Controlling and Measuring Heat Transport in Ion Traps

April 22nd 2015

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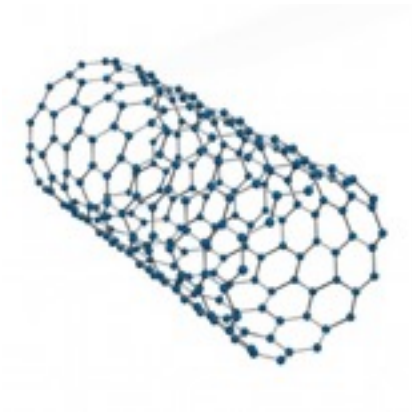
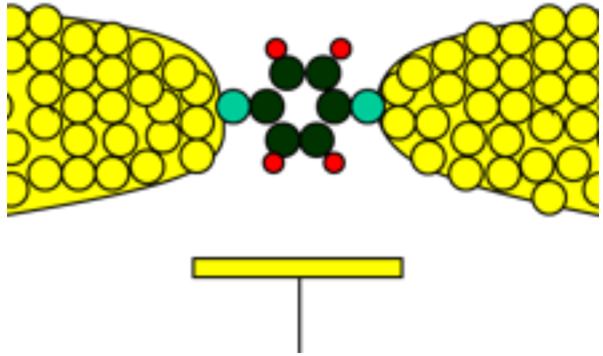
Stiftung / Foundation

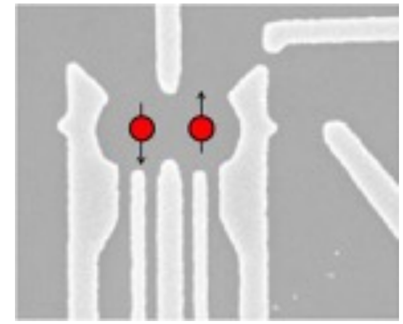
Heat Transport on the Nanoscale



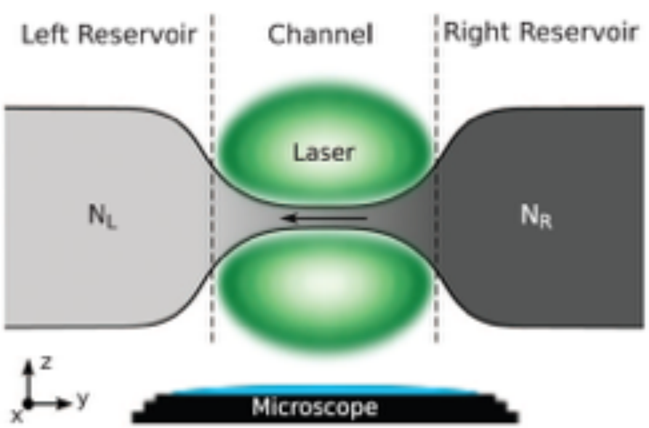
Carbon Nanotubes



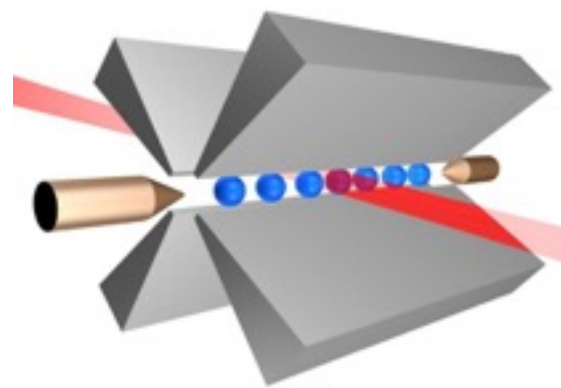
Molecular Nanojunctions



Quantum Dots
& Tunnel Junctions

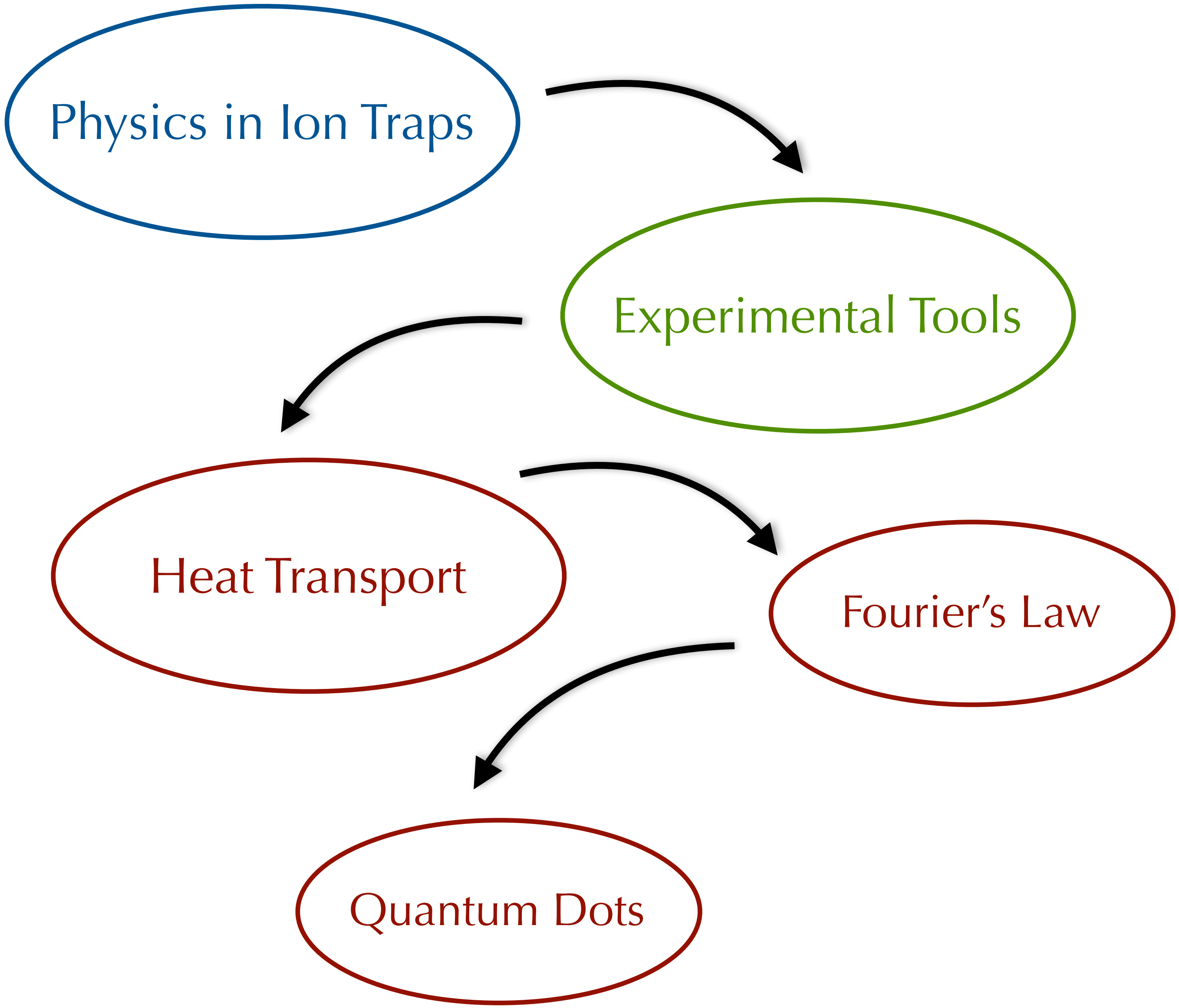


Ultracold Atomic Gases

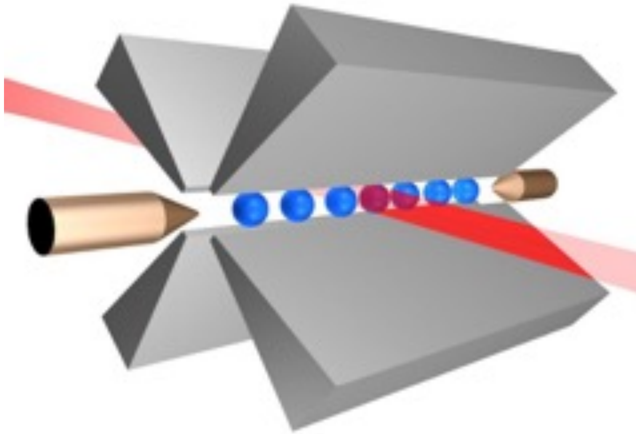


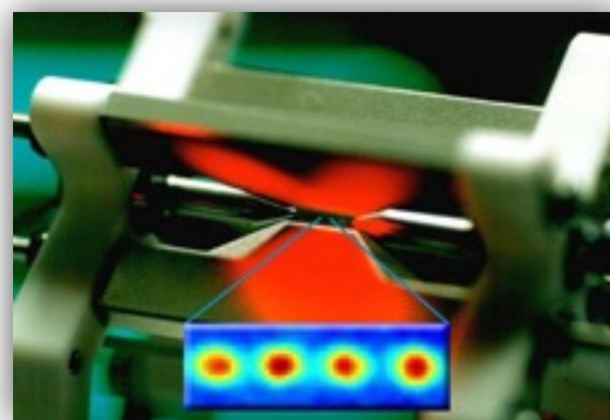
Ion Crystals

Overview


Physics in Ion Traps
Experimental Tools
Heat Transport
Fourier's Law
Quantum Dots

Trapped Ion Set-up



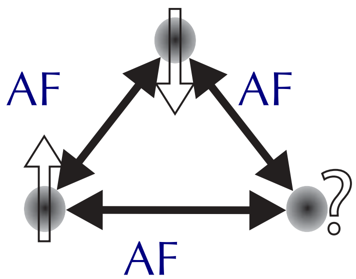


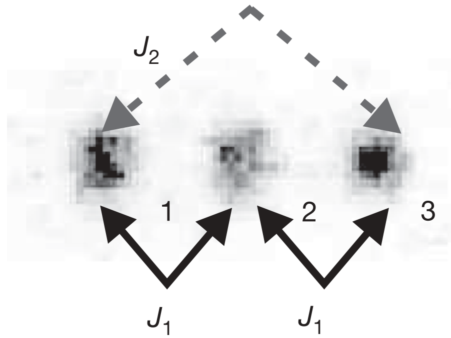
- Singly-charged ions (Be^+, Ca^+, Mg^+, …)
- Coulomb interaction between ions
- Single ions & ion crystals
- Control over motional & internal states (QIP)

H.C. Nägerl, W. Bechter, J. Eschner, F. Schmidt-Kaler & R. Blatt, Appl. Phys. B **66**, 603 (1998)
J. Eschner, G. Morigi, F. Schmidt-Kaler & R. Blatt, J. Opt. Soc. Am. B **20**, 1003 (2003)

Examples – Experiments

Observing Frustration in Spin Systems



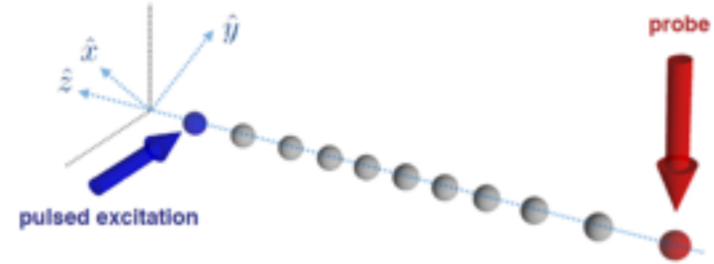


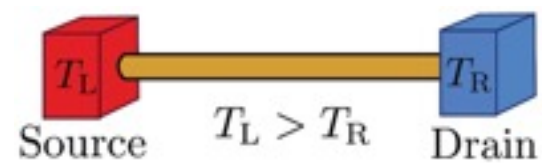
$$H = \sum_{i<j} J_{i,j}\sigma_x^{(i)}\sigma_x^{(j)} + B_y \sum_i \sigma_y^{(i)}$$

- Internal states = effective spins
- Interactions via collective modes
- Determine ground states

K. Kim et al., Nature **465**, 590 (2010)
R. Islam et al., Science **340**, 583 (2013)
A. Bermudez et al., New J. Phys. **14**, 093042 (2012)

Experiments on Heat Transport

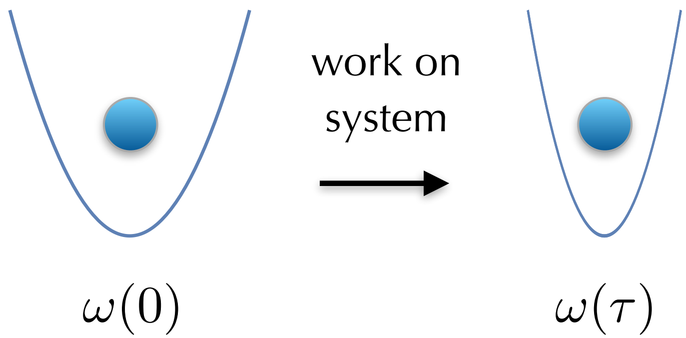




M. Ramm et al., New J. Phys. **16**, 063062 (2014)
A. Bermudez et al., PRL **111**, 040601 (2013)

Examples – Proposals

Verifying the Quantum Jarzynski Equality

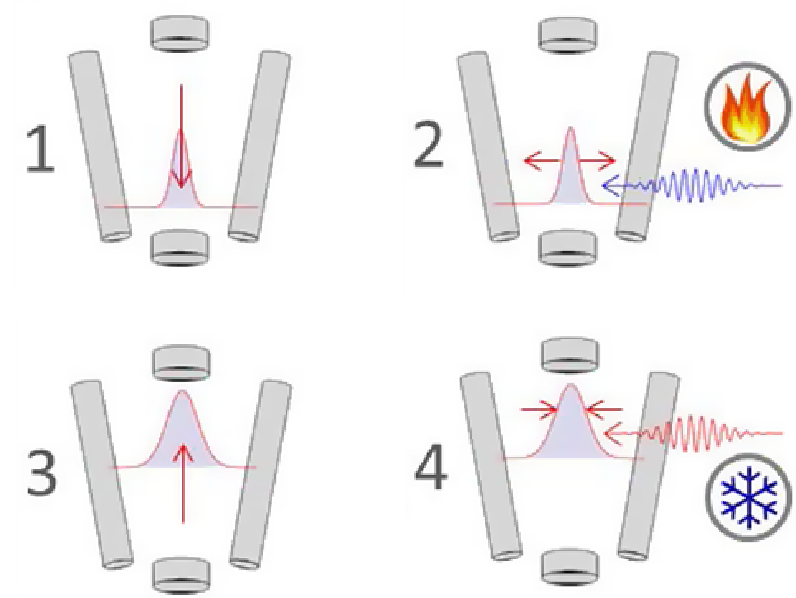


$$\Delta F = -kT \ln \langle \mathrm{e}^{-W/kT} \rangle$$

$$\langle \mathrm{e}^{-W/kT} \rangle = \int \mathrm{d}W p(W) \mathrm{e}^{-W/kT}$$

- Control over initial & final states
- Resolve single Fock states
- Determine work done on system

Single Ion Heat Engine (Otto Cycle)



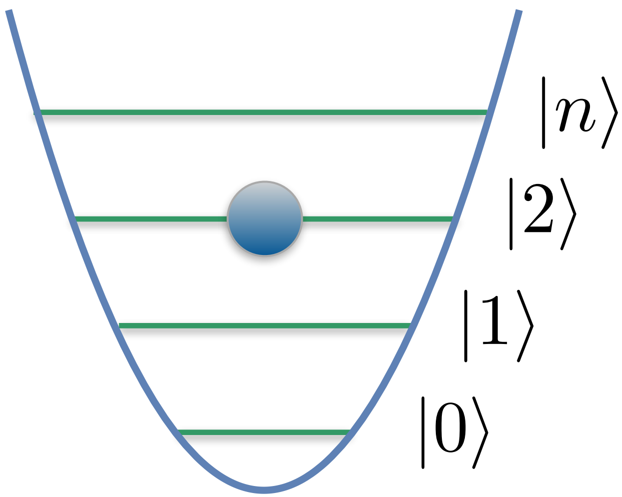
- Control over trapping
- Heating and cooling of single ions
- Generate motion of axial mode

G. Huber, F. Schmidt-Kaler, S. Deffner & E. Lutz, PRL **101,** 070403 (2008)
R. Dorner, S.R. Clark, L. Heaney, R. Fazio, J. Goold & V. Vedral, PRL **110**, 230601 (2013)
O. Abah, ..., F. Schmdt-Kaler, ... & E. Lutz, PRL **109**, 203006 (2012)

What are the basic tools?

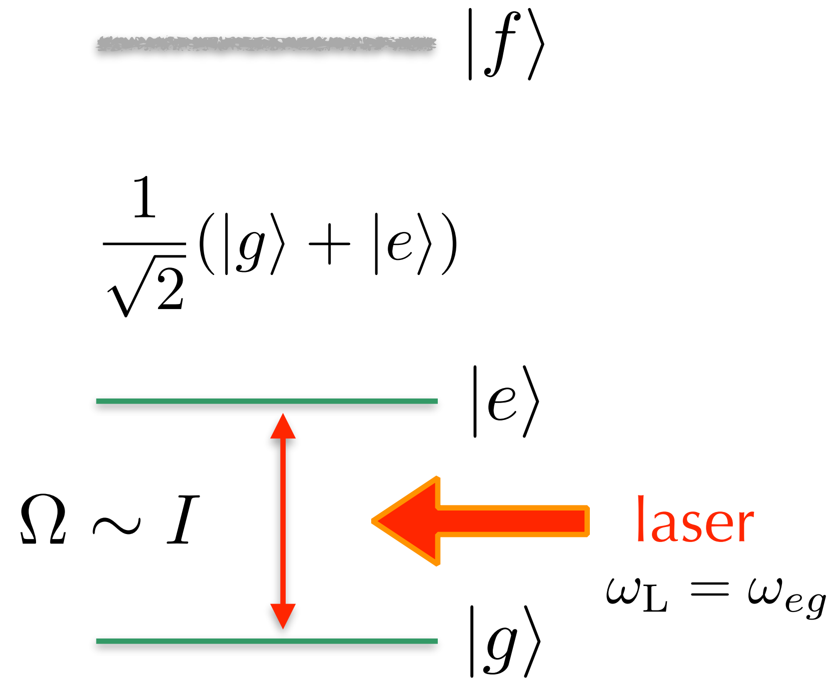
Motional and Internal States

- Motional and internal states are decoupled
- Manipulate internal states via laser coupling



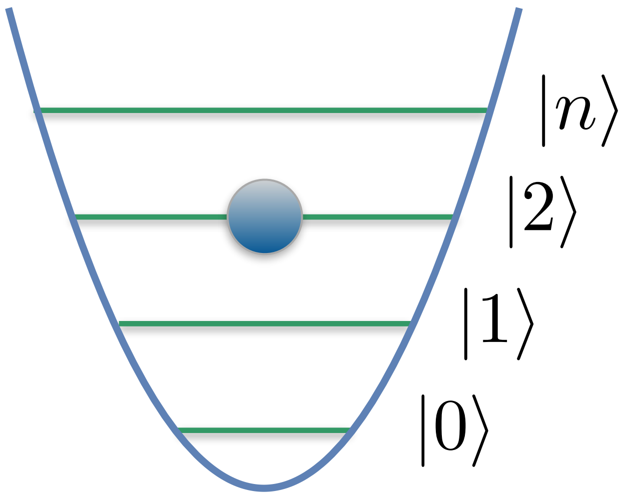
$$V(x) = \frac{1}{2} m\omega^2 x^2$$

motional states
(harmonic trap)



internal states
(electron configuration)

- Resonant driving of transition $|e\rangle$ to $|f\rangle$
- Radiative decay into $|e\rangle$ and $|g\rangle$



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motional states
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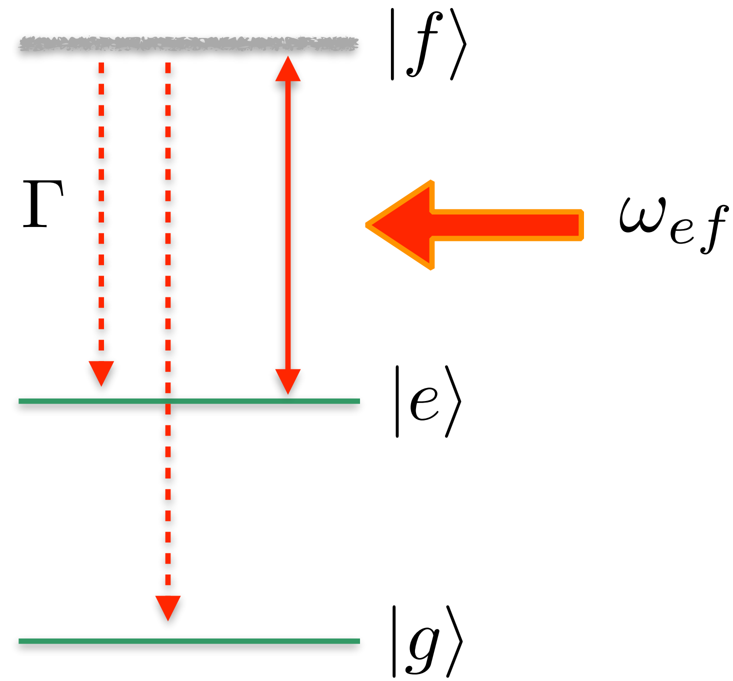
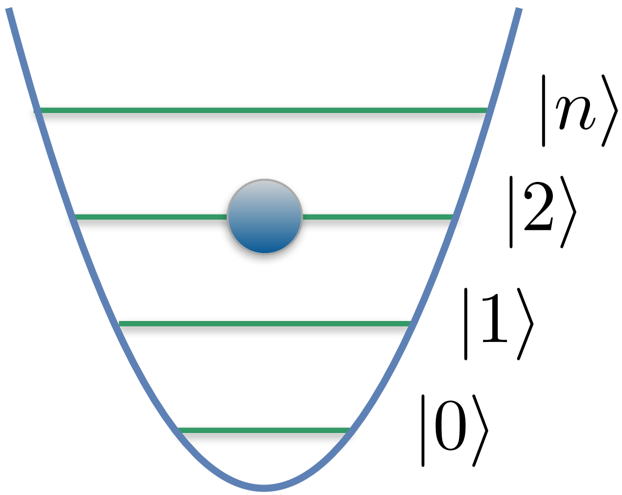


internal states
(electron configuration)

Resonance Fluorescence - Read Out

- Resonant driving of transition $|e\rangle$ to $|f\rangle$
- See that level $|e\rangle$ is populated



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motional states
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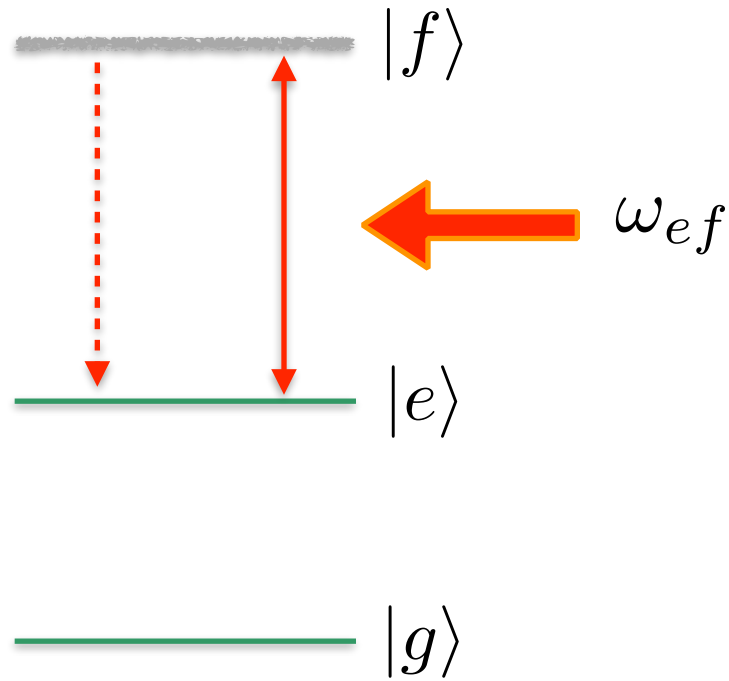

internal states
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Resonance Fluorescence - Read Out

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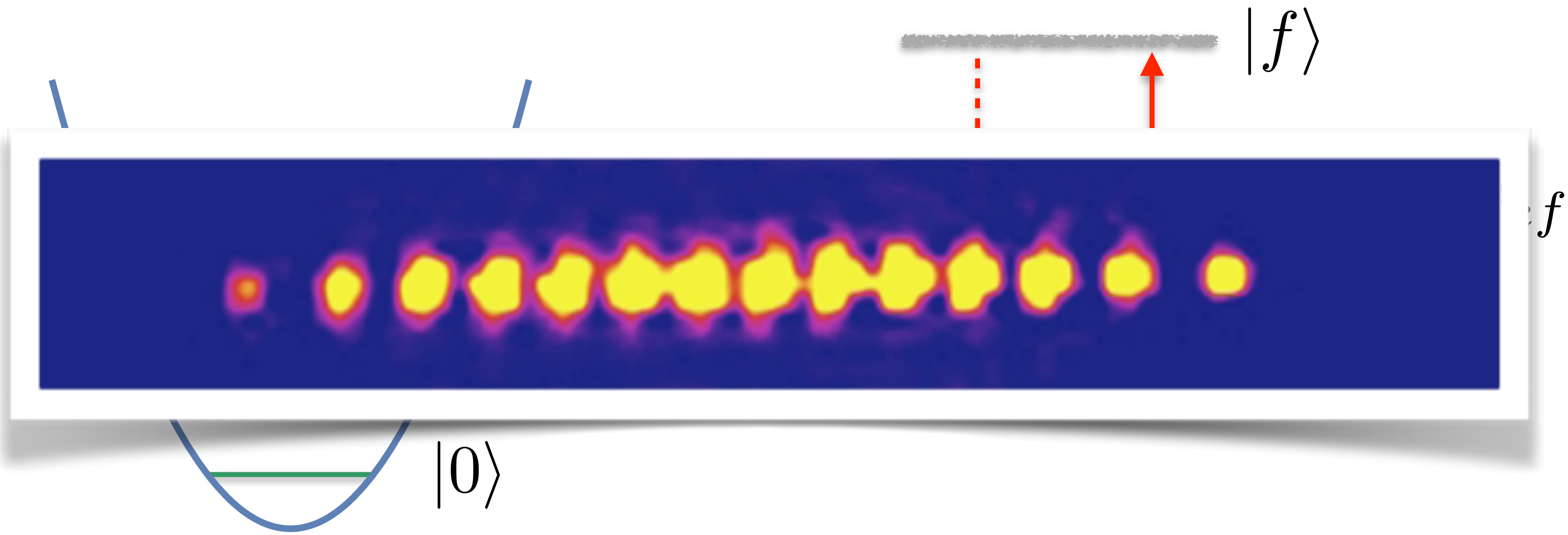


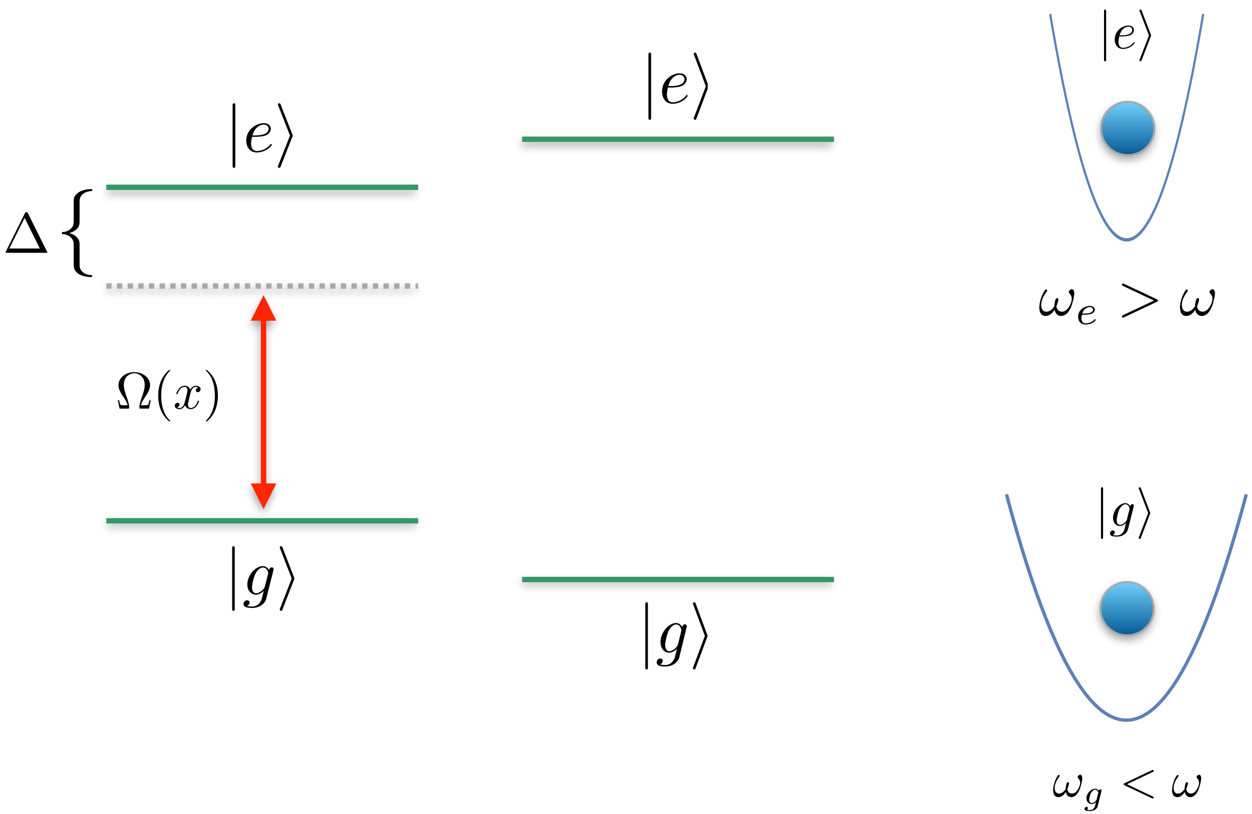
$$V(x) = \frac{1}{2} m \omega^2 x^2$$

motional states
(harmonic trap)

internal states
(electron configuration)

State-dependent Potential

- AC-Stark shift from far-detuned driving
- Position-dependent intensity $\Omega(x)$

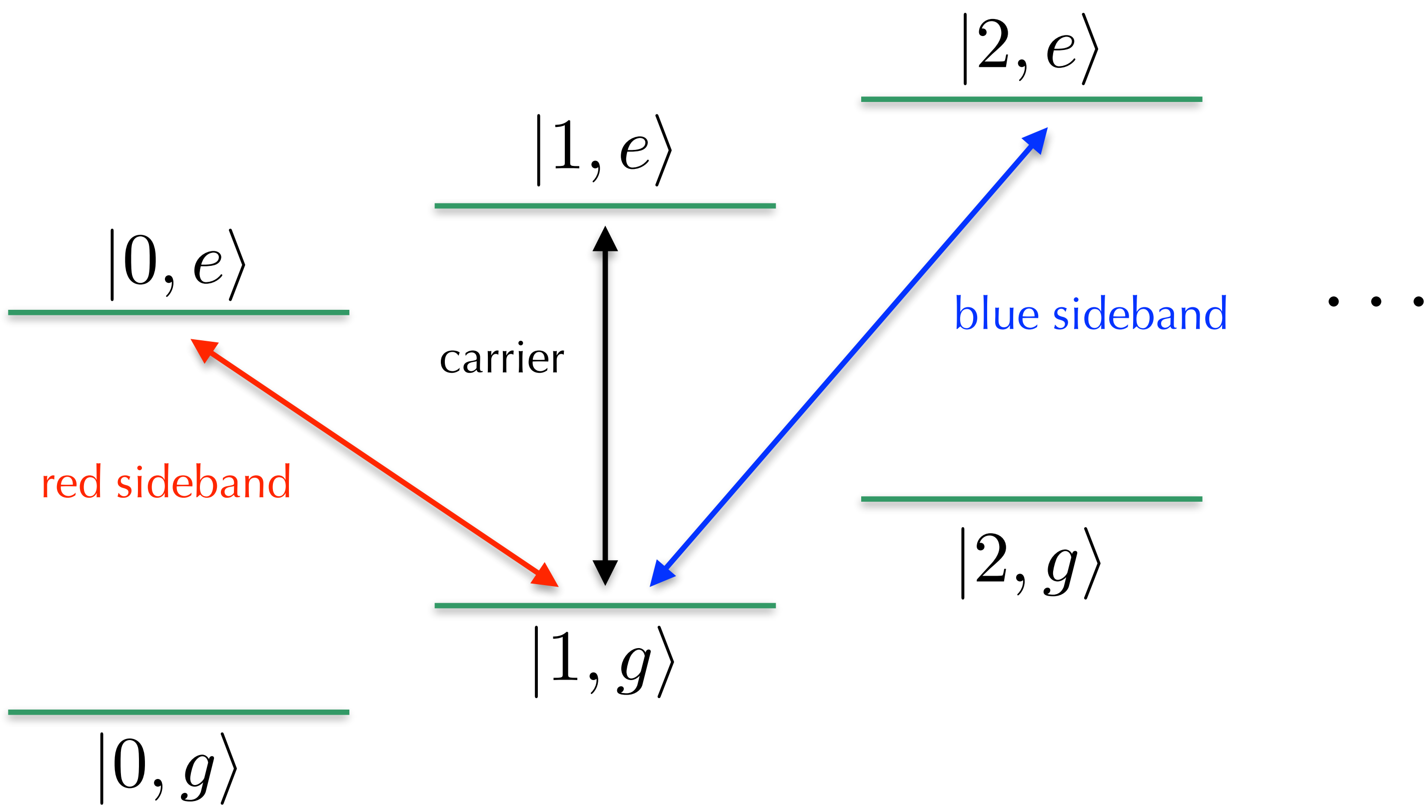


$$V_e(x) = \frac{\Omega^2(x)}{2\Delta}$$

$$V_g(x) = -\frac{\Omega^2(x)}{2\Delta}$$

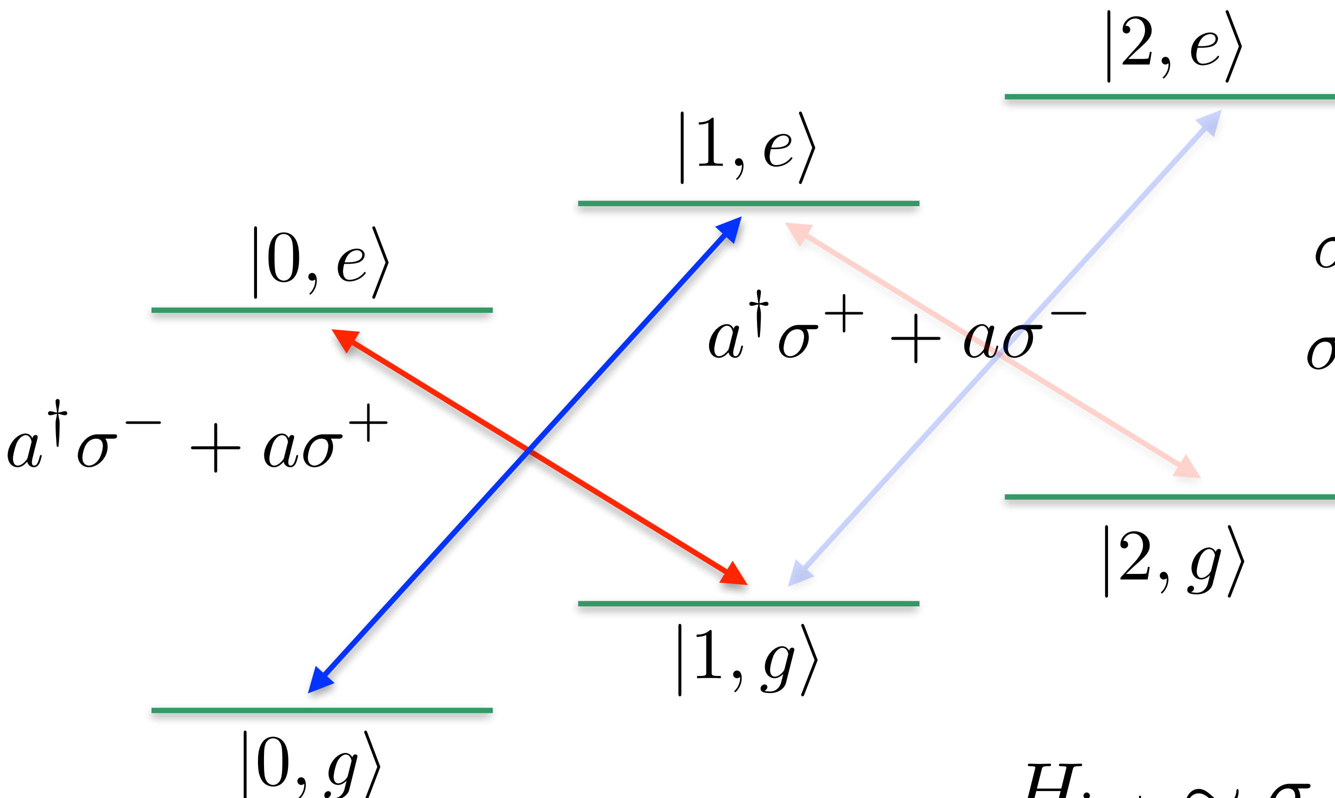
Couple Motional and Internal States

- Drive transitions that change HO state
- Requires narrow HO states



Spin-Mode Coupling

- Internal states act as spins $|e\rangle \equiv |\uparrow\rangle \quad |g\rangle \equiv |\downarrow\rangle$
- Spin-spin interactions via common mode)

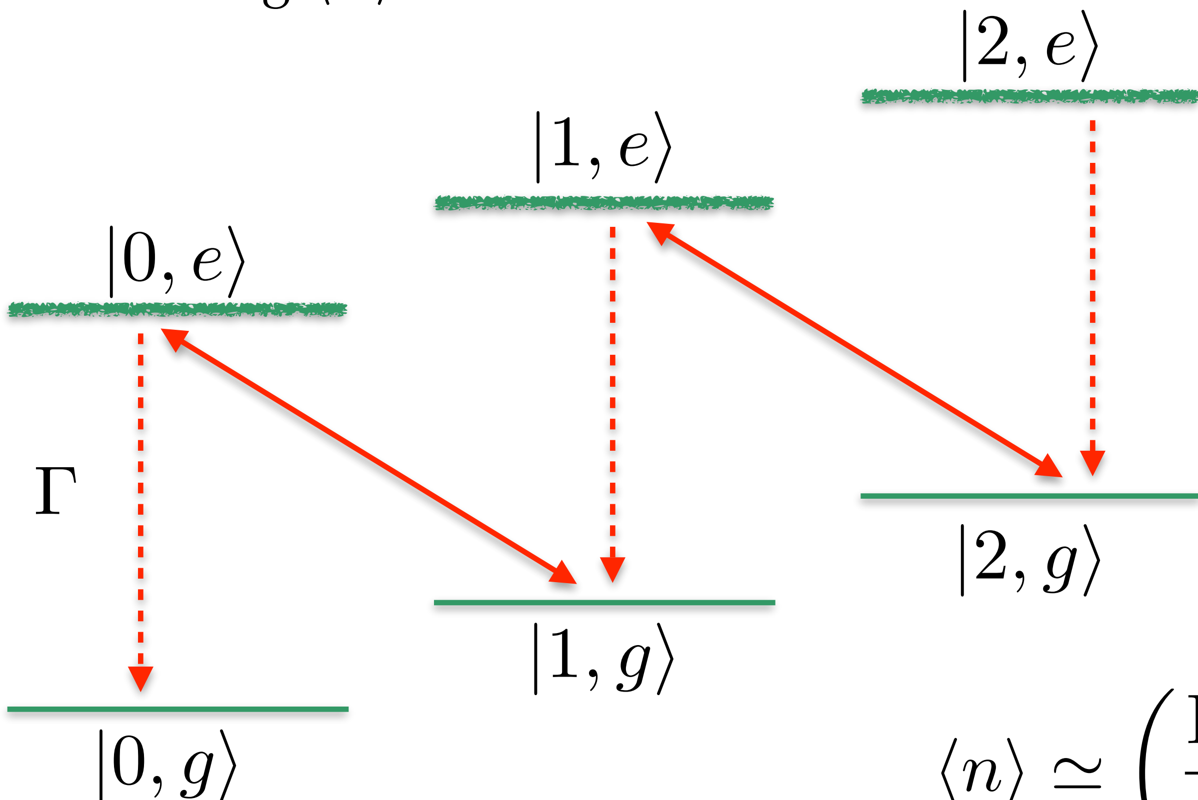


$$\sigma^+ = |e\rangle\langle g|$$
$$\sigma^- = |g\rangle\langle e|$$

$$H_{\mathrm{int}} \sim \sigma_x(a + a^\dagger)$$

Sideband Cooling

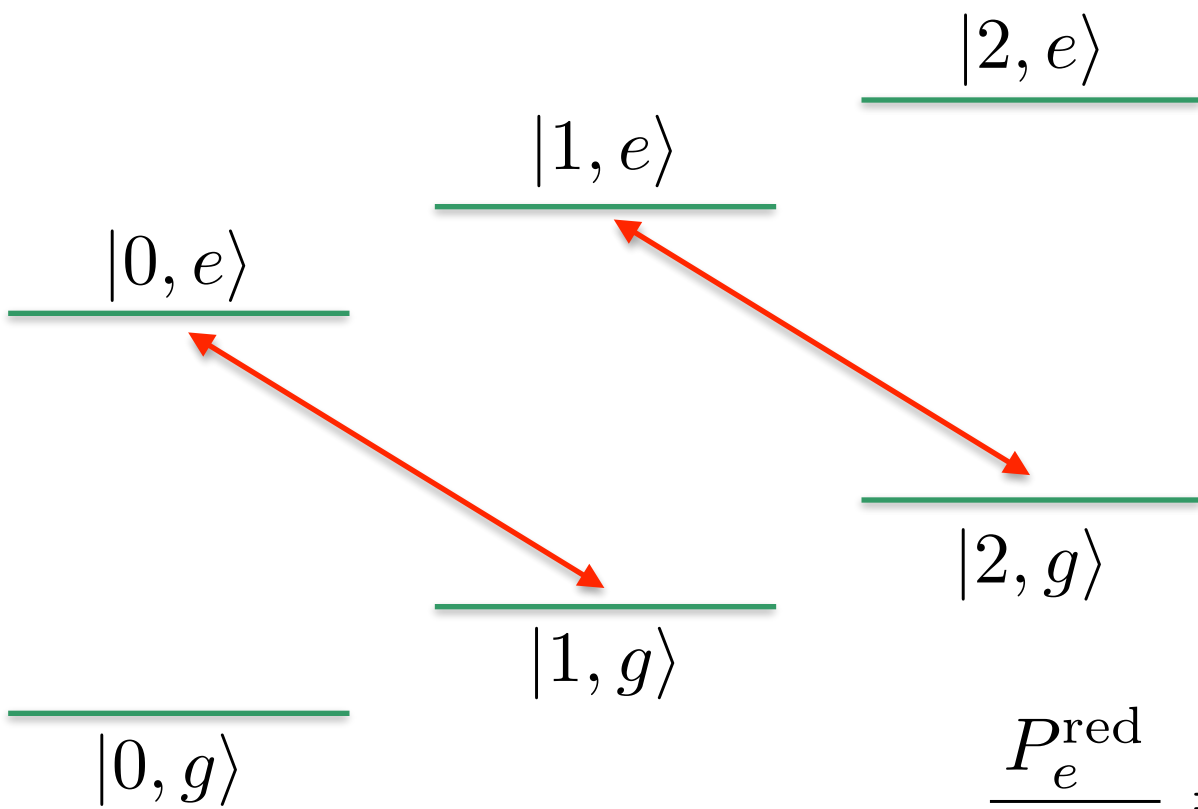
- Combine coherent driving & spontaneous decay
- Control temperature down to $\langle n \rangle$ ~ 0.05
- Doppler cooling $\langle n \rangle$ ~ 3 - 10



$$\langle n \rangle \simeq \left(\frac{\Gamma}{\omega}\right)^2 \ll 1$$

Temperature Measurement

- Thermal population of motional state $\langle n \rangle$
- Start in $|g\rangle$ and observe population in $|e\rangle$ after fixed time



$$\frac{P_e^{\mathrm{red}}}{P_e^{\mathrm{blue}}} = \frac{\langle n \rangle}{1 + \langle n \rangle}$$

Temperature Measurement

- Thermal population of motional state $\langle n \rangle$
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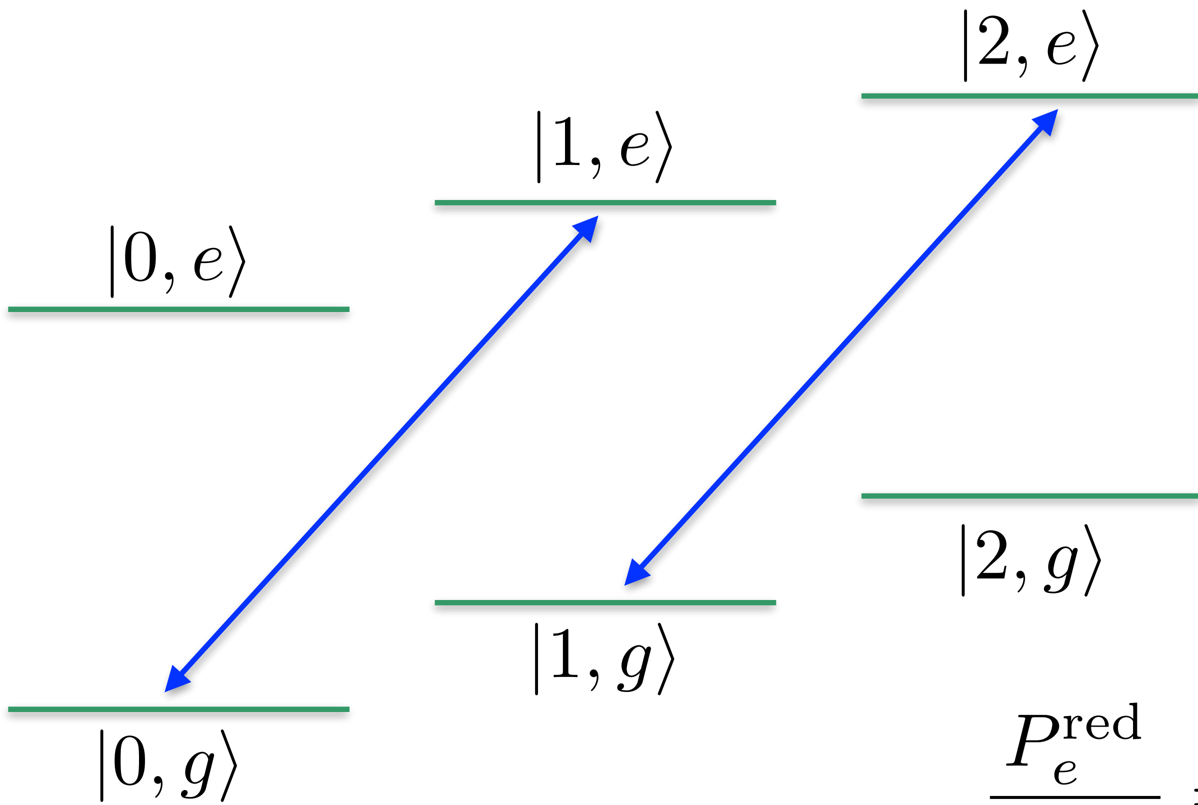

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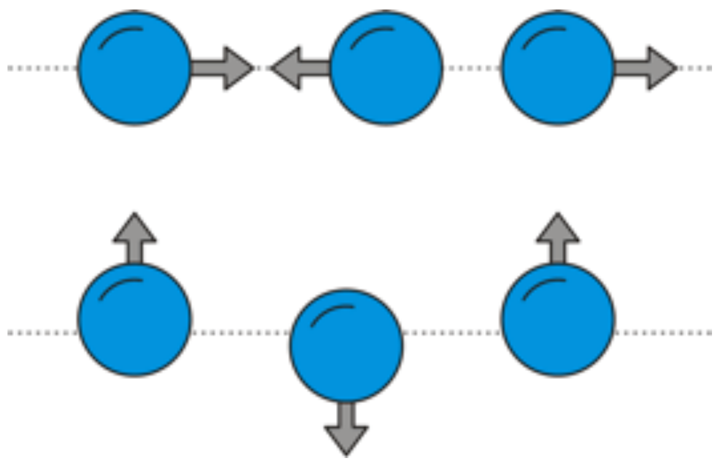
Hamiltonian for motional ion states

$$H = \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} \mathbf{r}_i \boldsymbol{\omega}^2 \mathbf{r}_i \right) + \frac{e^2}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$



Axial modes

Radial modes

harmonic oscillator modes ~ collective modes.

Fourier's Law

Fourier's Law

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pendant l'unité de temps, passe à travers une étendue égale à l'unité de surface prise sur une section parallèle à la base.

Ainsi l'état thermométrique d'un solide compris entre deux bases parallèles infinies dont la distance perpendiculaire est e, et qui sont maintenues à des températures fixes a et b, est représenté par les deux équations:

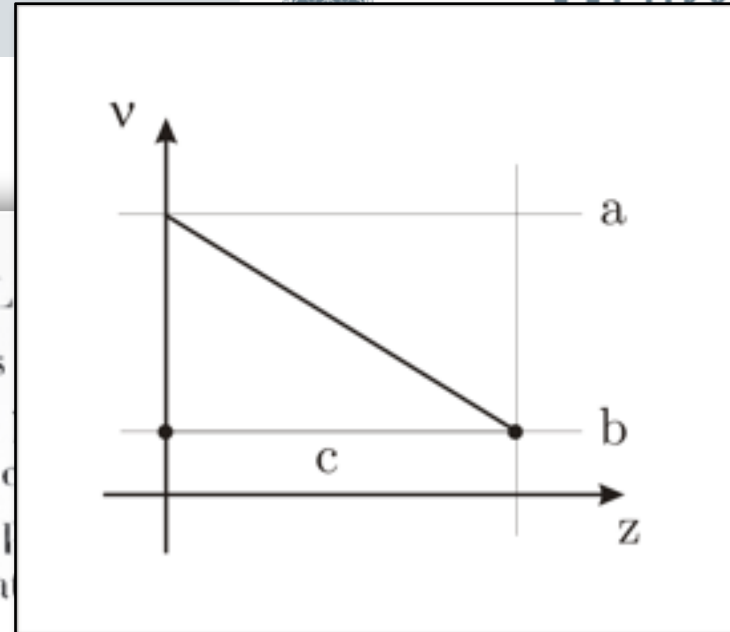
$$v = a + \frac{b-a}{e} z, \text{ et } F = K \frac{a-b}{e} \text{ ou } F = -K \frac{dv}{dz}.$$

La première de ces équations exprime la loi suivant laquelle les températures décroissent depuis la base inférieure jusqu'à la face opposée; la seconde fait connaître la quantité de chaleur qui traverse, pendant un temps donné, une partie déterminée d'une section parallèle à la base.

69.

J. Fourier, Théorie Analytique de la Chaleur (1822)

Fourier's Law



pendant l'unité de temps, passe à travers
à l'unité de surface prise sur une section

Ainsi l'état thermométrique d'un solide c
bases parallèles infinies dont la distance p
e, et qui sont maintenues à des températ
est représenté par les deux équations:

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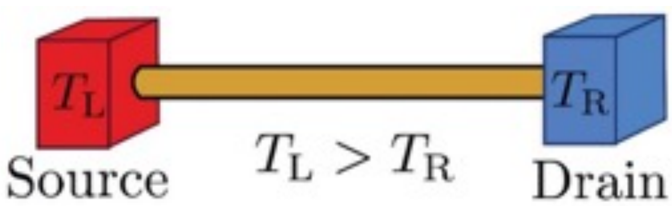
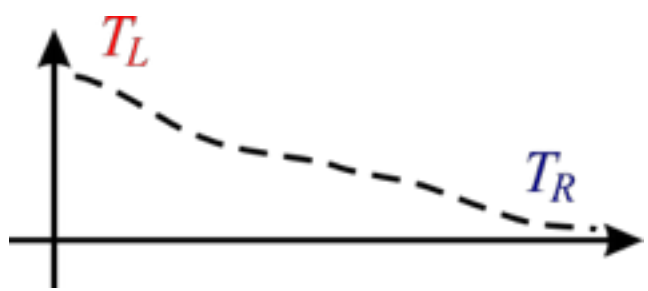
69.

J. Fourier, Théorie Analytique de la Chaleur (1822)

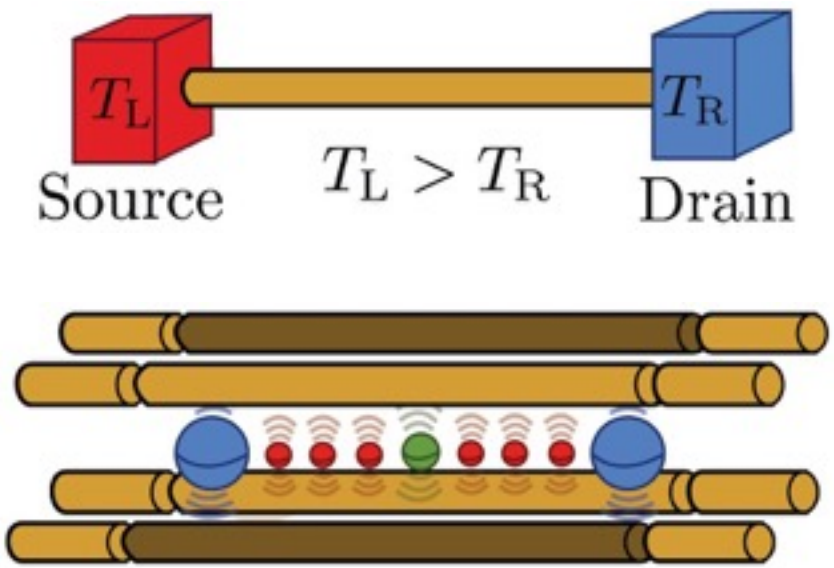
- How does Fourier's law emerge from microscopic laws?
- When do temperature gradients occur?
- Observe Fourier's law on nanoscales?





Z. Rieder, J. L. Lebowitz & E. Lieb, J. Math. Phys. **8**, 1073 (1967)
M. Michel, G. Mahler & J. Gemmer PRL **95**, 180602 (2005)
A. Asadian, D. Manzano, M. Tiersch & H. J. Briegel, Phys. Rev. E **87**, 012109 (2013)

Trapped-Ion Crystal Toolbox



Functionalities of ions

- Bulk ions
- Heat reservoir ions
- Multi purpose ions

- Full control over internal states
- Separation of time scales
- Different atomic species

Vibron Hopping Model

Hamiltonian for motional ion states

$$H = \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} \mathbf{r}_i \boldsymbol{\omega}^2 \mathbf{r}_i \right) + \frac{e^2}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Tight-binding model for vibron hopping

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i \neq j} \left(J_{ij} a_i^\dagger a_j + \text{h.c.} \right)$$

$$\omega \sim 1\text{MHz}$$
$$J \sim 10\text{kHz}$$

- Small radial oscillations above ground state (vibrons)
- Coupling from dipole-dipole interaction $J \sim 1/d^3$
- Heat transport by vibron hopping

chain of weakly coupled harmonic oscillators

D. Porras & J. I. Cirac, PRL **93**, 263602 (2004)

Heat Reservoirs

Tight-binding model for vibron hopping

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i \neq j} \left(J_{ij} a_i^\dagger a_j + \text{h.c.} \right)$$

$\bar{n}_L$  $\bar{n}_R$

Continuous heating/cooling of edge ions

$$\mathcal{L}_\alpha \rho_\alpha = \frac{\gamma_\alpha}{2}(\bar{n}_\alpha + 1)(2a\rho_\alpha a^\dagger - \{a^\dagger a, \rho_\alpha\}) + \frac{\gamma_\alpha}{2}\bar{n}_\alpha(a \leftrightarrow a^\dagger)$$

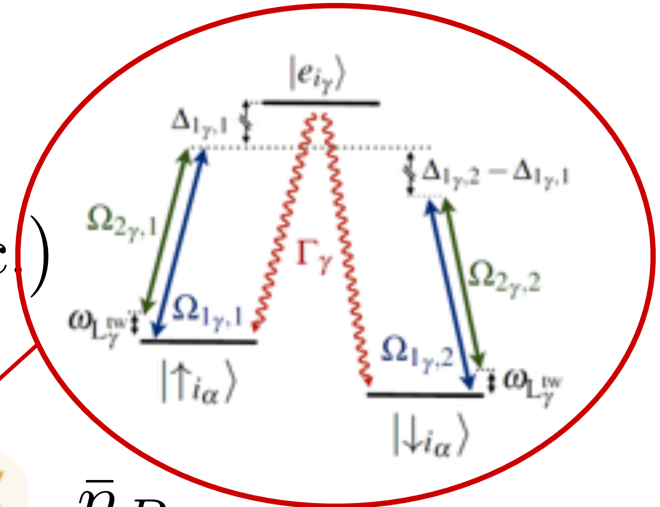
$$\alpha = \{L, R\}$$

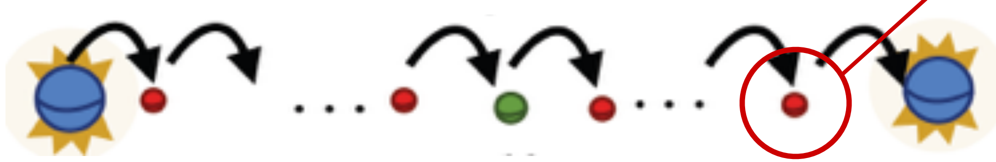
- Effective cooling rate γ_α
- Reservoirs at constant temperature $\bar{n}_\alpha$
- Cooling much faster than hopping $\gamma_\alpha \gg J_{ij}$

Spin–Vibron Coupling

Tight-binding model for vibron hopping

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i \neq j} \left(J_{ij} a_i^\dagger a_j + \text{h.c.} \right)$$



$\bar{n}_L$  $\bar{n}_R$

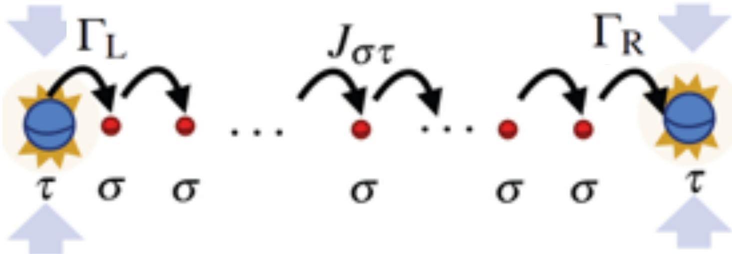
Couple internal states to vibrons

$$H_i^{\text{SV}} = \frac{1}{2}(A + \Delta\sigma_i^z)\cos(\nu t - \phi) a_i^\dagger a_i$$

(1) $$H_i^{\text{SV}} = \frac{1}{2} A \cos(\nu t) a_i^\dagger a_i$$ ▻ Photon-assisted tunneling

(2) $$H_i^{\text{SV}} = \frac{1}{2} \Delta\sigma_i^z a_i^\dagger a_i$$ ▻ Probing & Disorder

Ballistic Transport



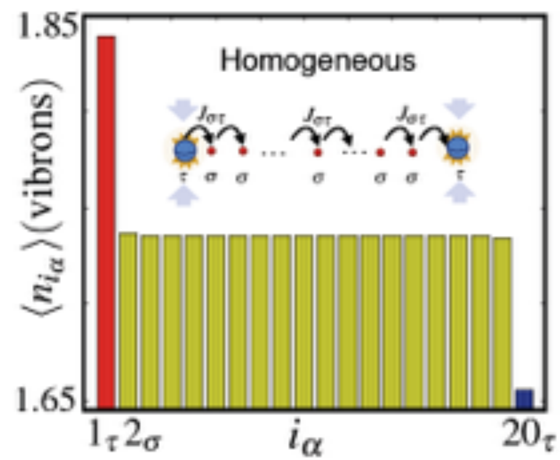
Assume $\gamma_\alpha \gg J_{ij}$ and project dynamics onto state $\rho_L^{\mathrm{th}} \otimes \rho_{\mathrm{bulk}} \otimes \rho_R^{\mathrm{th}}$

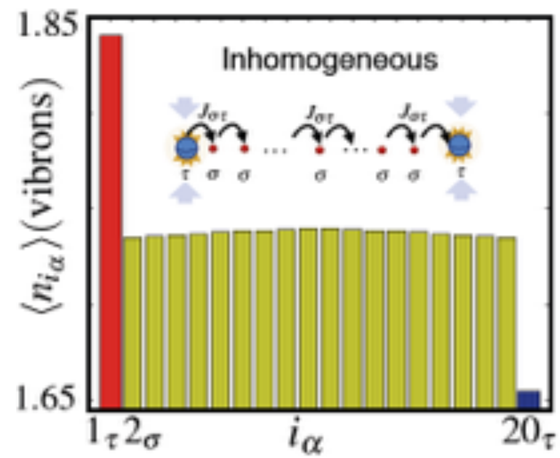
$$\langle n_i \rangle_{\mathrm{ss}} = \frac{\Gamma_L \bar{n}_L + \Gamma_R \bar{n}_R}{\Gamma_L + \Gamma_R}$$

$$\langle I^{\mathrm{vib}} \rangle_{\mathrm{ss}} = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} (\bar{n}_L - \bar{n}_R)$$

- Ballistic transport of vibrons
- Anomalous heat transport

Vibron occupations





Spin-Induced Disorder

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Strong spin-vibron coupling

$$H_i^{\mathrm{SV}} = \frac{1}{2}\Delta\sigma_i^z a_i^\dagger a_i$$

Spins in superposition

$$|+\rangle_i = \frac{1}{\sqrt{2}}(|\uparrow\rangle_i + |\downarrow\rangle_i)$$

Random binary alloy (RBA) model (bosonic)

$$H^{\mathrm{RBA}} = \sum_i \varepsilon_i a_i^\dagger a_i + \sum_{i\neq j}\left(J_{ij}a_i^\dagger a_j + \mathrm{h.c.}\right)$$

Binary diagonal disorder

$\varepsilon_i \in \{\omega_i - \frac{1}{2}\Delta, \omega_i + \frac{1}{2}\Delta\}$

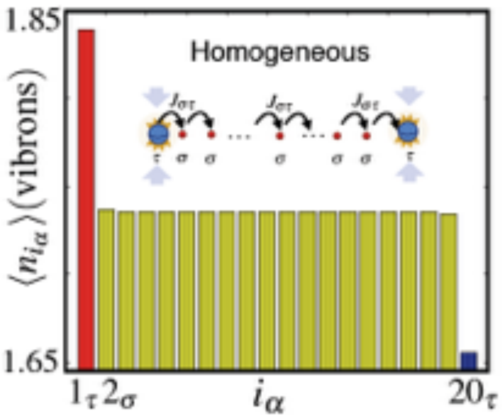
Exploit "quantum parallelism"

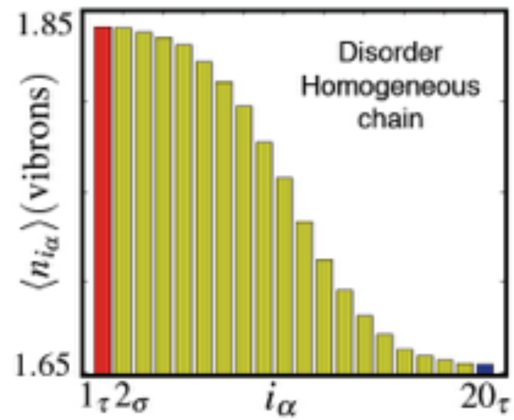
B. Velicky, S. Kirkpatrick & H. Ehrenreich, Phys. Rev. **175**, 747 (1968)

B. Paredes, F. Verstraete & J. I. Cirac, PRL **95,** 140501 (2005)

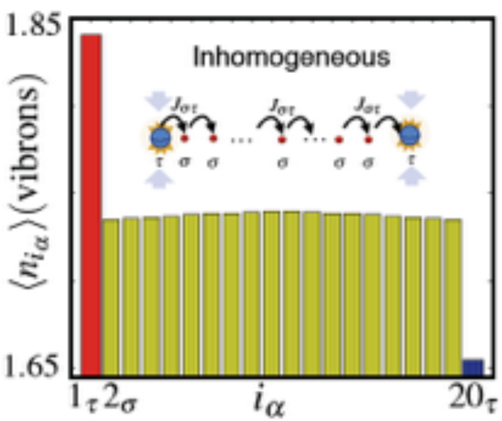
Fourier’s Law

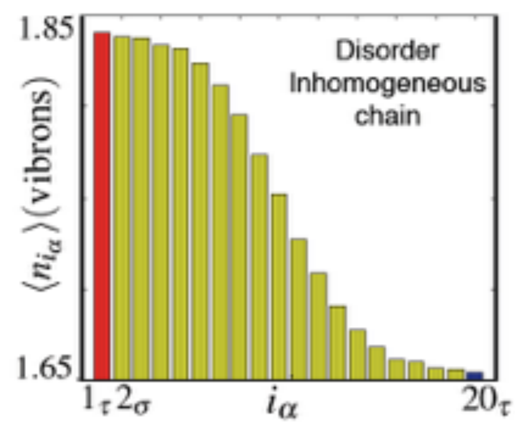
Ballistic | Disorder





Homogeneous



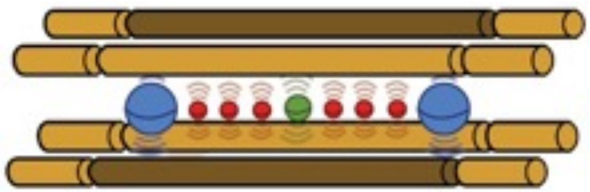


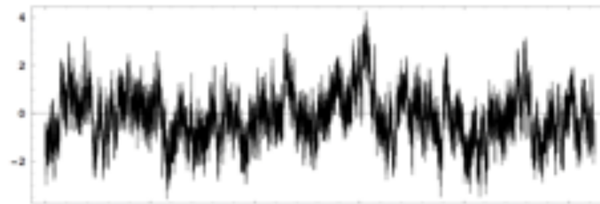
Inhomogeneous

Clear signature of Fourier’s law

Dephasing from Electrodes

Dynamic fluctuations from noisy electrodes





Tight-binding model with dephasing noise

$$H^{\phi} = \sum_i \varepsilon_i(t) a_i^\dagger a_i + \sum_{i \neq j} \left(J_{ij} a_i^\dagger a_j + \mathrm{h.c.} \right)$$

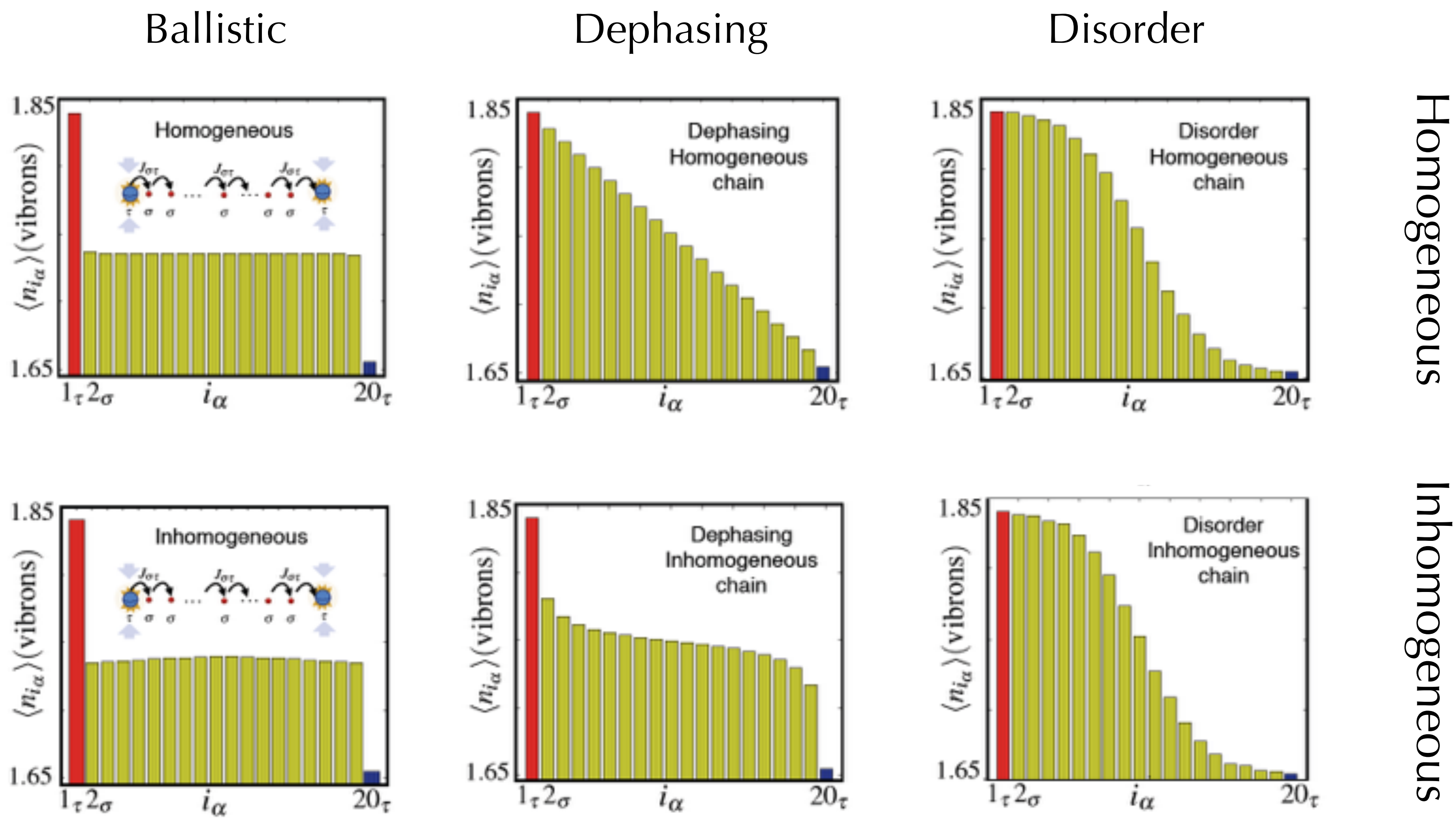
Markovian noise with correlation length ξ_c

$$\varepsilon_i(t) = \omega_i + \delta\omega(t)$$

Description as Lindblad operator

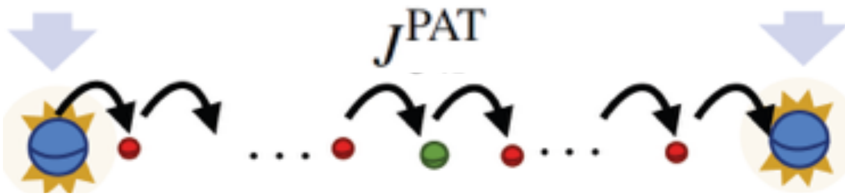
$$\mathcal{L}_\phi \rho = \sum_{i,j} \Gamma_\phi \mathrm{e}^{-d_{ij}/\xi_\mathrm{c}} \left(2 n_i \rho n_j - \{ n_j n_i, \rho \} \right)$$

Fourier's Law



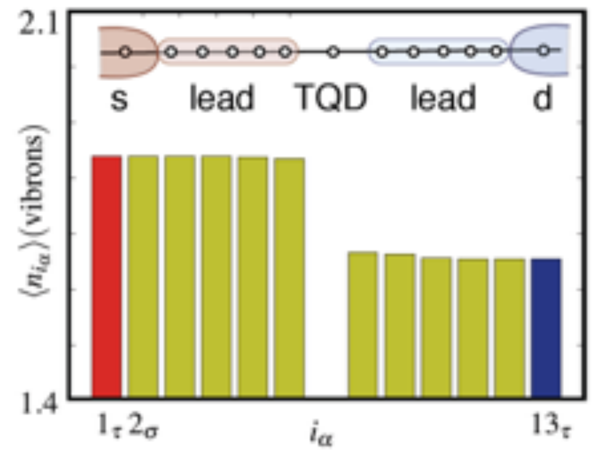
Thermal Quantum Dot

Single site & thermal leads



$$|\downarrow \cdots \downarrow\rangle|\phi\rangle|\uparrow \cdots \uparrow\rangle$$

$$\omega_\downarrow \ll \omega \qquad \omega_\uparrow \gg \omega$$



Photon-assisted tunneling $\Delta = A$

$$H_i^{\mathrm{SV}} = \frac{1}{2}A(1+\sigma_i^z)\cos(\nu t)a_i^\dagger a_i$$

$$H^{\mathrm{PAT}} = \sum_i \left(J_i^{\mathrm{PAT}} a_i^\dagger a_\kappa + \mathrm{h.c}\right)$$

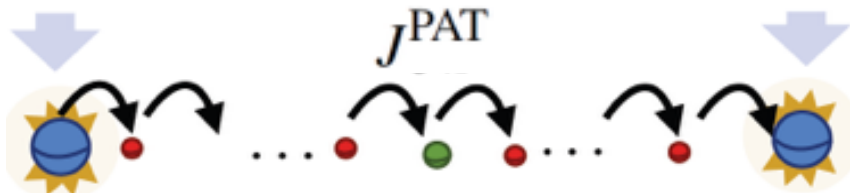
Single-spin heat switch

$$|\phi\rangle = |\uparrow\rangle$$

$$|\phi\rangle = |\downarrow\rangle$$

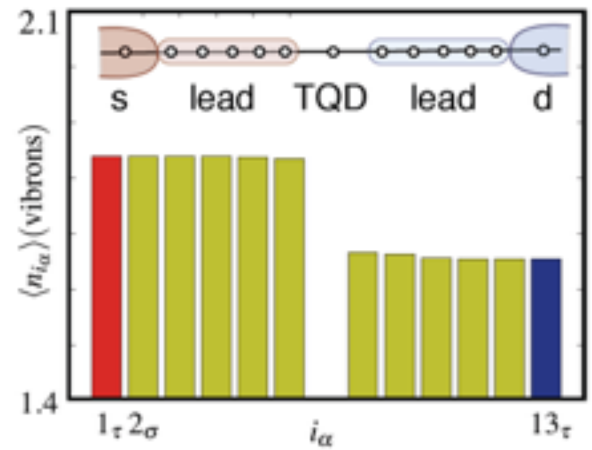
Full control of heat current through TQD

Single site & thermal leads



$$|\downarrow \ \cdots \ \downarrow\rangle|\phi\rangle|\uparrow \ \cdots \ \uparrow\rangle$$

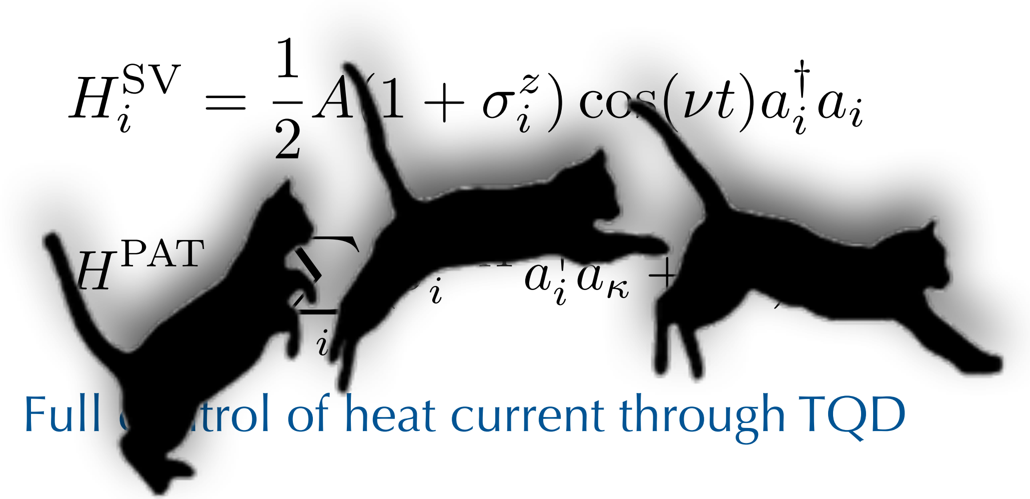
$$\omega_\downarrow \ll \omega \qquad \omega_\uparrow \gg \omega$$



Photon-assisted tunneling $\Delta = A$

$$H_i^{\mathrm{SV}} = \frac{1}{2} A(1 + \sigma_i^z)\cos(\nu t) a_i^\dagger a_i$$

$$H^{\mathrm{PAT}} \ \sum_i \ \ldots_i \ a_i^\dagger a_\kappa + \ldots$$



Full control of heat current through TQD

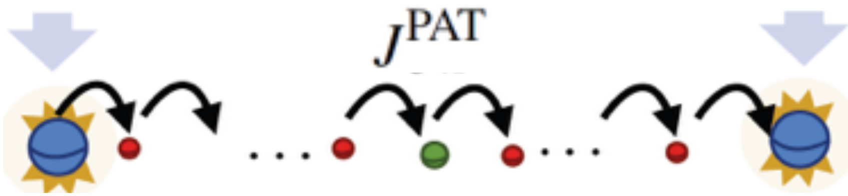
Single-spin heat switch

$$|\phi\rangle = |\uparrow\rangle$$

$$|\phi\rangle = |\downarrow\rangle$$

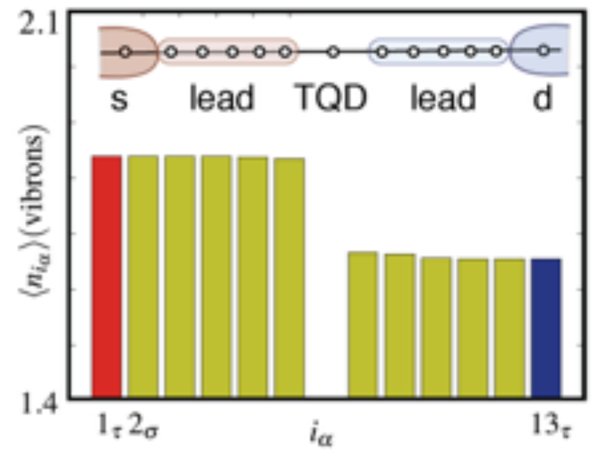
$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

Single site & thermal leads



$$|\downarrow \cdots \downarrow\rangle|\phi\rangle|\uparrow \cdots \uparrow\rangle$$

$$\omega_{\downarrow} \ll \omega \qquad \omega_{\uparrow} \gg \omega$$



Verify fluctuation theorems for bosons (switch on)

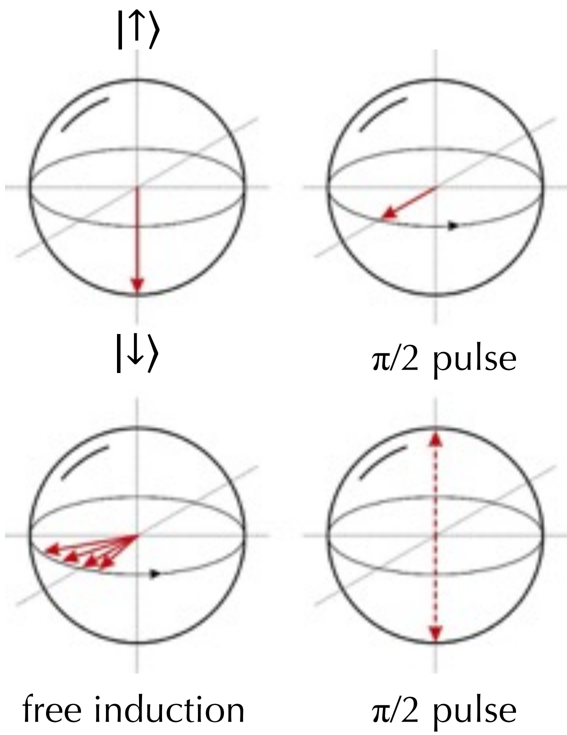
$$\lim_{t\to\infty} \frac{p(N,t)}{p(-N,t)} = \mathrm{e}^{\omega_\kappa N(\beta_D - \beta_S)}$$

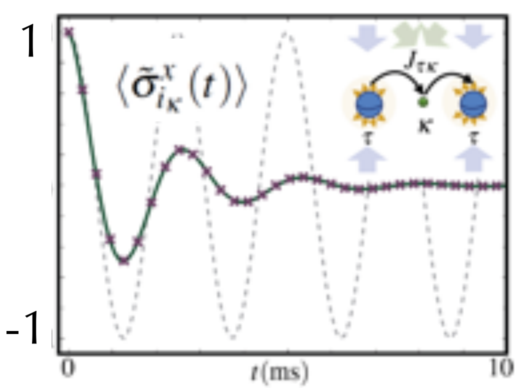
N = number of vibrons

Use Ramsey probe to measure current

M. Esposito, U. Harbola & S. Mukamel, Rev. Mod. Phys. **81**, 1665 (2009)

Non-Invasive Ramsey Probe





Operator couples weakly to spin

$$H_i^{\mathrm{SV}} = \frac{1}{2}\lambda O_i \sigma_i^z$$

Spin evolution

$$\langle \sigma_i^x \rangle = \cos\left(\lambda \langle O_i \rangle_{\mathrm{ss}} t\right) \mathrm{e}^{-\lambda^2 S(0) t}$$

$$S(\omega) = \int_0^\infty \mathrm{d}t \, \langle\langle O_i(t) O_i(0) \rangle\rangle_{\mathrm{ss}} \, \mathrm{e}^{-\mathrm{i}\omega t}$$

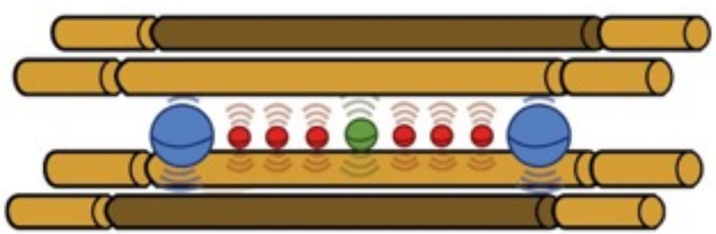
- Oscillations with frequency ∼ ⟨O⟩
- Damping by fluctuations ∼ ⟨δO²⟩

Measure occupations and thermal currents

MB and D. Jaksch, New J. Phys. **8**, 87 (2006)

G. B. Lesovik, F. Hassler & G. Blatter, PRL **96**, 106801 (2006)

Summary



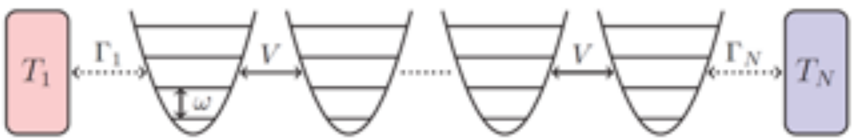
Implement thermal reservoirs

Measure currents & local temperatures

Control & manipulate heat flow

Analytical Results

Local vibron hopping and dephasing



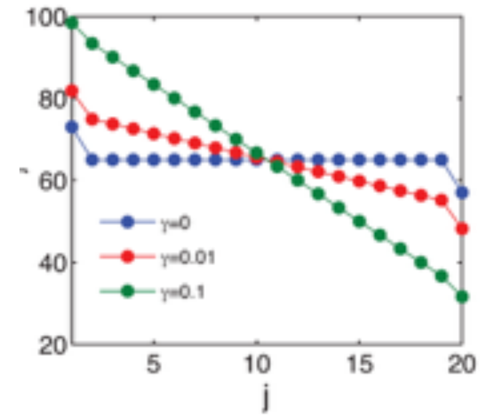
$$H = \sum_{j=1}^{N} \omega a_j^\dagger a_j + \sum_{j=1}^{N-1} V_{j,j+1}(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j)$$

$$\mathcal{L}_{\mathrm{deph}}\rho = \sum_{j=1}^{N} \gamma_j \left(a_j^\dagger a_j \rho a_j^\dagger a_j - \frac{1}{2}\{(a_j^\dagger a_j)^2, \rho\} \right)$$

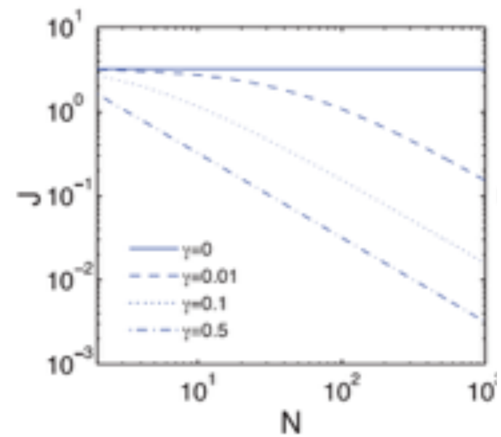
$$J = \frac{4\omega V^2 \Gamma_1 \Gamma_N (n_1 - n_N)}{(4V^2 + \Gamma_1\Gamma_N)(\Gamma_1 + \Gamma_N) + 2(N-1)\gamma\Gamma_1\Gamma_N}$$

Long-range coupling 1/d³ probably not essential

Vibron occupations



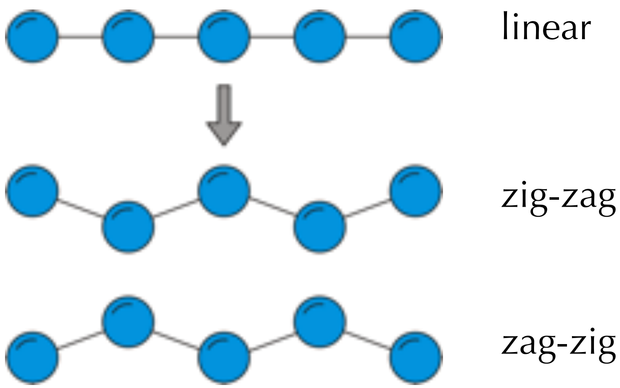
Vibron current

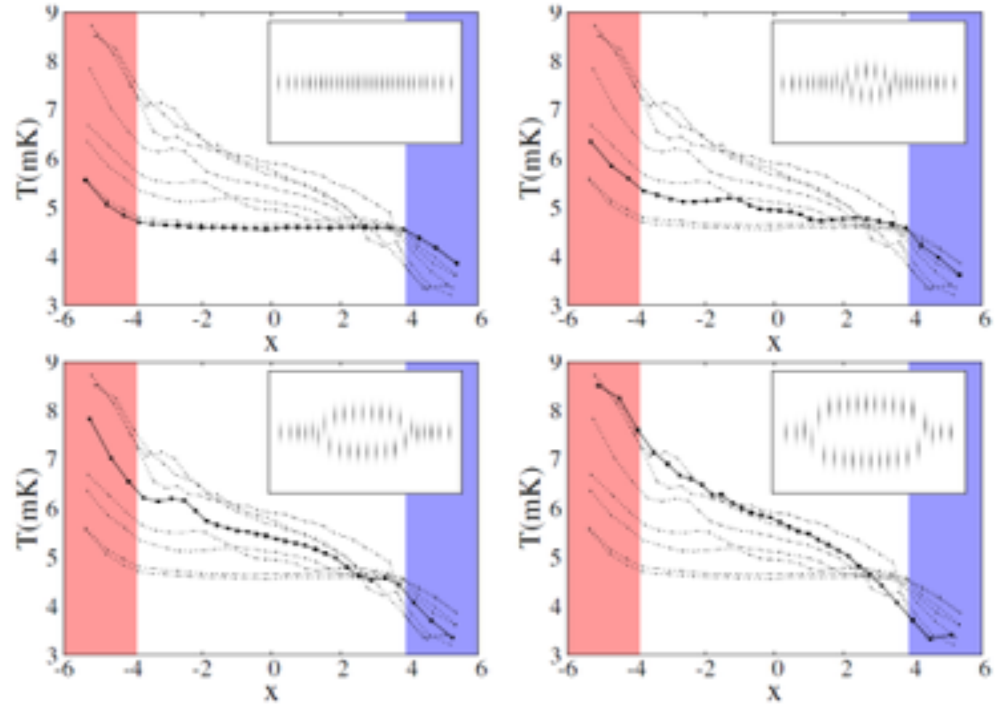


A. Asadian, D. Manzano, M. Tiersch & H. J. Briegel, Phys. Rev. E **87**, 012109 (2013)

Zig-zag Crossover

Reduce radial trapping
for zig-zag crossover

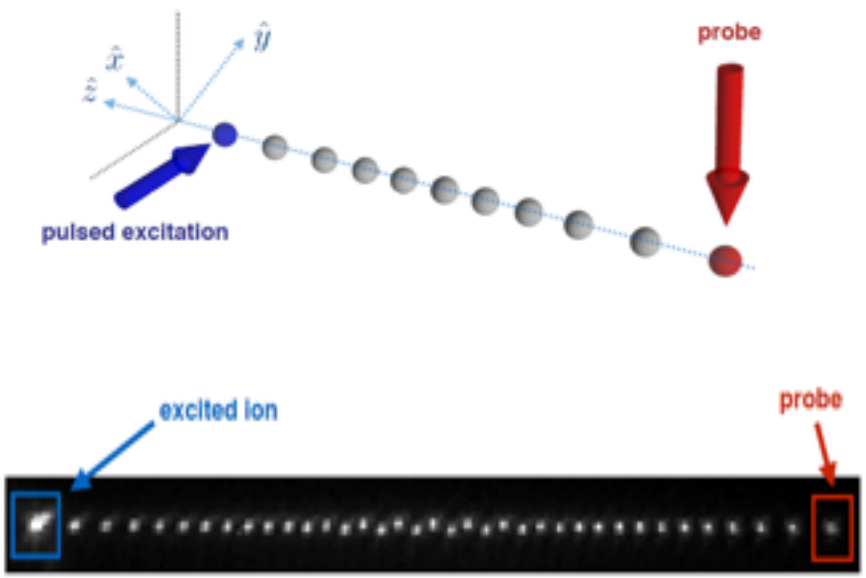


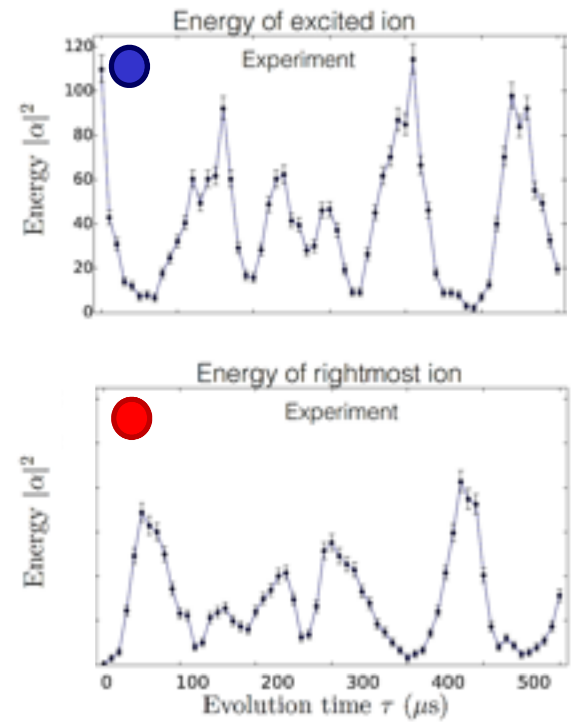


- Anomalous transport for linear chain
- Zig-zag results in temperature gradient and suppressed heat current
- Nonlinearity & coupling of axial and radial modes

K. Pyka et al., Nat. Commun. **4**, 2291 (2013)
A. Ruiz, D. Alonso, M. B. Plenio & A. del Campo, Phys. Rev. B **89**, 214305 (2014)
N. Freitas, E. Martinez & J. P. Paz, Preprint arXiv:1312.6644 (2013)

Experimental Status





- Heat transport in chain with up to 37 ions
- Measure vibron occupation via cooling time
- Coupling ~ 10kHz stronger than cooling rate (need $\gamma_\alpha \gg J_{ij}$)

M. Ramm, T. Pruttivarasin & H. Häffner, New J. Phys. **16**, 063062 (2014)

T. Pruttivarasin, M. Ramm, I. Talukdar, A. Kreuter & H. Häffner, New J. Phys. **13** 075012, (2011)