^* = \sum_{i=1}^{d} \sqrt{\frac{e^{-\beta e_i}}{\mathcal{Z}}} |e_1 \ldots . e_N\rangle$$

Extractable work from correlations.

- Maximal work extraction in a unitary transformation:

$$W = \mathrm{Tr}(\mathrm{H}(\rho - \rho_{\mathrm{passive}}))$$

with $\rho_{\mathrm{passive}} = \sum_i \lambda_i |E_i\rangle\langle E_i|$, for $\lambda_{i+1} \leq \lambda_i$, $E_{i+1} \geq E_i$.

- Given the restriction of local thermal marginals, the optimal ρ is,

$$\rho^* = \sum_{i=1}^{d} \sqrt{\frac{e^{-\beta e_i}}{\mathcal{Z}}} |e_1 .... e_N\rangle$$

- ρ^* is entangled, the optimal separable state can be shown to be,

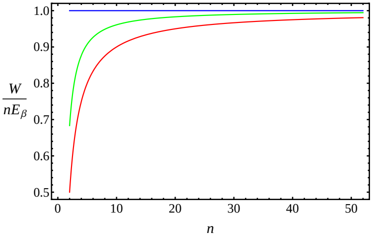
$$\rho^*_{\mathrm{sep}} = \sum_{i=1}^{d} \frac{e^{-\beta e_i}}{\mathcal{Z}} |e_1 .... e_N\rangle\langle e_1 ... e_N|.$$

Extractable work from correlations II

- Entangled states allow for a smaller global entropy than separable states.

Extractable work from correlations II

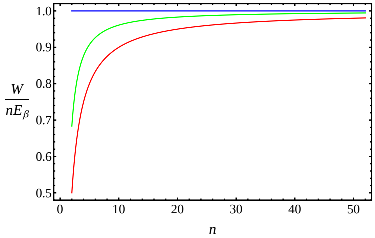
- Entangled states allow for a smaller global entropy than separable states.
- For a fixed global entropy, entanglement still increases the extractable work.



- The gain vanishes in the thermodynamic limit.

Extractable work from correlations II

- Entangled states allow for a smaller global entropy than separable states.
- For a fixed global entropy, entanglement still increases the extractable work.



- The gain vanishes in the thermodynamic limit.
- When given access to a bath, the extractable work reads [7]

$$W = T(NS_{\rm local} - S_{\rm global}).$$

[7] J. Oppenheim, Phys. Rev. Lett. **89**, 180402 (2002).; S. Jevtic et al, Phys. Rev. Lett. **108**, 110403 (2012).

Conclusions

- Maximal temperatures for entanglement and GME generation.
- Energy cost of correlations entanglement.
- Extractable work from correlations.

Conclusions

- Maximal temperatures for entanglement and GME generation.
- Energy cost of correlations entanglement.
- Extractable work from correlations.

References

- Maximal temperatures and energy cost: *arXiv:1404.2169*, *arXiv:1409.4647*.
- Extracting work from correlations *arXiv:1407.7765*.

Conclusions

- Maximal temperatures for entanglement and GME generation.
- Energy cost of correlations entanglement.
- Extractable work from correlations.

References

- Maximal temperatures and energy cost: *arXiv:1404.2169*, *arXiv:1409.4647*.
- Extracting work from correlations *arXiv:1407.7765*.

Thank you for your attention