Work cost of quantum measurements

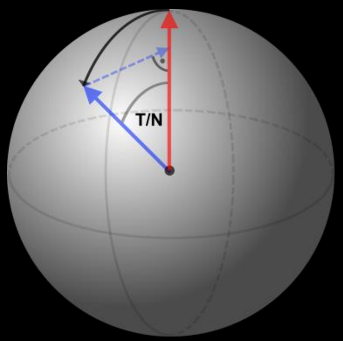
Kais Abdelkhalek, David Reeb, Yoshifumi Nakata

Institute for Theoretical Physics
Leibniz Universität Hannover

24.04.15

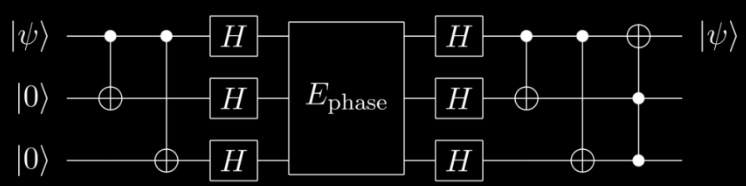
Why are measurements important?

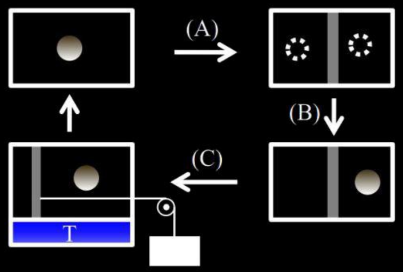
1. Non-unitary state transformation



Zeno stabilisation

Quantum error correction





Szilard engine

2. Feedback:
   action based on
   measurement outcome

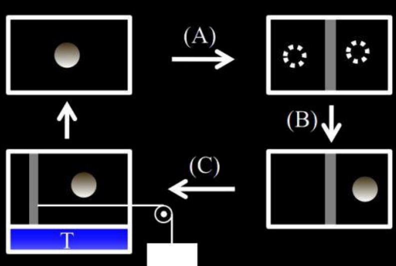
„Didn't Charlie Bennett solve this problem already?"

Notes on Landauer's Principle, Reversible Computation, and Maxwell's Demon

Charles H. Bennett
IBM Research Division, Yorktown Heights, NY 10598, USA — bennetc@watson.ibm.com
(January 12, 2011)

Landauer's principle, often regarded as the basic principle of the thermodynamics of information processing, holds that any logically irreversible manipulation of information, such as the erasure of a bit or the merging of two computation paths, must be accompanied by a corresponding entropy increase in non- information-bearing degrees of freedom of the information processing apparatus or its environment. Conversely, it is generally accepted that any logically reversible transformation of information can in principle be accomplished by an appropriate physical mechanism operating in a thermodynamically reversible fashion. These notions have sometimes been criticized either as

- ✓ measurements can be implemented reversibly
- ✗ only true for classical systems



Szilard engine

Important:
Do not forget any system that is involved in the process

Recent work by Kammerlander/Anders (2015):

Quantum measurement and its role in thermodynamics

Philipp Kammerlander*
Institute for Theoretical Physics, ETH Zurich, Wolfgang-Pauli-Strasse 27, 8093 Zurich, Switzerland.

Janet Anders†
Department of Physics and Astronomy, University of Exeter, Stocker Road, EX4 4QL, United Kingdom.

A central goal of the research effort in quantum thermodynamics is the extension of standard thermodynamics to include small-scale and quantum effects. Here we lay out consequences of seeing measurement, one of the central pillars of quantum theory, not merely as a mathematical projection but as a thermodynamic process. We uncover that measurement, a component of any experimental realisation, is accompanied by work and heat contributions and that these are distinct

$$\rho^S \mapsto \rho'^S = \sum_k P_k \rho^S P_k \qquad \boxed{\forall \rho^S}$$

$$\Rightarrow \Delta W_{\text{cost}} < 0$$ possible

- ✓ clear axiomatic framework
- ✗ only projective measurements
- ✗ dephasing only required for particular ρ^S
- ✗ measurement outcome is not stored

Goal:

✓ describe general measurements

Inefficiency of measurement

$$\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i} \rho^S A_{k,i}^\dagger\}$$

✓ with a microscopic model that incorporates a memory

$$\rho^S \otimes \rho^M \mapsto \rho'^{SM}$$

$$\boxed{\mathcal{H}^M = \bigoplus_k \mathcal{H}_k^M}$$
$\{Q_k\}_k$ projections

✓ describe the erasure process to allow for repetitive use of memory

✓ Quantify work cost in terms of $\boxed{\rho^S,\ H^S,\ \{A_{k,i}\}}$

Measurement model for: $\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i}\rho^S A_{k,i}^\dagger\}$

$\rho'^{SM} = \qquad\qquad \rho^S \otimes \rho^M$

Measurement model for: $\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i}\rho^S A_{k,i}^\dagger\}$

$$\rho'^{SM} = \qquad U(\rho^S \otimes \rho^M)U^\dagger$$

Measurement model for: $\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i}\rho^S A_{k,i}^\dagger\}$

$$\rho'^{SM} = \sum_k (\mathbb{1} \otimes Q_k) U (\rho^S \otimes \rho^M) U^\dagger (\mathbb{1} \otimes Q_k)$$

specifies a measurement device

Measurement model for: $\rho^S \mapsto \{p_k, \rho^S_k = \sum_i^I A_{k,i}\rho^S A^\dagger_{k,i}\}$

$$\rho'^{SM} = \sum_k (\mathbb{1} \otimes Q_k) U (\rho^S \otimes \rho^M) U^\dagger (\mathbb{1} \otimes Q_k) \quad \boxed{\forall \rho^S}$$

specifies a measurement device

Measurement model for: $\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i}\rho^S A_{k,i}^\dagger\}$

$$\rho'^{SM} = \sum_k (\mathbb{1}\otimes Q_k)U(\rho^S\otimes\rho^M)U^\dagger(\mathbb{1}\otimes Q_k) \quad \forall\rho^S$$

specifies a measurement device

For all measurements such a device exists!

Outcome is now stored in ρ_k^M.

Work cost:

$$\Delta W = \mathrm{tr}[H^S(\rho'^S - \rho^S)] + \mathrm{tr}[H^M(\rho'^M - \rho^M)]$$

Note: $\{Q_k\} \stackrel{!}{=} \sum Q_k \otimes U_k^B$

Measurement model for: $\rho^S \mapsto \{p_k, \rho_k^S = \sum_i^I A_{k,i} \rho^S A_{k,i}^\dagger\}$

$$\rho'^{SM} = \sum_k (\mathbb{1} \otimes Q_k) U (\rho^S \otimes \rho^M) U^\dagger (\mathbb{1} \otimes Q_k) \quad \forall \rho^S$$

specifies a measurement device

For all measurements such a device exists!

Outcome is now stored in ρ_k^M.

Work cost:

$$\Delta W = \mathrm{tr}[H^S(\rho'^S - \rho^S)] + \overbrace{\mathrm{tr}[H^M(\rho'^M - \rho^M)]}^{=:\, \Delta W^M_{\mathrm{meas}}}$$

Note: $\{Q_k\} \overset{!}{=} \sum Q_k \otimes U_k^B$

Erasure model:

$$\rho^M = \mathrm{tr}_B[V\rho'^M \otimes \tau^B V^\dagger]$$

Landauer erasure

Work cost:

$$\Delta W^{MB}_{\mathrm{eras}} := \mathrm{tr}[H^M(\rho^M - \rho'^M)] + \mathrm{tr}[H^B(\rho'^B - \tau^B)]$$

Total work cost: $\Delta W_{\mathrm{cost}} = \Delta W^M_{\mathrm{meas}} + \Delta W^{MB}_{\mathrm{eras}}$

“work cost for operating the measurement device“

Work cost of quantum measurements

Minimal Energy Cost of Thermodynamic Information Processing: Measurement and Information Erasure

Takahiro Sagawa[1] and Masahito Ueda[1,2]
[1] *Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8654, Japan*
[2] *ERATO Macroscopic Quantum Control Project, JST, 2-11-16 Yayoi, Bunkyo-ku, Tokyo 113-8656, Japan*
(Dated: May 31, 2009)

The fundamental lower bounds on the thermodynamic energy cost of measurement and information erasure are determined. The lower bound on the erasure validates Landauer's principle for

Same microscopic model:

$$\rho'^{SM} = \sum_k (\mathbb{1} \otimes Q_k) U (\rho^S \otimes \rho^M) U^\dagger (\mathbb{1} \otimes Q_k) \quad \forall \rho^S$$

Additional assumption by Sagawa/Ueda:

$$\rho'^{SM} = \sum_k A_k \rho^S A_k^\dagger \otimes \rho_k^M \qquad \forall \rho^S$$

no entanglement

only efficient measurements

arbitrary $A_{k,i}$

$\beta\Delta W_{\mathrm{cost}} \geq ?$

efficient A_k + no entanglement

$\beta\Delta W_{\mathrm{cost}} \geq I_{\mathrm{QC}} \geq 0$ Sagawa/Ueda

projective P_k

$\beta\Delta W_{\mathrm{cost}} \geq 0$

arbitrary $A_{k,i}$

$\beta \Delta W_{\text{cost}} \geq ?$

efficient A_k + no entanglement

$\beta \Delta W_{\text{cost}} \geq I_{\text{QC}} \geq 0$ Sagawa/Ueda

Our main result:
Work cost of operating a measurement device

$$\beta \Delta W_{\text{cost}} \geq S(\rho^S) - \sum_k p_k S(\rho_k^S)$$

arbitrary $A_{k,i}$

$$\beta \Delta W_{\text{cost}} \geq \boxed{S(\rho^S) - \sum_k p_k S(\rho_k^S)}$$

efficient A_k + no entanglement

$$\beta \Delta W_{\text{cost}} \geq I_{\text{QC}} \geq 0$$ Sagawa/Ueda

Our main result:
Work cost of operating a measurement device

$$\beta \Delta W_{\text{cost}} \geq S(\rho^S) - \sum_k p_k S(\rho_k^S)$$

Properties of our bound:

- for efficient measurements: $S(\rho^S) - \sum_k p_k S(\rho_k^S) = I_{\mathrm{QC}} \geq 0$

Correct generalisation of previous bound!

- for arbitrary measurements: $\rho_k^S = \sum_i^I A_{k,i} \rho^S A_{k,i}^\dagger$

$$\Rightarrow \beta \Delta W_{\mathrm{cost}} \geq S(\rho^S) - \sum_k p_k S(\rho_k^S) \geq -\log I$$

can construct equality!

Inefficient measurements can yield work!

arbitrary $A_{k,i}$

$$\beta \Delta W_{\text{cost}} \geq S(\rho^S) - \sum_k p_k S(\rho_k^S) \geq \boxed{-\log I}$$

Work cost can be negative!

efficient A_k + no entanglement

$$\beta \Delta W_{\text{cost}} \geq I_{\text{QC}} \geq 0$$

Sagawa/Ueda

projective P_k

$$\beta \Delta W_{\text{cost}} \geq 0$$

Precise knowledge over the gap terms:

$$\beta \Delta W_{\mathrm{cost}} = \left[S(\rho^S) - \sum_k p_k S(\rho_k^S) \right] + \mathcal{I}(S:M) + \Delta S_{\mathrm{total}}$$

Even better bound for projective measurements:

$$\rho'^S = \textstyle\sum_k P_k \rho^S P_k \quad \forall \rho^S$$

Equality result for the
work cost of projective measurements:

$$\beta \Delta W_{\mathrm{cost}} = H(\{p_k\})$$

$\Rightarrow$ **Projective measurements always cost work!**

arbitrary $A_{k,i}$

$$\beta \Delta W_{\text{cost}} \geq \boxed{S(\rho^S) - \sum_k p_k S(\rho_k^S)} \geq \boxed{-\log I}$$

Work cost can be negative!

efficient A_k + no entanglement

$$\beta \Delta W_{\text{cost}} \geq I_{\text{QC}} \geq 0$$ Sagawa/Ueda

projective P_k

$$\beta \Delta W_{\text{cost}} = \boxed{H(\{p_k\})} \geq 0$$

Thank you for your attention!