— NONEQUILIBRIUM FLUCTUATIONS IN QUANTUM HEAT ENGINES THEORY, EXAMPLE, AND SOLID STATE EXPERIMENTS —

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[1] M. Campisi, J. Phys. A: Math. Theor. **47**, 245001 (2014)

[2] M. Campisi, J. Pekola, R. Fazio, New J. Phys. **17** 035012 (2015)

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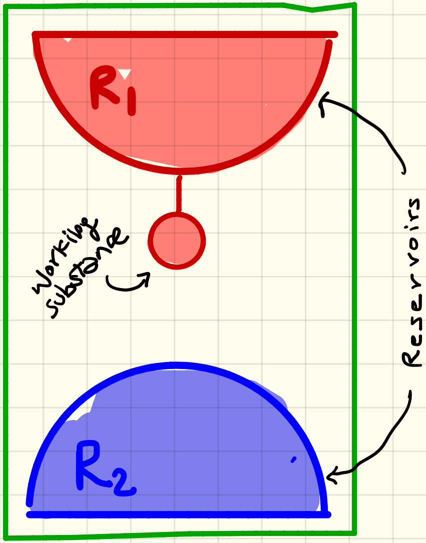
SCUOLA NORMALE SUPERIORE

NEST National Enterprise for nanoScience and nanoTechnology

MARIE CURIE ACTIONS

cost

OBSERVATION: HEAT ENGINE AS A DRIVEN BIPARTITE SYSTEM



Subsystem 1 = R_1 + WS

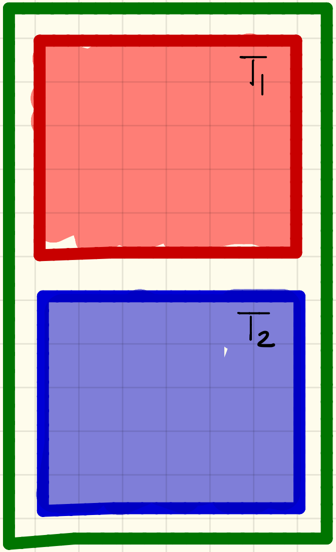
Subsystem 2 = R_2

Driving = Operations on WS and switch ON/OFF couplings

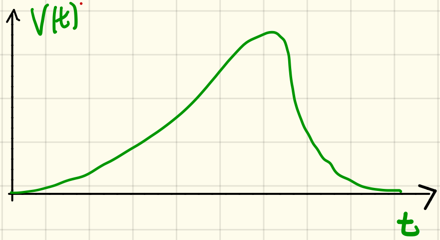
Examples: 4 stroke engines (Carnot, Diesel, etc)

Continuous engines

FLUCTUATION RELATION FOR HEAT ENGINES



$$H(t) = H_1 + H_2 + V(t)$$ ← Work Source



$$\rho_0 = \frac{e^{-\beta_1 H_1}}{Z_1} \otimes \frac{e^{-\beta_2 H_2}}{Z_1}$$

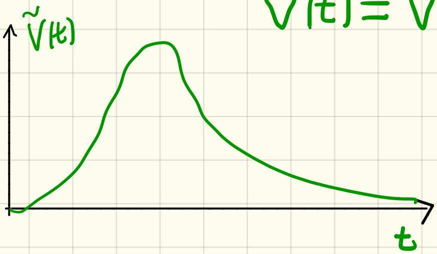
$t = 0$ $\qquad E^1_{n_1}$ $\qquad E^2_{n_2}$ $\qquad P_{n_1, n_2} = \frac{e^{-\beta_1 E^1_{n_1}}}{Z_1} \; \frac{e^{-\beta_2 E^2_{n_2}}}{Z_2}$

$t = \tau$ $\qquad E^1_{m_1}$ $\qquad E^2_{m_2}$ $\qquad P_{m_1, m_2 | n_1, n_2} = |\langle m_1 \, m_2 | U | n_1 \, n_2 \rangle|^2$

$$\begin{array}{ccccc} E^1_{m_1} & & E^2_{m_2} & & \\ \downarrow & & \downarrow & & \\ \Delta E_1 & + & \Delta E_2 & = & W \end{array}$$

$$P(\Delta E_1, \Delta E_2) = \sum_{\substack{n_1 n_2 \\ m_1 m_2}} P_{n_1 n_2} \, P_{m_1 m_2 | n_1 n_2} \, \delta\left[\Delta E_1 - (E^1_{m_1} - E^1_{n_1})\right] \times$$
$$\times \, \delta\left[\Delta E_2 - (E^2_{m_2} - E^2_{n_2})\right]$$

$$\tilde{V}(t) = V(t - \tau)$$



$$\tilde{P}_{m_1 m_2 | n_1 n_2} = P_{n_1 n_2 | m_1 m_2}$$

$$\Downarrow$$

$$\frac{P(\Delta E_1, \Delta E_2)}{\tilde{P}(-\Delta E_1, -\Delta E_2)} = e^{+\beta_1 \Delta E_1} \, e^{+\beta_2 \Delta E_2}$$

EXCHANGE FLUCTUATION THEOREM

Jarzynski & Wójcik PRL **92** 230602 (2004)

Change of variables $W = \Delta E_1 + \Delta E_2$

$$\frac{P(\Delta E_1, W)}{\tilde{P}(-\Delta E_1, -W)} = e^{+(\beta_1 - \beta_2)\Delta E_1 + \beta_2 W}$$

← HEAT ENGINE FLUCTUATION RELATION

$$\Rightarrow \left\langle e^{-(\beta_1 - \beta_2)\Delta E_1 - \beta_2 W} \right\rangle = 1$$

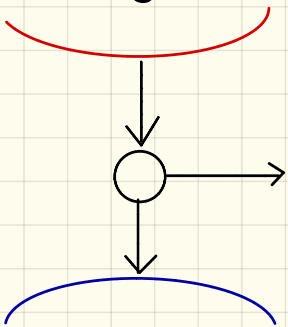
$$\Rightarrow (\beta_1 - \beta_2)\langle \Delta E_1 \rangle - \beta_2 \langle W \rangle \geq 0$$

Sinitsyn JPA 44 405001 (2011)

Campisi JPA 47 245001 (2014)

Campisi et al. NJP in press (2015)

Heat engine operation



$$\langle W \rangle \lesssim 0$$

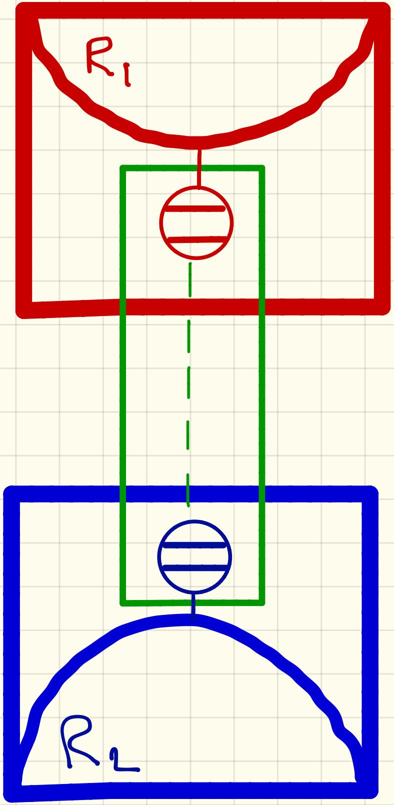
$$\langle \Delta E_1 \rangle \lesssim 0$$

$$(\beta_1 - \beta_2)\langle \Delta E_1 \rangle - \beta_2 \langle W \rangle \gtrsim 0$$

$$\Rightarrow \quad \eta \doteq \frac{\langle W \rangle}{\langle \Delta E_1 \rangle} \lesssim 1 - \frac{\beta_2}{\beta_1} \doteq \eta^c$$

Fluctuation Theorem $\Rightarrow$ $\eta \leq \eta^c$

EXAMPLE: TWO QUBIT ENGINE



SIMPLIFYING ASSUMPTION: Driving FAST compared to thermal relaxation

$$\frac{\omega_1}{2}\sigma_1^z \qquad \frac{\omega_2}{2}\sigma_2^z$$

$$H = H_1 + H_2 + V(t) \rightarrow U$$

$$\langle \Delta E_1 \rangle = \mathrm{Tr}_1 \mathrm{Tr}_2 H_1 (U \rho U^\dagger - \rho)$$

$$+$$

$$\langle \Delta E_2 \rangle = \mathrm{Tr}_1 \mathrm{Tr}_2 H_2 (U \rho U^\dagger - \rho)$$

$$=$$

$$\langle W \rangle$$

MAXIMIZATION OF WORK OUTPUT

$$U = \left(\begin{array}{c|cc|c} e^{i\varphi_1} & & & \\ \hline & 0 & e^{i\varphi_3} & \\ & e^{i\varphi_2} & 0 & \\ \hline & & & e^{i\varphi_4} \end{array}\right) \longleftarrow \text{SWAP}$$

Basis: $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$

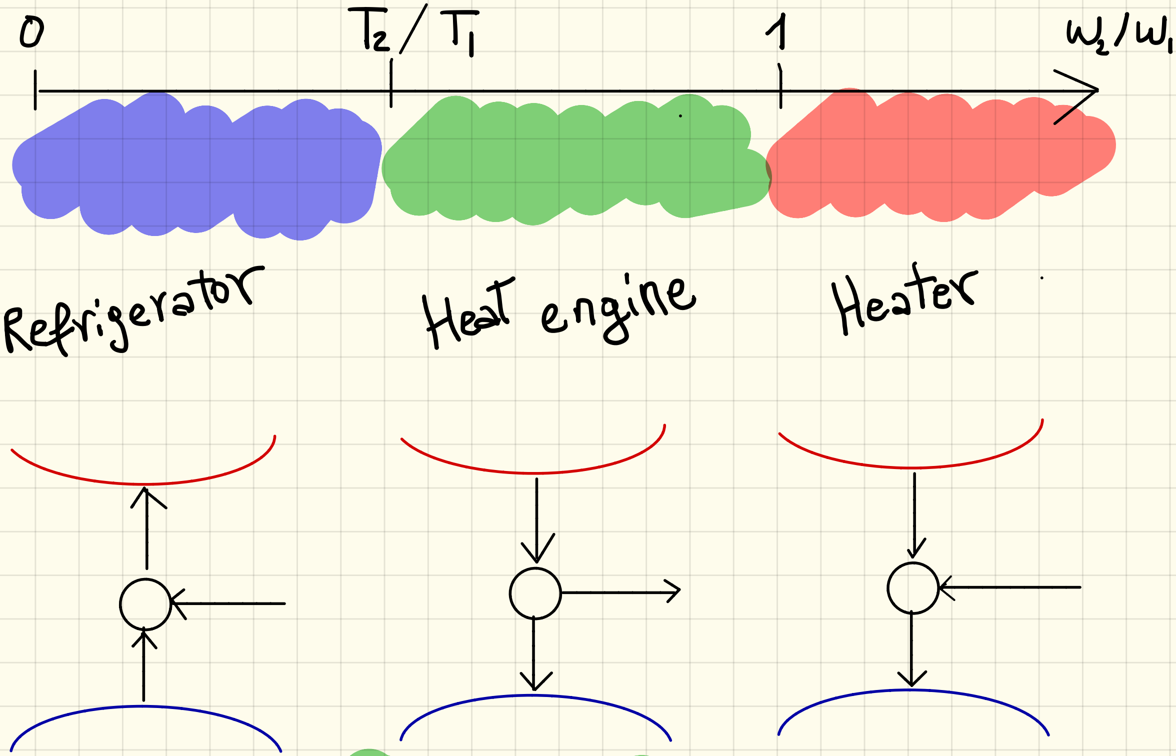
$$U|+-\rangle = e^{i\varphi_3}|-+\rangle$$

$$U|-+\rangle = e^{i\varphi_2}|+-\rangle$$

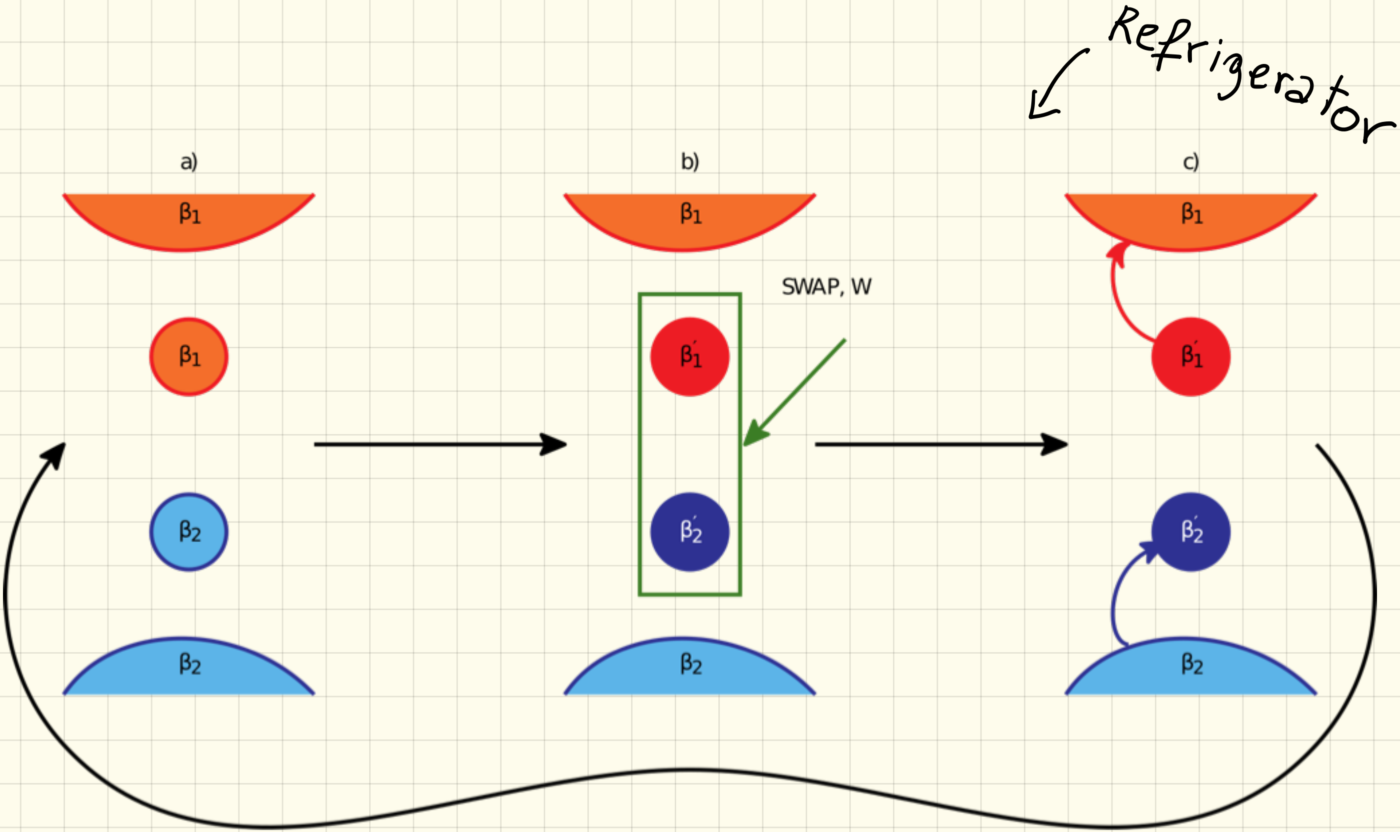
$$\langle \Delta E_1 \rangle = -\left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right)\omega_1$$

$$\langle \Delta E_2 \rangle = \left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right)\omega_2$$

$$\langle W \rangle = \left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right)(\omega_2 - \omega_1)$$



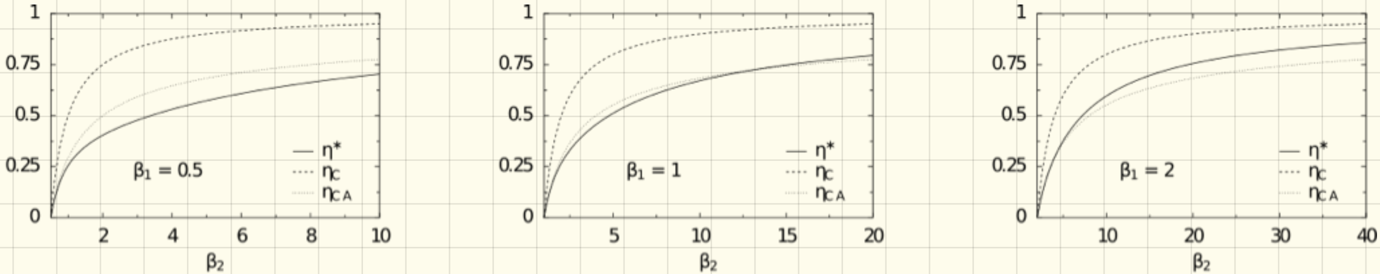
$$\eta = 1 - \frac{W_2}{W_1} \leqslant 1 - \frac{T_2}{T_1}$$

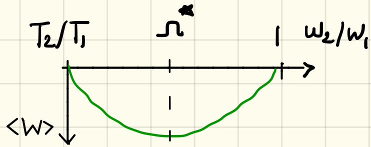


$$\beta_1' = \beta_2 \frac{\omega_2}{\omega_1} \qquad \beta_2' = \beta_1 \frac{\omega_1}{\omega_2}$$

RESULTS

Efficiency @ max. work output





$$\eta_c = 1 - \frac{\beta_1}{\beta_2}$$

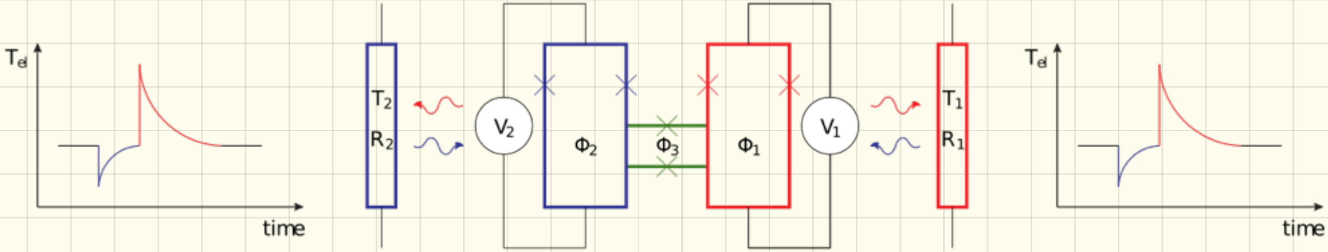
$$\eta_{CA} = 1 - \sqrt{\frac{\beta_1}{\beta_2}}$$

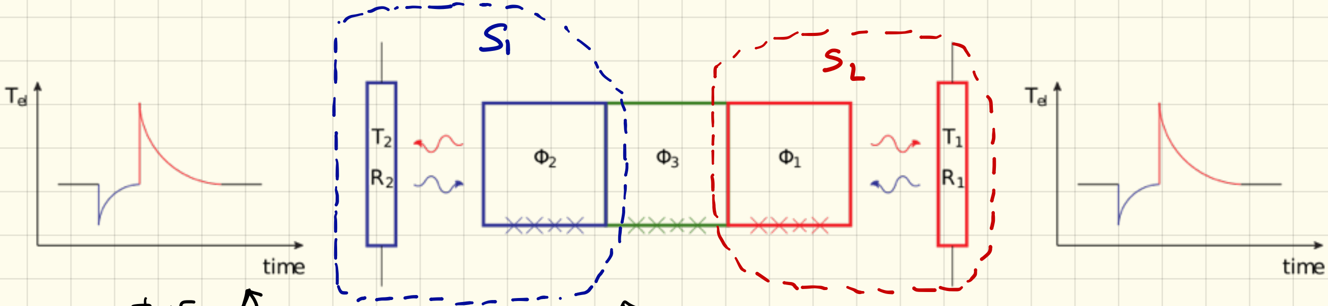
$$\eta^* = 1 - \Omega^*$$

IMPLEMENTATION

Cooper pair Boxes

ECHTERNACH et al.
Quantum Info. Comput. 1 143 (2001)



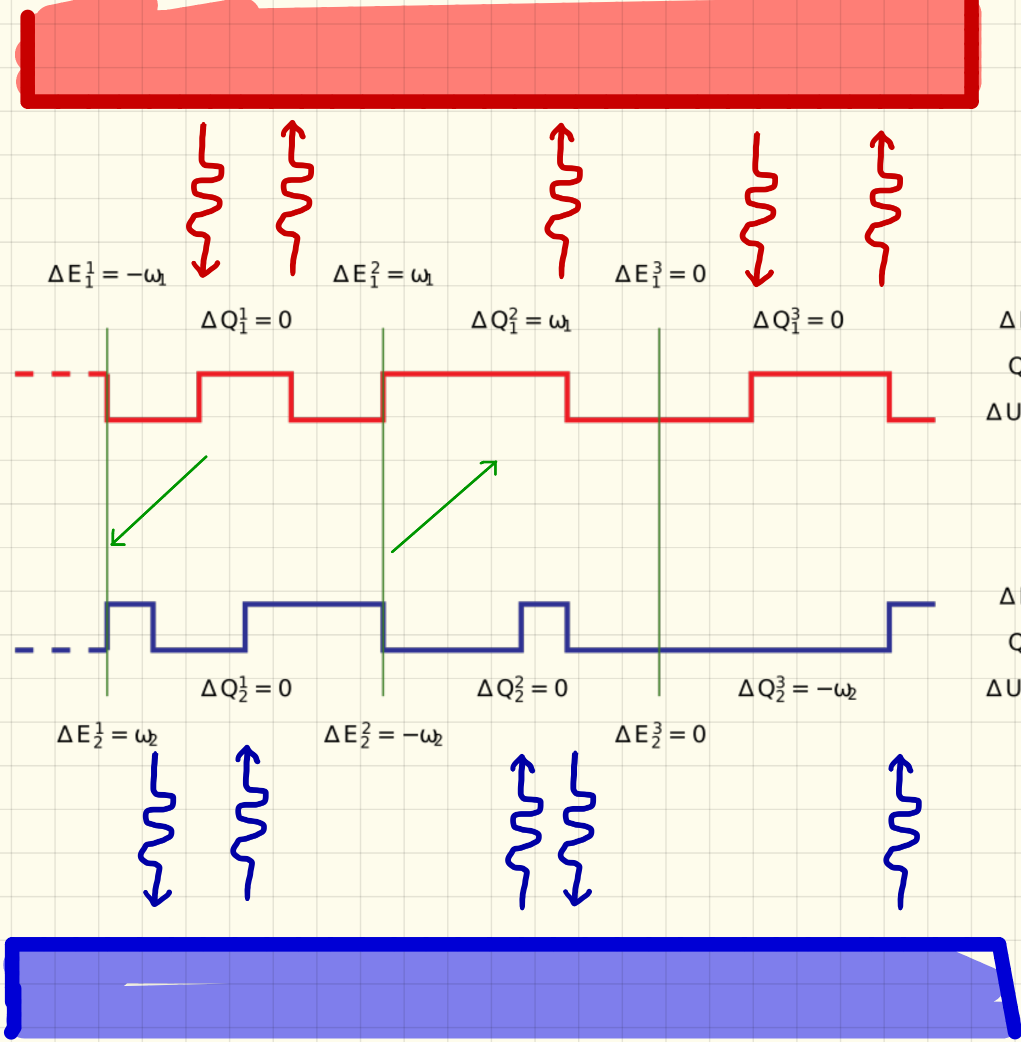


Calorimeter

Flux qubits

GASPARINETTI et al
Phys. Rev. App. 3 014007 (2015)

NISKANEN et al
Science 316 723 (2007)



$\Delta E_1 = \sum_i \Delta E_1^i = -N\omega_1$

$Q_1 = \sum_i \Delta Q_1^i$

$\Delta U_1 = \Delta E_1 - Q_1$

$\Delta E_2 = \sum_i \Delta E_2^i = N\omega_2$

$Q_2 = \sum_i \Delta Q_2^i$

$\Delta U_2 = \Delta E_2 - Q_2$

$\Delta E_2 = -\frac{\omega_2}{\omega_1} \Delta E_1$

$\frac{W}{\Delta E_1} = 1 - \frac{\omega_2}{\omega_1}$

MODELLING: MONTECARLO WAVE-FUNCTION METHOD

$$\dot{\rho}_i = -i[H_{q,i}, \rho] + \mathcal{L}_i \rho \tag{12}$$

$$\mathcal{L}_i \rho = \gamma(n_i + 1) D[\sigma_i]\rho + \gamma n_i D[\sigma_i^\dagger]\rho_i, \qquad i = 1, 2 \tag{13}$$

$$n_i = \frac{1}{e^{\beta_i \omega_i} - 1} \qquad D[c]\rho = c\rho c^\dagger - \frac{1}{2} c^\dagger c \rho - \frac{1}{2} \rho c^\dagger c \tag{14}$$

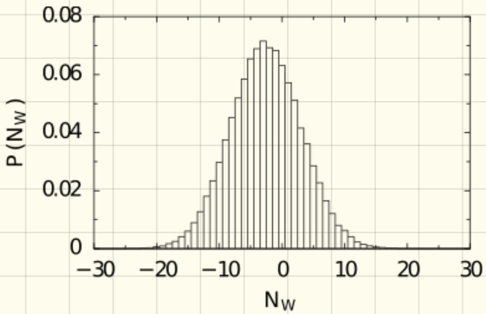
$$d|\psi_i\rangle = -iG_i(|\psi_i\rangle)dt + \left(\frac{\sigma_i |\psi_i\rangle}{||\sigma_i|\psi_i\rangle||} - |\psi_i\rangle \right) dN_i^+ + \left(\frac{\sigma_i^\dagger |\psi_i\rangle}{||\sigma_i^\dagger|\psi_i\rangle||} - |\psi_i\rangle \right) dN_i^-$$

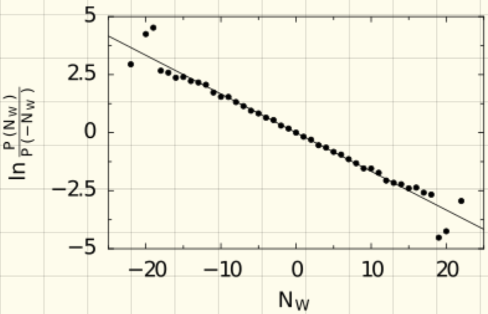
$$G_i(|\psi_i\rangle) = H_{q,i}^{\text{eff}} |\psi_i\rangle + \frac{i}{2}\gamma(n_i + 1)||\sigma_i|\psi_i\rangle||^2 |\psi_i\rangle + \frac{i}{2}\gamma(n_i)||\sigma_i^\dagger|\psi_i\rangle||^2 |\psi_i\rangle$$

$$H_{q,i}^{\text{eff}} = H_{q,i} - \frac{i}{2}\gamma(n_i + 1)\sigma_i^\dagger \sigma_i - \frac{i}{2}\gamma n_i \sigma_i \sigma_i^\dagger \tag{15}$$

$$\Gamma_i^- = \gamma n_i\,, \quad \Gamma_i^+ = \gamma(n_i + 1)\,, \quad \frac{\Gamma_i^-}{\Gamma_i^+} = e^{-\beta_i \omega_i} \tag{16}$$

RESULTS

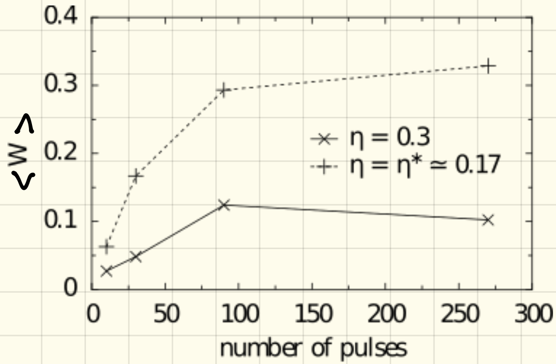




$$\frac{P(\Delta E_1, \Delta E_2)}{P(-\Delta E_1, -\Delta E_2)} = e^{+\beta_1 \Delta E_1 + \beta_2 \Delta E_2}$$

$$\Delta E_2 = -\frac{w_2}{w_1} \Delta E_1 \quad , \qquad \Delta E_1 = N_W \, w_1 \quad \Longrightarrow \quad \frac{P(N_W)}{P(-N_W)} = e^{(\beta_1 w_1 - \beta_2 w_2) N_W}$$

RESULTS

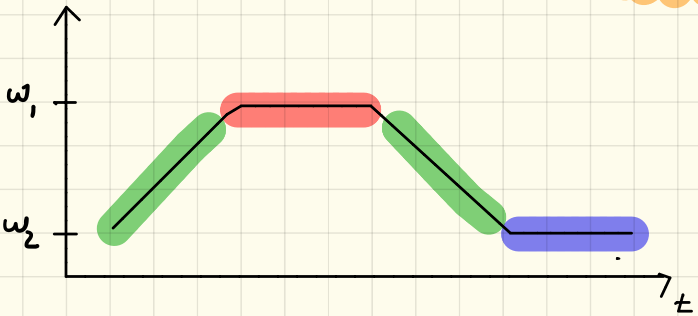


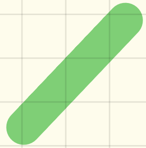
Increase Power without compromising efficiency !!

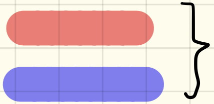
COMPARISON

Single qubit "Otto Engine"

$\eta = 1 - \frac{\omega_2}{\omega_1}$



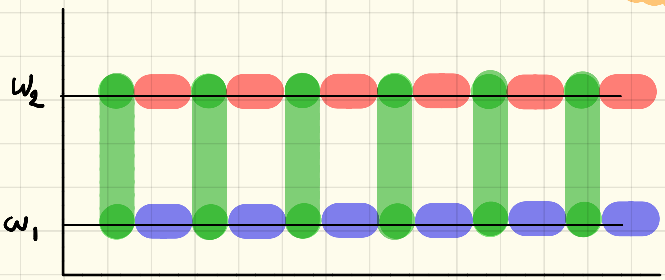
 = slow adiabatic ramp *

 = slow FULL thermalization

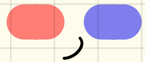
[* can be replaced by fast shortcut to adiabaticity — Del Campo, Sci. Rep 4 6208 (2014)]

Double qubit SWAP engine

$\eta = 1 - \frac{\omega_2}{\omega_1}$



 = Fast SWAP gate

 = Fast PARTIAL thermalization

Summary

1. THEORY: Fluctuation Theorem $\Rightarrow$ $\eta \leq \eta^c$

2. EXAMPLE: Optimal two qubit engine

3. IMPLEMENTATION: Superconducting qubits + Calorimetric scheme

UPCOMING CONFERENCES IN ERICE, ITALY

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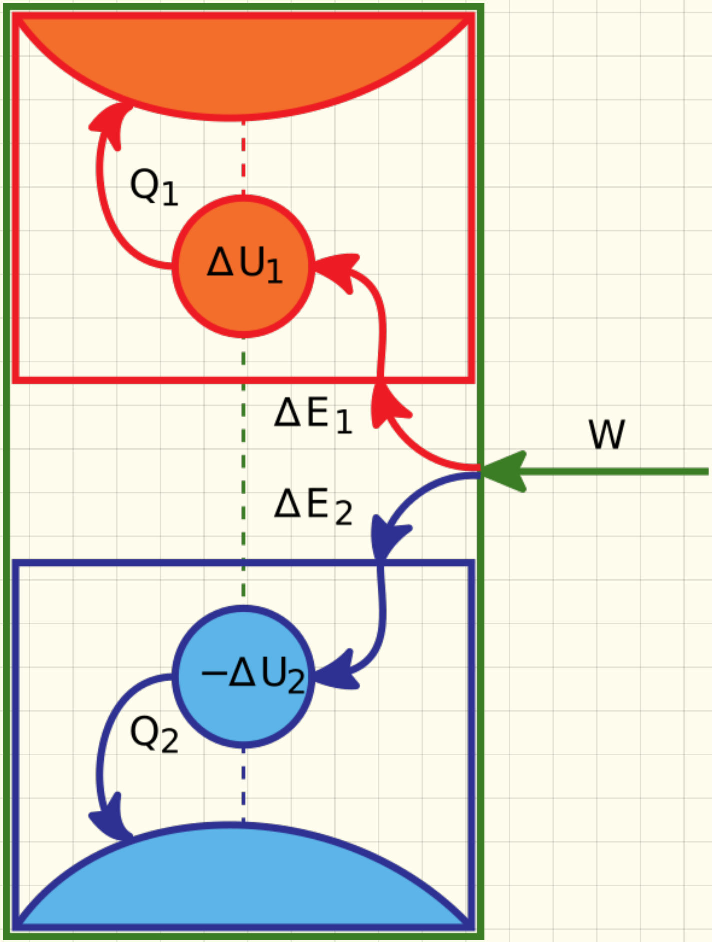
OCT. 26-30 2015 6th Course of the School of statistical physics
"NEW HORIZONS IN NONEQUILIBRIUM THERMODYNAMICS"

https://sites.google.com/site/newhorizonsnetd2015/home

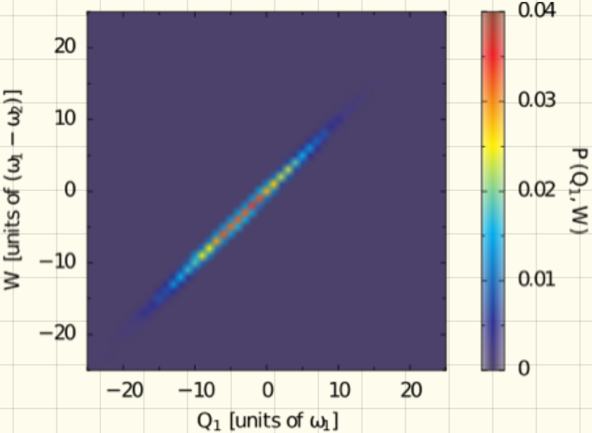
MAY 8-13 2016
"4th COST MP1209 QUANTUM THERMODYNAMICS CONFERENCE"

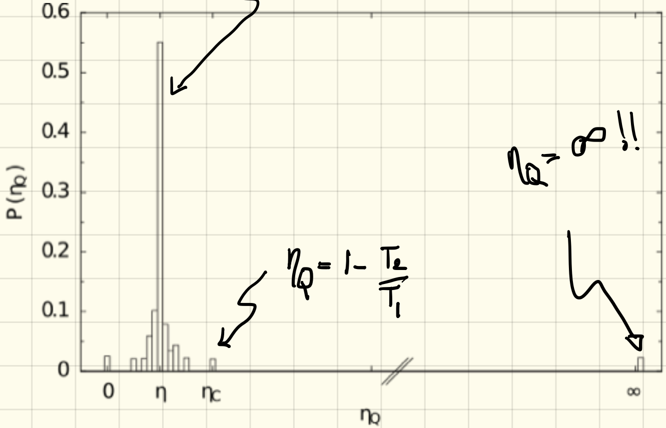


Q1
ΔU1
ΔE 1
W
ΔE 2
−ΔU2
Q2

RESULTS

$$\eta_Q = 1 - \frac{\omega_2}{\omega_1}$$





$$\eta_Q = \infty \,!!$$

$$\eta_Q = 1 - \frac{T_2}{T_1}$$

$$Q_1 = \Delta E_1 - \Delta U_1$$

into Reservoir

into system

$$\eta_Q = \frac{Q_1}{W} \qquad \langle \eta_Q \rangle = \infty \,!!$$

$$\frac{\langle Q_1 \rangle}{\langle W \rangle} \longrightarrow 1 - \frac{W_2}{W_1}$$