Thermodynamics beyond free energy relations or "Quantum" Quantum Thermodynamics

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EPSRC

cost
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Kamil
Korzekwa

This is wind, he is actually slim

David Jennings

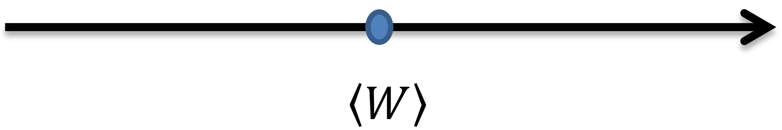
Terry Rudolph

What makes quantum thermodynamics really quantum?

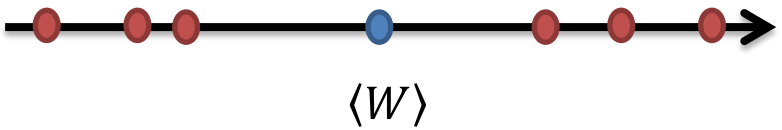
Can we develop a general framework to understand coherence?

Single-shot

Try to maximize
average work

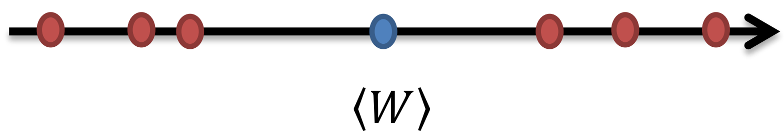


Single-shot



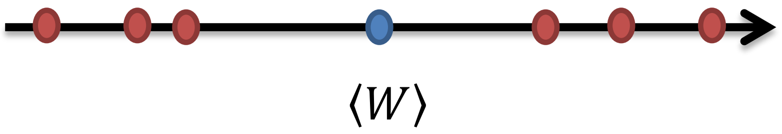
Truly work-like work extraction
Johan Aberg
Nature Communications 4, 1925 (2013)

Single-shot

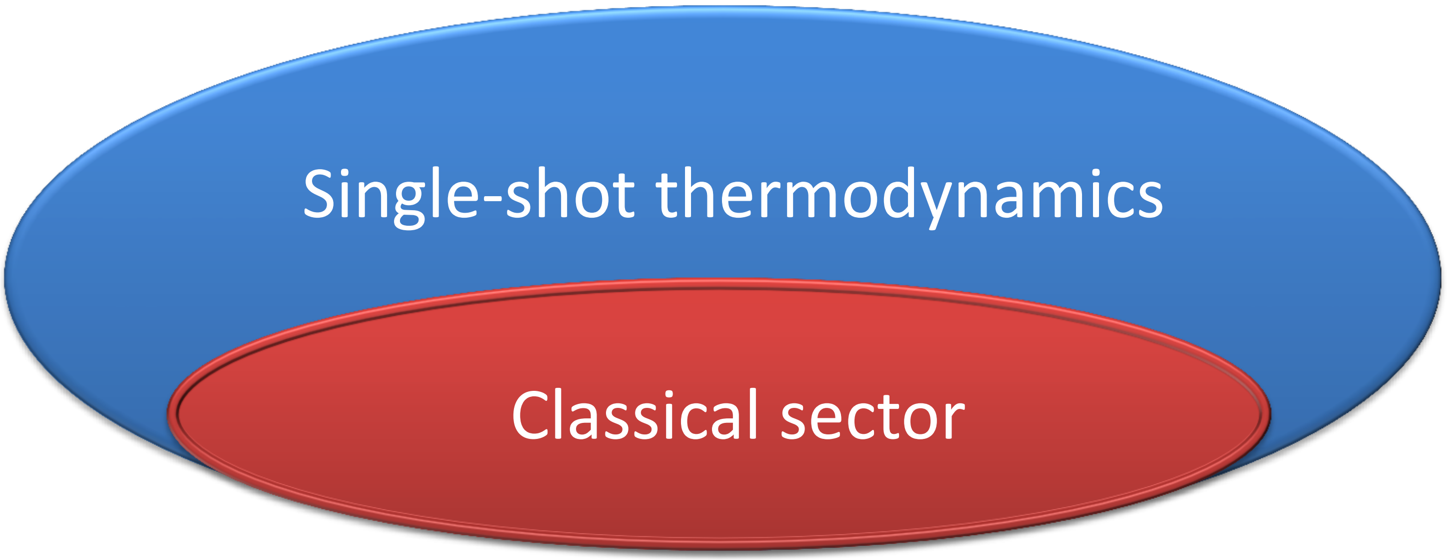


Single-shot thermodynamics

Single-shot



$\langle W \rangle$



Single-shot thermodynamics
Classical sector

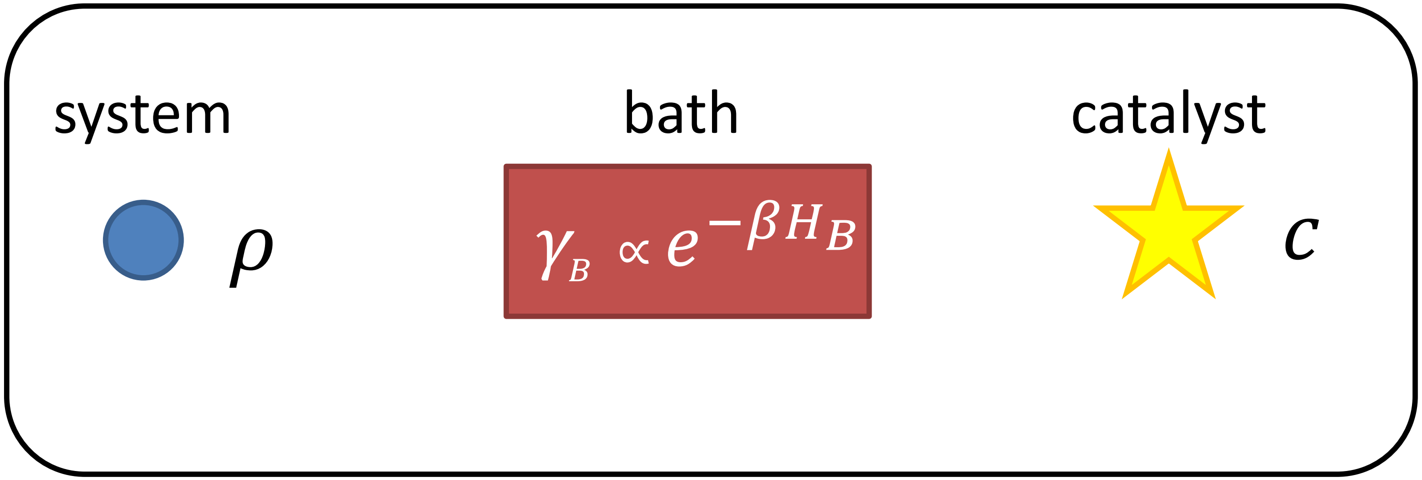
“Classical” Quantum Thermodynamics

system ρ

bath $\gamma_B \propto e^{-\beta H_B}$

catalyst c

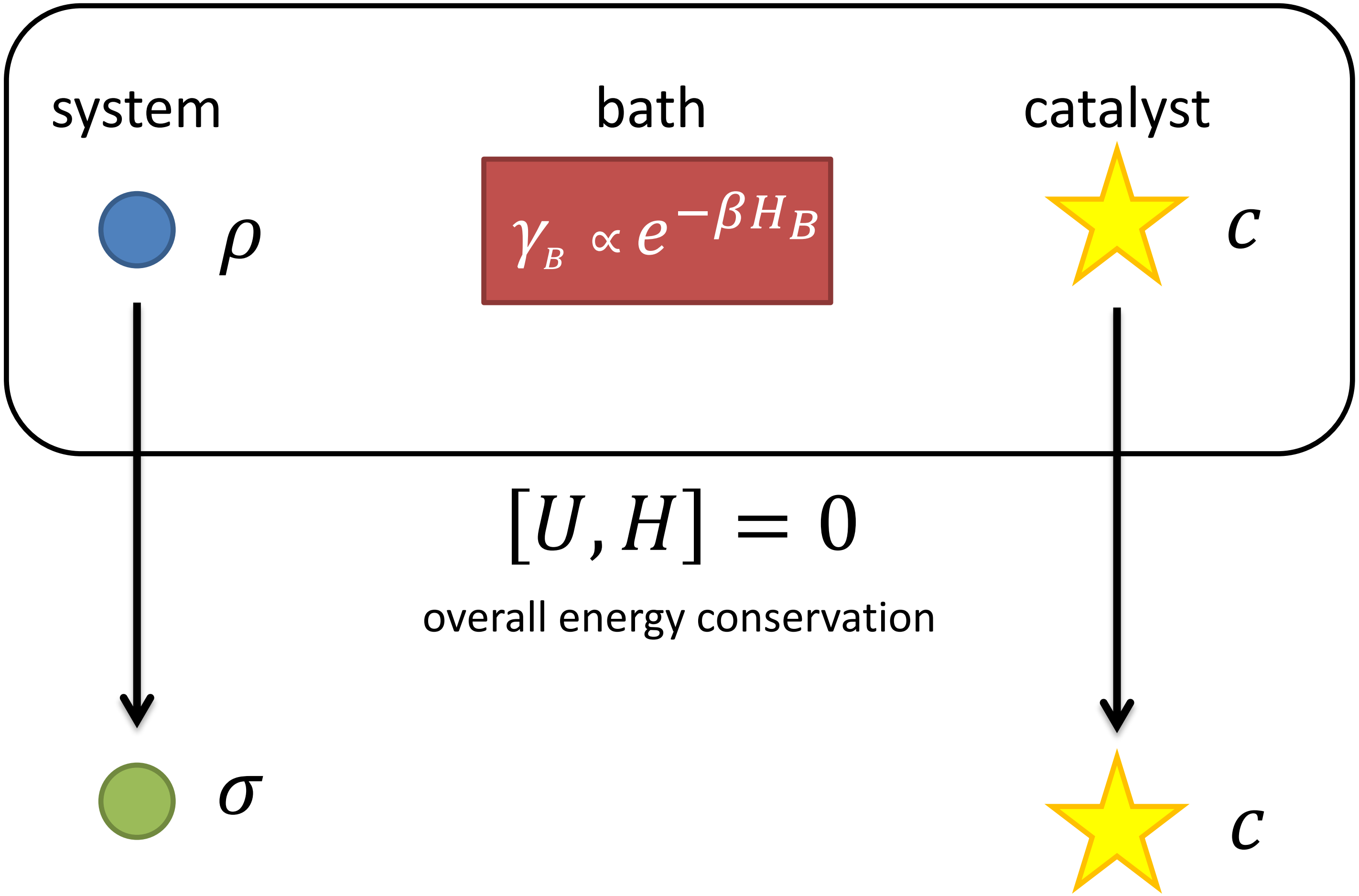
“Classical” Quantum Thermodynamics



$$[U, H] = 0$$

overall energy conservation

“Classical” Quantum Thermodynamics



“Classical” Quantum Thermodynamics

$$[\rho, H_S] = 0$$

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Theory becomes ``classical’’ :

1. Diagonalise everything in energy eigenbasis.
2. Hence, only probabilities of occupying different energies matters.
3. Thermal operations are particular classical stochastic processes.

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c.f. two-measurement protocol in fluctuation theorems (?)

“Classical” Quantum Thermodynamics

If $[\rho, H_S] = 0,$

$$\rho \to \sigma \qquad \Leftrightarrow \qquad F_\alpha(\rho) \geq F_\alpha(\sigma) \qquad \forall \alpha$$

The second laws of quantum thermodynamics
Fernando G.S.L. Brandao, Michał Horodecki, Nelly Huei Ying Ng, Jonathan Oppenheim, Stephanie Wehner
PNAS 112, 3275 (2015)

"Classical" Quantum Thermodynamics

If $[\rho, H_S] = 0$

$$\rho \rightarrow \sigma \qquad \Leftrightarrow \qquad F_\alpha(\rho) \geq F_\alpha(\sigma) \qquad \forall\alpha$$

where

$$F_\alpha(\rho) = -kT \log Z + kT S_\alpha(\rho||\gamma)$$

$$S_\alpha(\rho||\gamma) = \frac{\alpha/|\alpha|}{\alpha - 1} \log Tr\rho^\alpha \gamma^{1-\alpha} \qquad \gamma = \frac{e^{-\beta H_S}}{Z_S}$$

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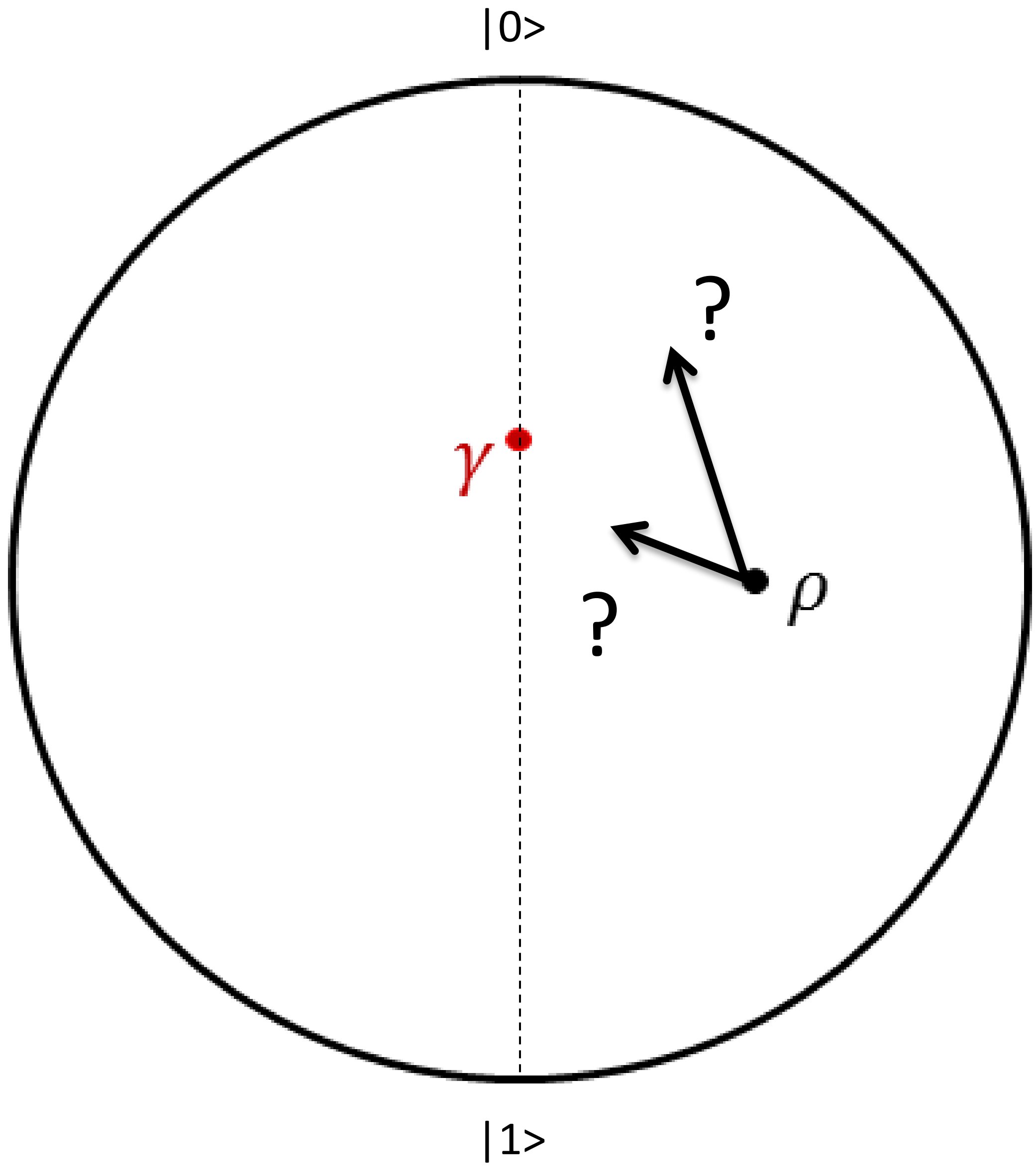
“Quantum” Quantum Thermodynamics

$$\sigma = \varepsilon_T(\rho) = Tr_B[U\left(\rho \otimes \frac{e^{-\beta H_B}}{Z_B}\right)U^\dagger]$$

$$[U, H] = 0$$

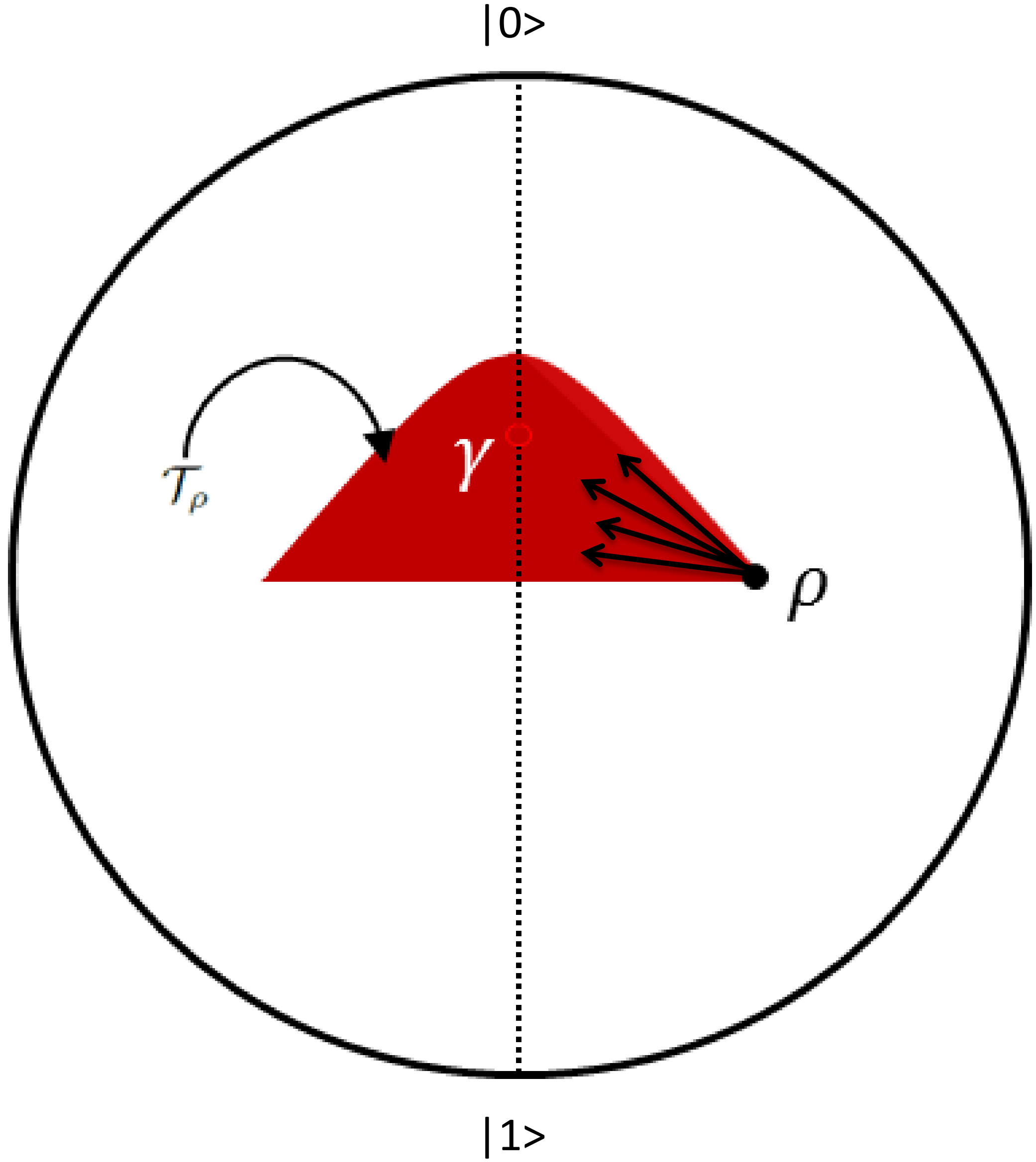
Now $[\rho, H_S\,] \neq 0$
(superpositions)

“Quantum” Quantum Thermodynamics



$$\gamma = \frac{e^{-\beta H_S}}{Z_S}$$

“Quantum” Quantum Thermodynamics



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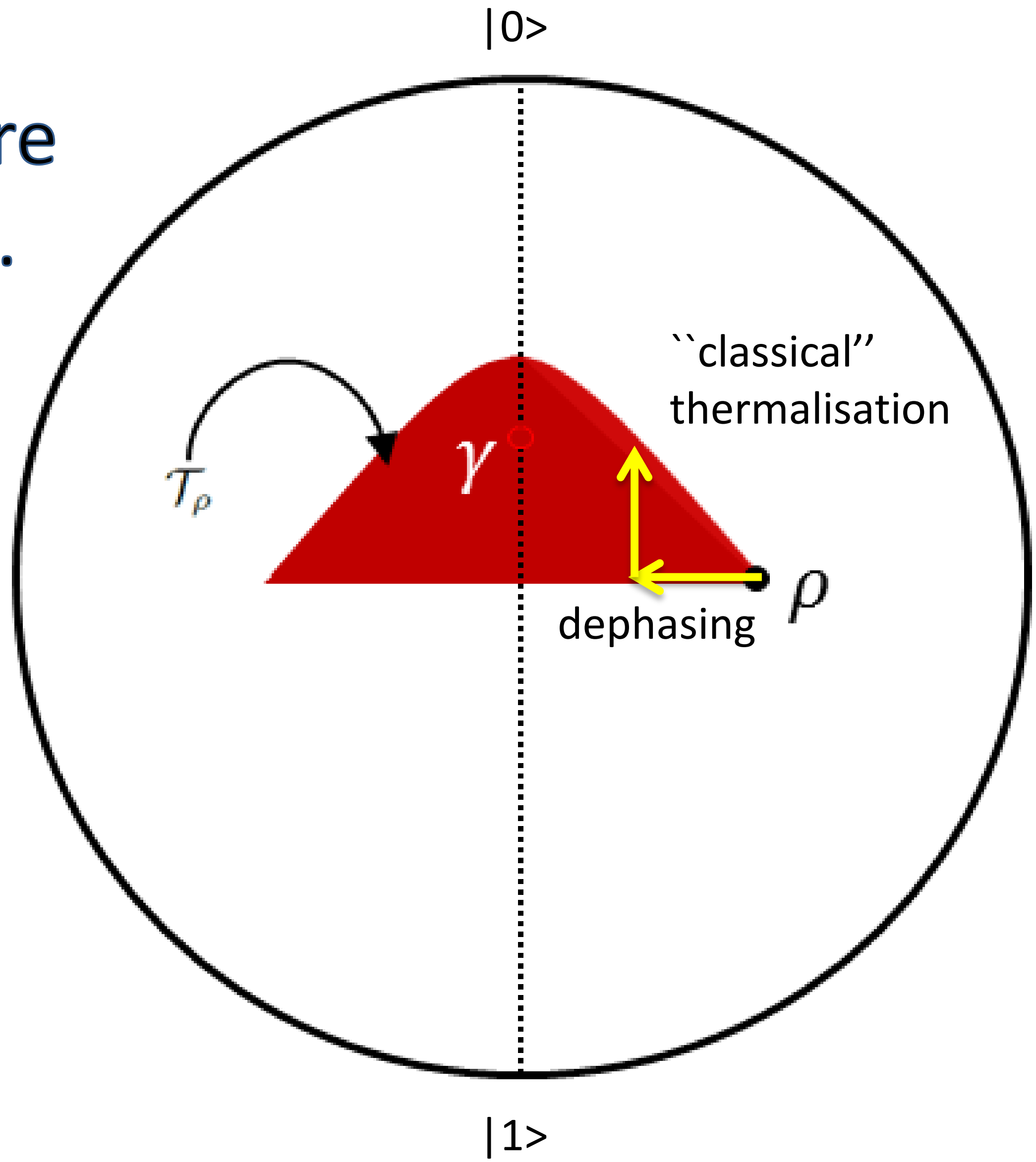
“Quantum” Quantum Thermodynamics

Is this more than just…



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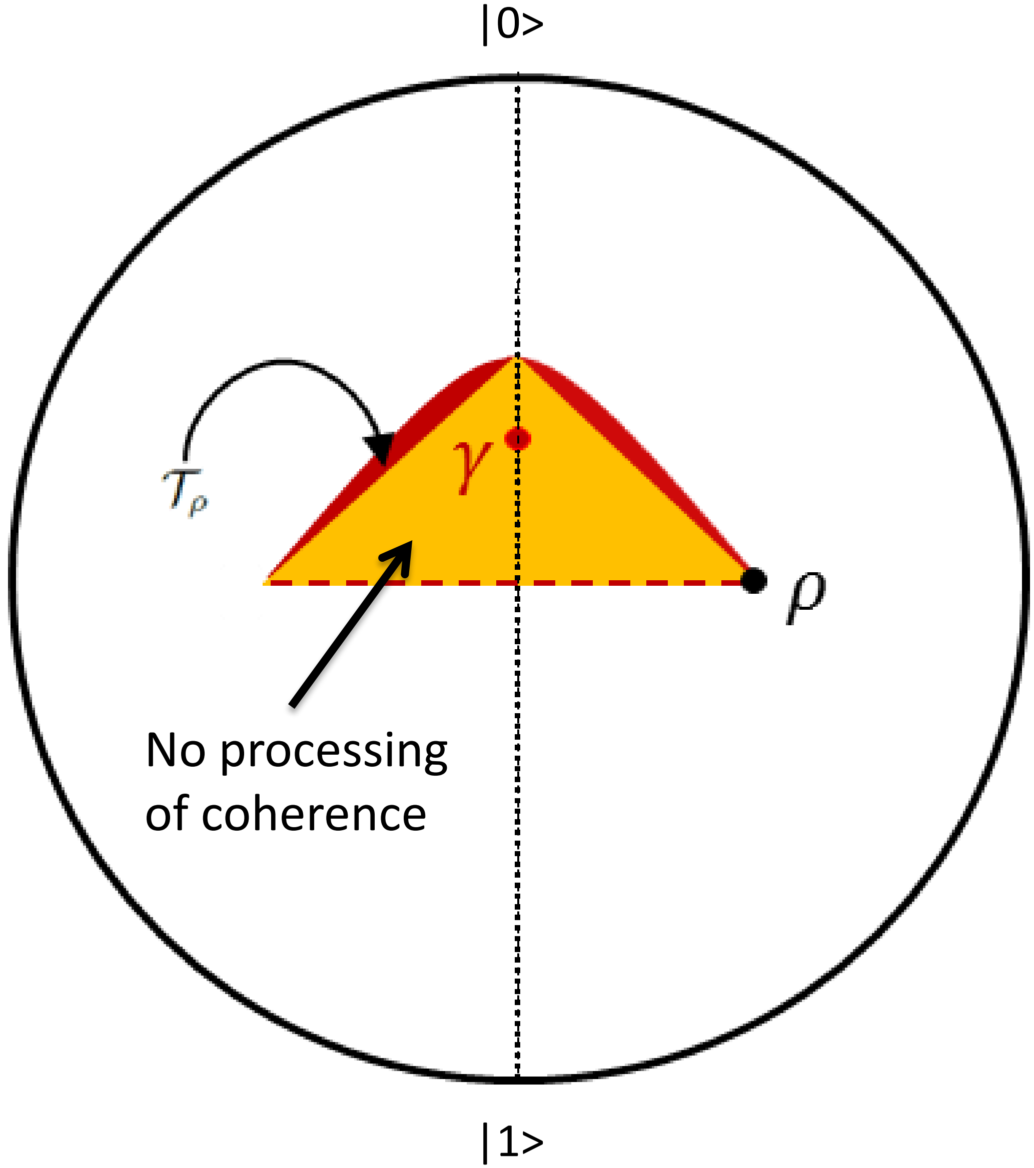
“Quantum” Quantum Thermodynamics

Yes!



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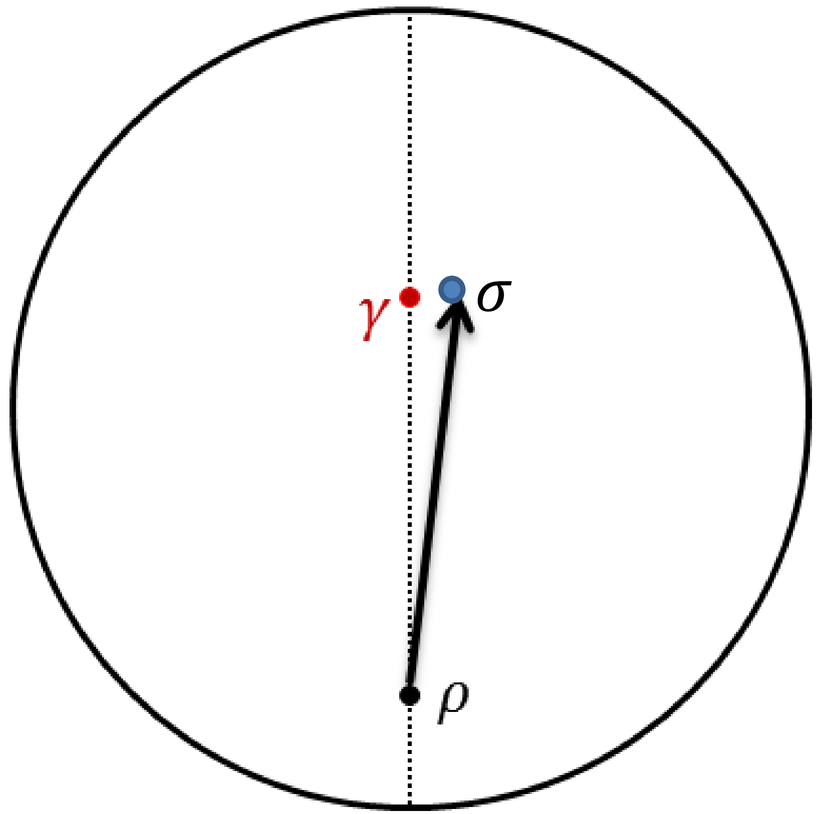
Free energy is not enough

If first law $[U, H] = 0$, this is impossible:



Despite any condition on F being satisfied.

What new ideas can we use to incorporate coherence in the description?

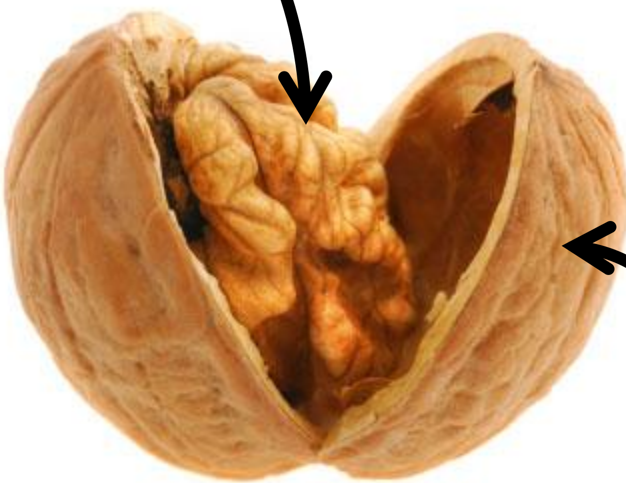
Description of quantum coherence in thermodynamic processes requires constraints beyond free energy
Matteo Lostaglio, David Jennings, Terry Rudolph
Nature Communications 6, 6383 (2015)
Quantum coherence, time-translation symmetry and thermodynamics
Matteo Lostaglio, Kamil Korzekwa, David Jennings, Terry Rudolph
Phys. Rev. X 5, 021001 (2015)

“Quantum” Quantum Thermodynamics

Simple but powerful symmetry consideration:

$$\mathcal{E}_T \subset \mathcal{E}_S$$

Non-equilibrium thermodynamics



Time-translation symmetry

“Quantum” Quantum Thermodynamics

Simple but powerful symmetry consideration:

$$\varepsilon_T \subset \varepsilon_S$$

$$\varepsilon_S\left(e^{-iH_S t}\rho e^{iH_S t}\right) = e^{-iH_S t}\varepsilon_S(\rho)e^{iH_S t}$$

time-translation symmetric maps

Extending Noether's theorem by quantifying the asymmetry of quantum states
Nature Communications 5, 3821 (2014)
Modes of asymmetry: the application of harmonic analysis to symmetric quantum dynamics and quantum reference frames
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Symmetry that describes how energy and coherence are NOT conserved in the system.

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1. Even if closed, $\langle H_S\rangle, \langle {H_S}^2\rangle, \dots, \langle {H_S}^k\rangle$ not enough for mixed states
2. Open dynamics

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2. **Open dynamics**

THERMODYNAMICS!

“Quantum” Quantum Thermodynamics

Some consequences

$$A_\alpha(\rho) = S_\alpha(\rho||D(\rho))$$

c.f. $F_\alpha(\rho) = -kT\log Z + kTS_\alpha(\rho||\gamma)$

Under any thermal operation:

$$\Delta A_\alpha(\rho) \leq 0$$

``second laws’’ for quantum coherence

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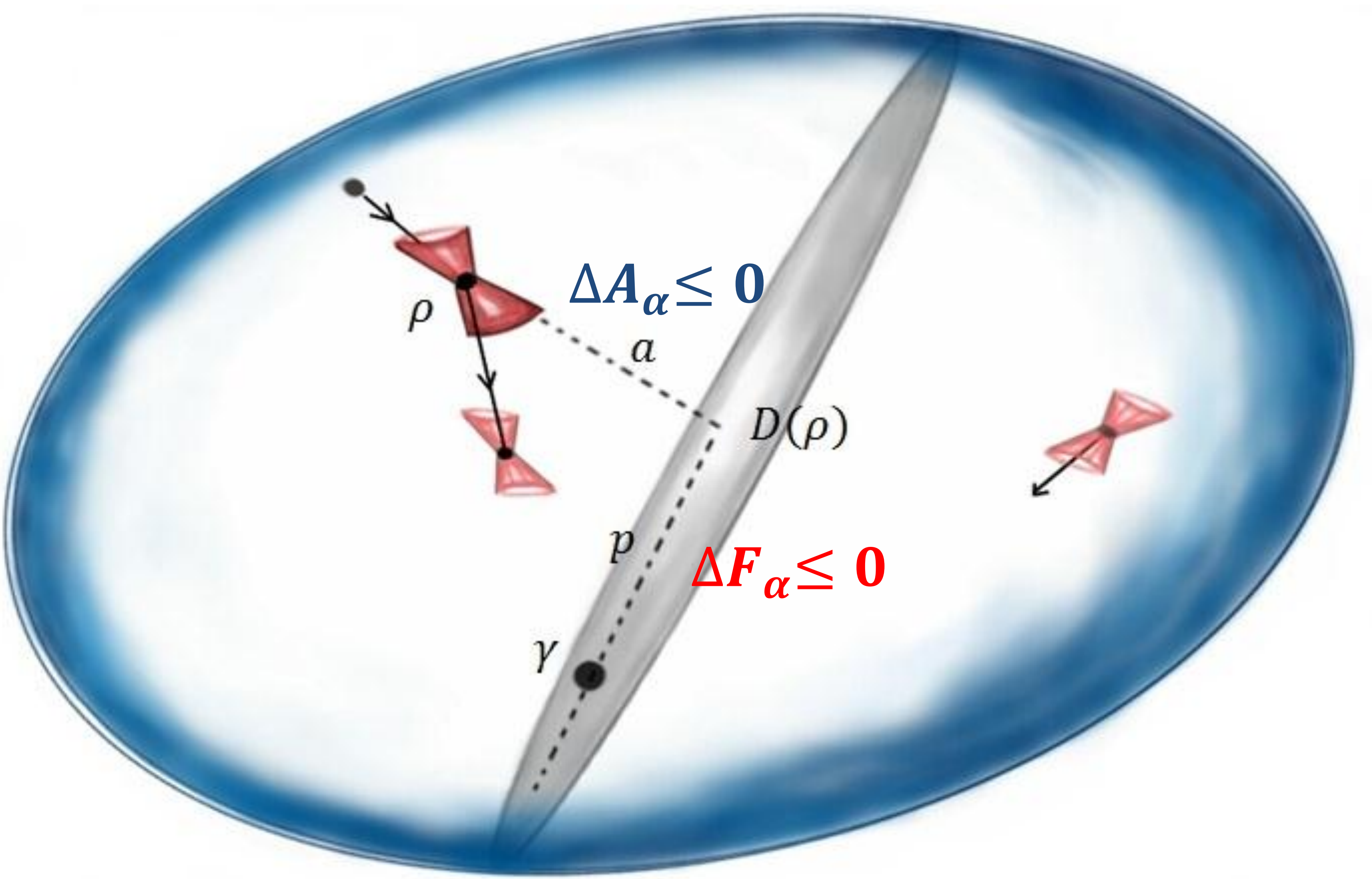
``second laws’’ for quantum coherence

Macroscopically irrelevant: $\lim_{N\to\infty} \frac{A_\alpha(\rho^{\otimes N})}{N} = 0$



$\Delta A_\alpha \leq 0$
ρ
a
$D(\rho)$
p
$\Delta F_\alpha \leq 0$
γ

“Quantum” Quantum Thermodynamics

Some consequences

Standard quantum free energy:

both decrease

$$F(\rho) = \langle H \rangle - \frac{S(\rho)}{\beta} = F\big(D(\rho)\big) + \frac{1}{\beta} A(\rho)$$

Work locked

“Quantum” Quantum Thermodynamics

Some consequences

both decrease

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Work locked

Activation:

$F\left(D\left(\rho^{\otimes 2}\right)\right) \geq 2F\big(D(\rho)\big)$ (strict if $[\rho, H_S\,] \neq 0$)

c.f. passive states…

“Quantum” Quantum Thermodynamics

Concrete bounds? If $\rho \to \sigma$

$$|\sigma_{nm}| \leq \sum_{\substack{k,l \\ E_k < E_n \\ E_k - E_l = E_n - E_m}} |\rho_{kl}|\, e^{-\beta(E_n - E_k)} + \sum_{\substack{k,l \\ E_k \geq E_n \\ E_k - E_l = E_n - E_m}} |\rho_{kl}|$$

“Quantum” Quantum Thermodynamics

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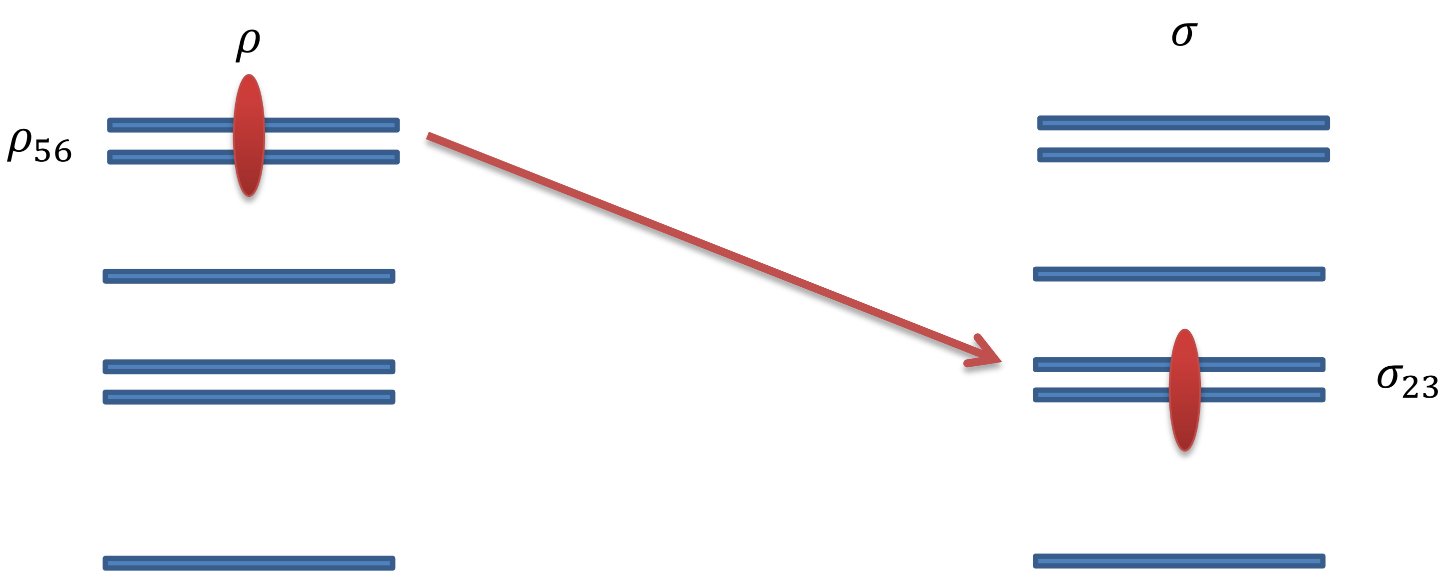

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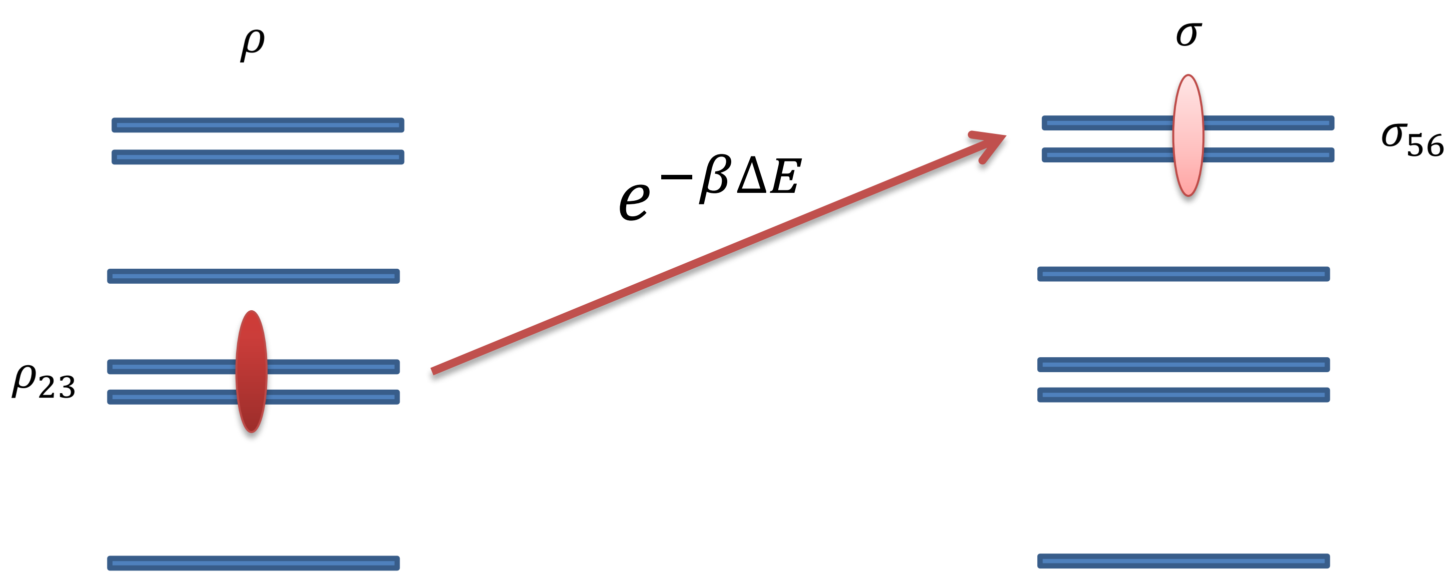
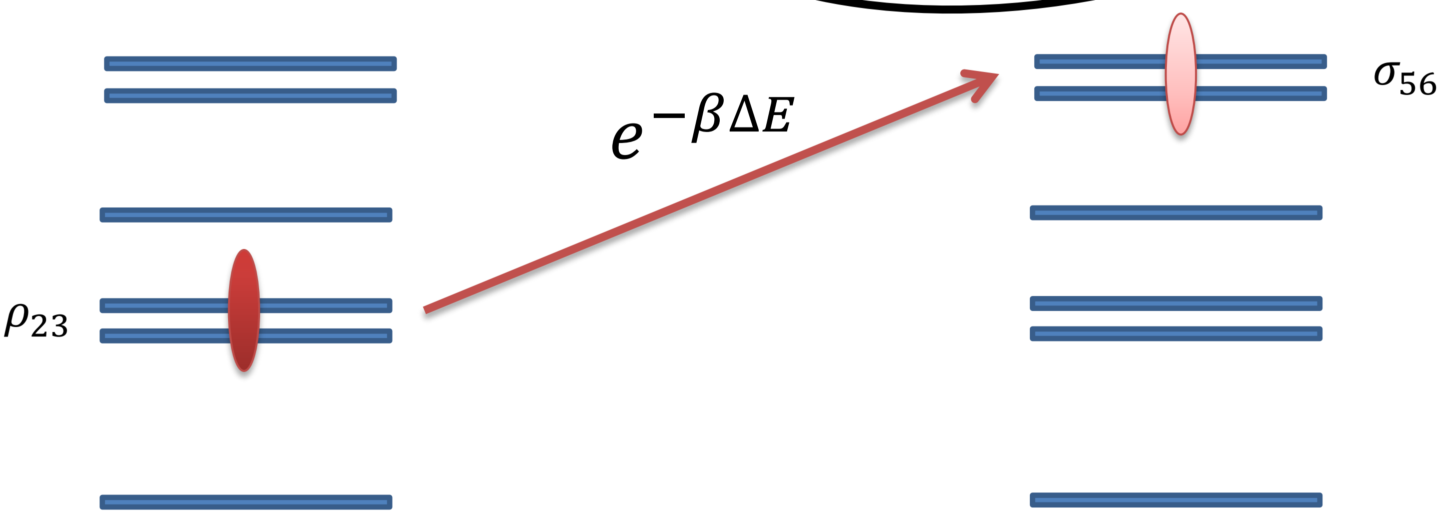


“Quantum” Quantum Thermodynamics

Concrete bounds? If $\rho \to \sigma$

Kamil: Uououo!! This is explained better in my poster!



Conclusions

1. **Coherence in thermodynamics can be understood through symmetry principles**
2. **Coherence + Thermo > Free energy consideration**
3. **Bias towards energetic considerations?**
4. **More to be done!**

Thank you for your attention!

More details:

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Phys. Rev. X 5, 021001 (2015)
Work extraction from coherence
Kamil Korzekwa, Matteo Lostaglio , Jonathan Oppenheim, David Jennings
Soon on ArXiv!

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$$S_\alpha(\rho||\gamma) = \frac{\alpha/|\alpha|}{\alpha - 1} \log Tr \begin{cases} \rho^\alpha \gamma^{1-\alpha} & 0 \leq \alpha \leq 1 \\ (\rho^{\frac{1-\alpha}{2\alpha}} \gamma \rho^{\frac{1-\alpha}{2\alpha}})^\alpha & \alpha > 1 \end{cases}$$

Quantum hypothesis testing and the operational interpretation of the quantum Renyi relative entropies
Milan Mosonyi, Tomohiro Ogawa
Communications in Mathematical Physics: Volume 334, Issue 3 (2015), Page 1617-1648