Strongly coupled quantum heat machines

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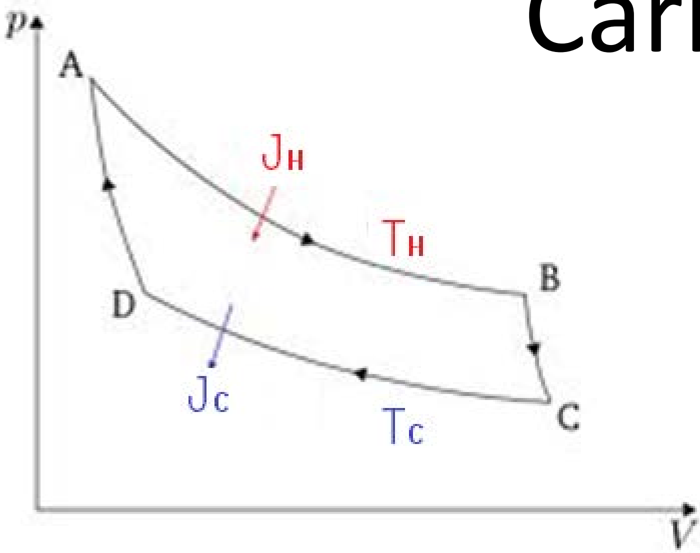
Palma, 2015

D. G.-K. Alán Aspuru-Guzik, arXiv:1504.04744

Carnot machines



Components

1. Working fluid, the system (Ex. Gas)
2. Hot and cold bath
3. External cyclic driving of the system. It extracts or invests work (Piston)

The second law limits the machine efficiency

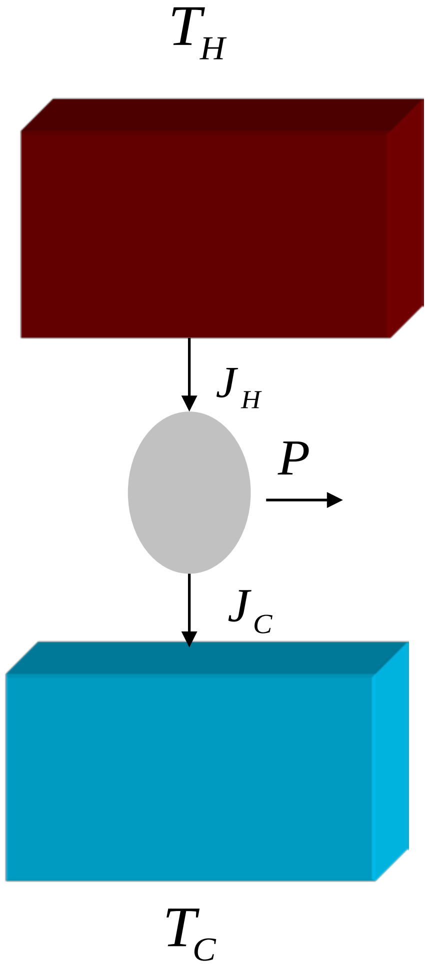
Engine

Extracted Power

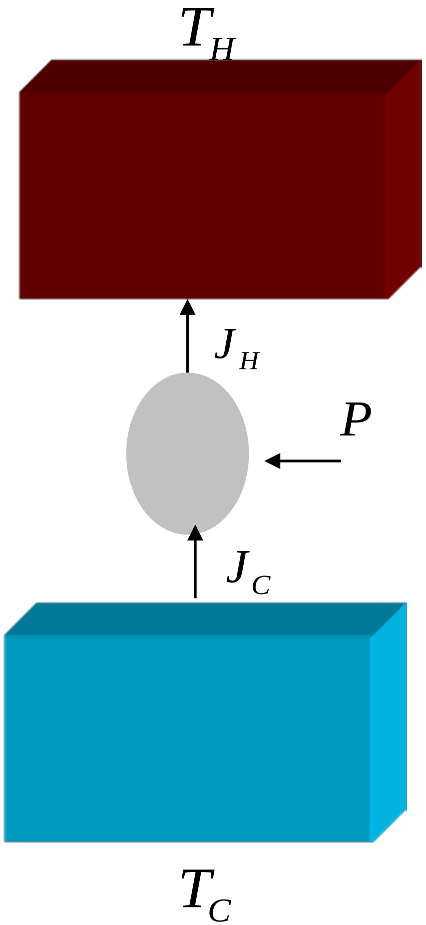
$$\eta \equiv \frac{-P}{J_H} \leq 1 - \frac{T_C}{T_H} \equiv \eta_{Carnot}$$

$$(P = \dot{W})$$

Absorbed heat (Invested energy)



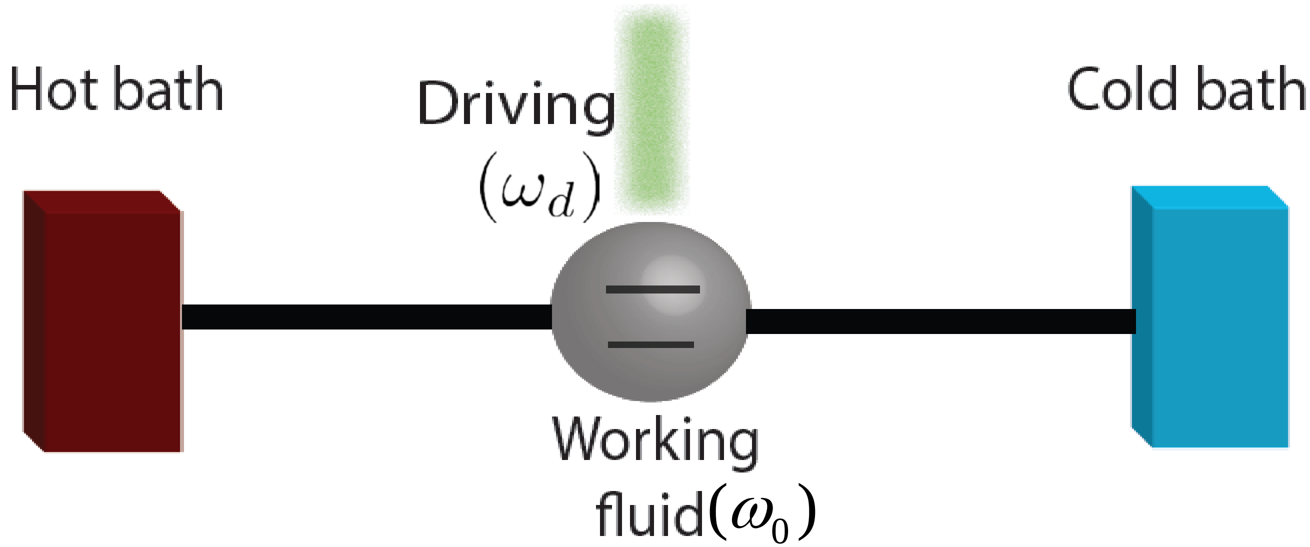
Engine



Refrigerator

Continuous quantum machine

K Szczygielski, D. G. –K., R Alicki Physical Review E 87 (1), 012120



Components

1. Working fluid: (qubit, TLS).
2. Hot and cold bath (normal modes), **permanently** coupled to the system (weak coupling).
3. A piston (External driving) periodically drives the system and gets or gives work.

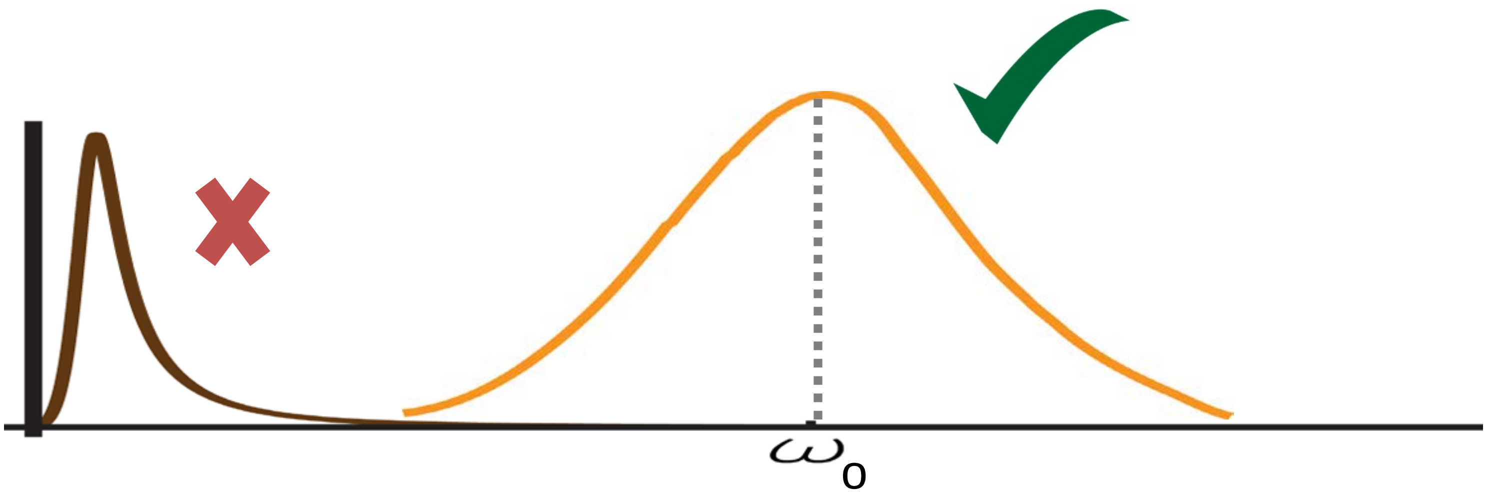
$$H_{Tot} = H_S(t) + \sum_i (H_{B_i} + \underline{\xi_i S} \otimes F_i) \quad i \in H, C$$

Review: D. G.-K, W. Niedenzu, G. Kurizki arXiv:1503.01195

Coupling specturm

$$G_i(\omega) = \int_{-\infty}^{\infty} e^{it\omega}\xi_i^2\langle F_i^\dagger(t)F_i(0)\rangle, \quad H_{SB} = \sum_{i\in H,C}\xi_i S\otimes F_i$$

$$G_i(-\omega) = e^{-\beta_i\omega}G_i(\omega)$$ KMS Condition



(resonant baths) $H_{Tot} = H_S(t) + \sum_i H_{B_i} + H_{SB_i} \quad i \in H, C$

Lindblad master equation
(weak coupling, “small” ξ_i)

$$\rho_S(t)$$

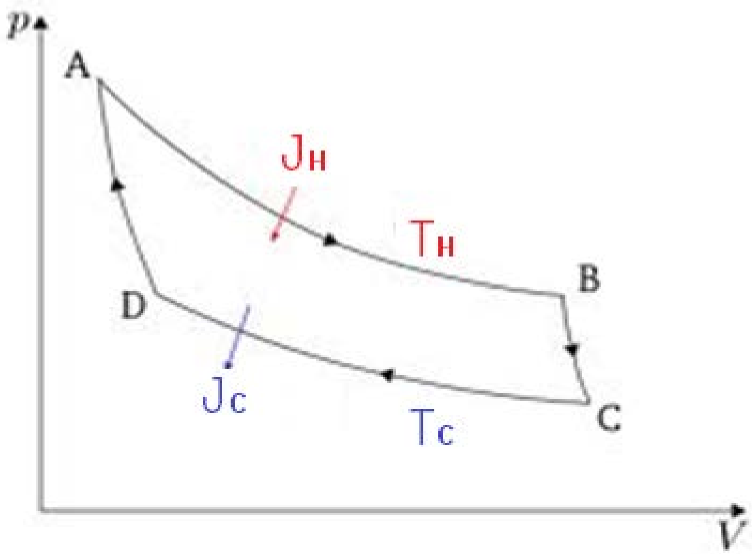
$J_i(G_i(\omega))$

Thermodynamic quantities
(analytic expressions)

Baths at equilibrium $\Rightarrow$ KMS condition $\Rightarrow$ $\eta \leq 1 - \frac{T_C}{T_H}$

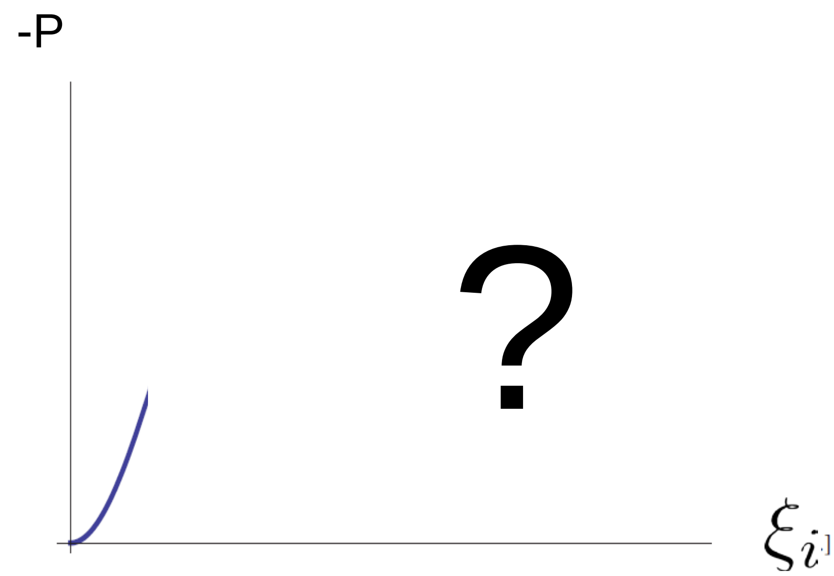
Weak coupling limitations

Weak coupling QHM $\quad P \propto \gamma \propto \xi_i^2 \quad (\xi_i \text{ coupling strength})$



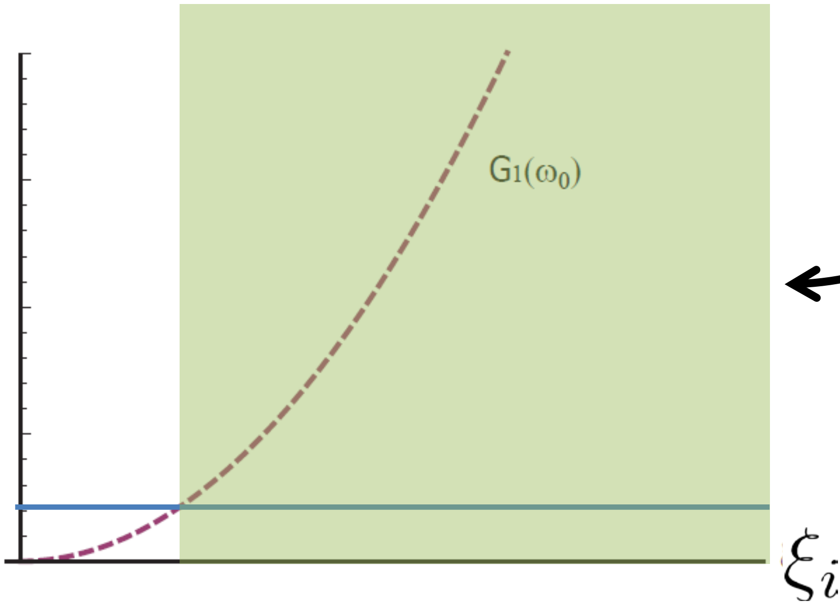
$$t \propto 1/\gamma$$

Also strokes QHMs are limited

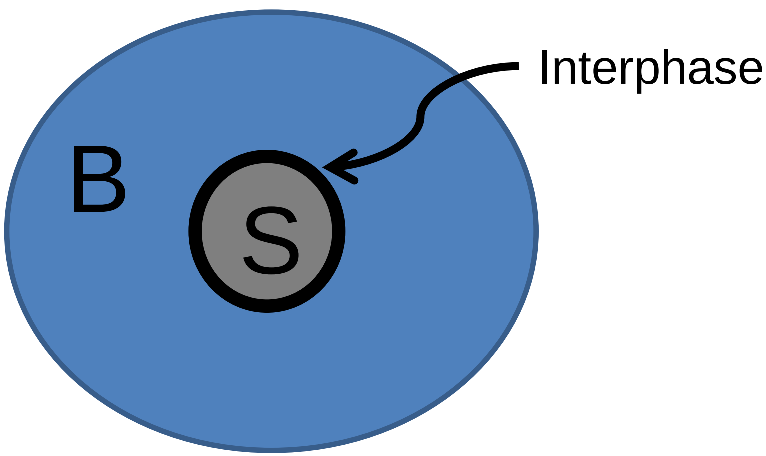


The coupling strength limits the output

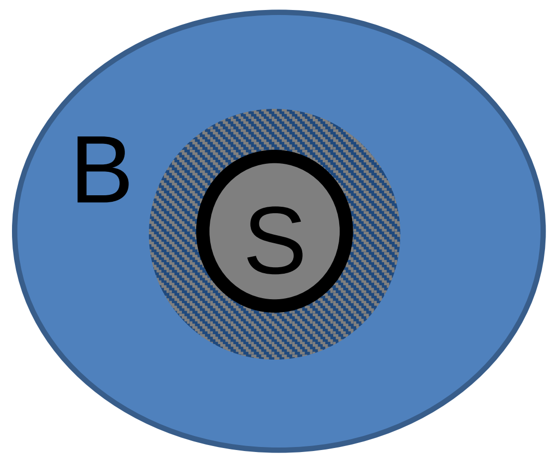
Strong coupling



Lindblad equations are no longer valid!

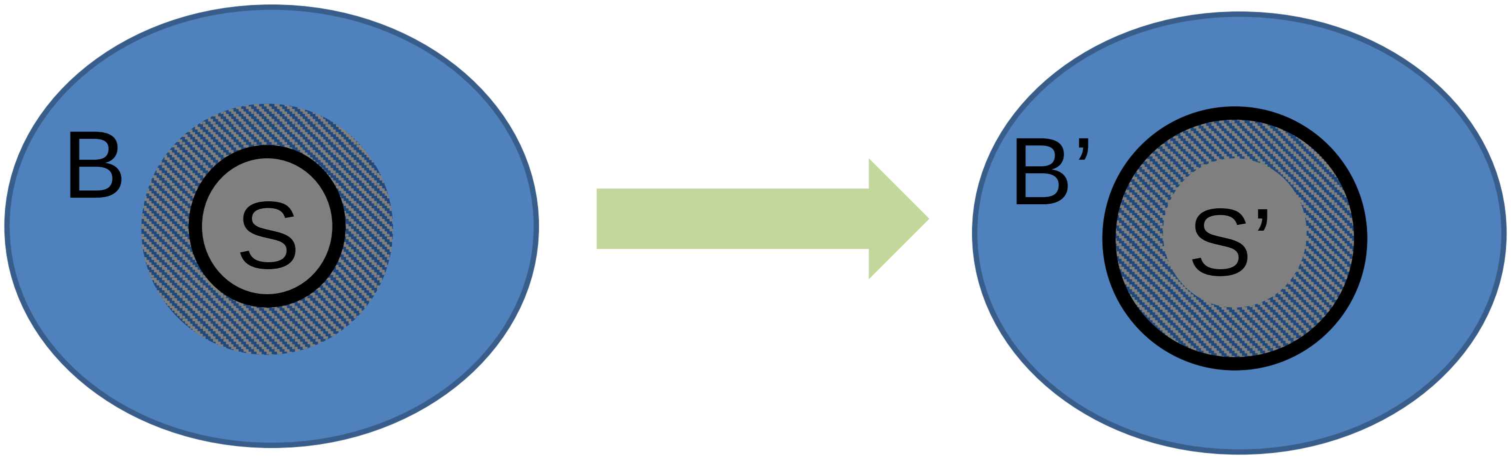


Weak Coupling



Strong Coupling

Redefining the system and the bath



S-B strong coupling

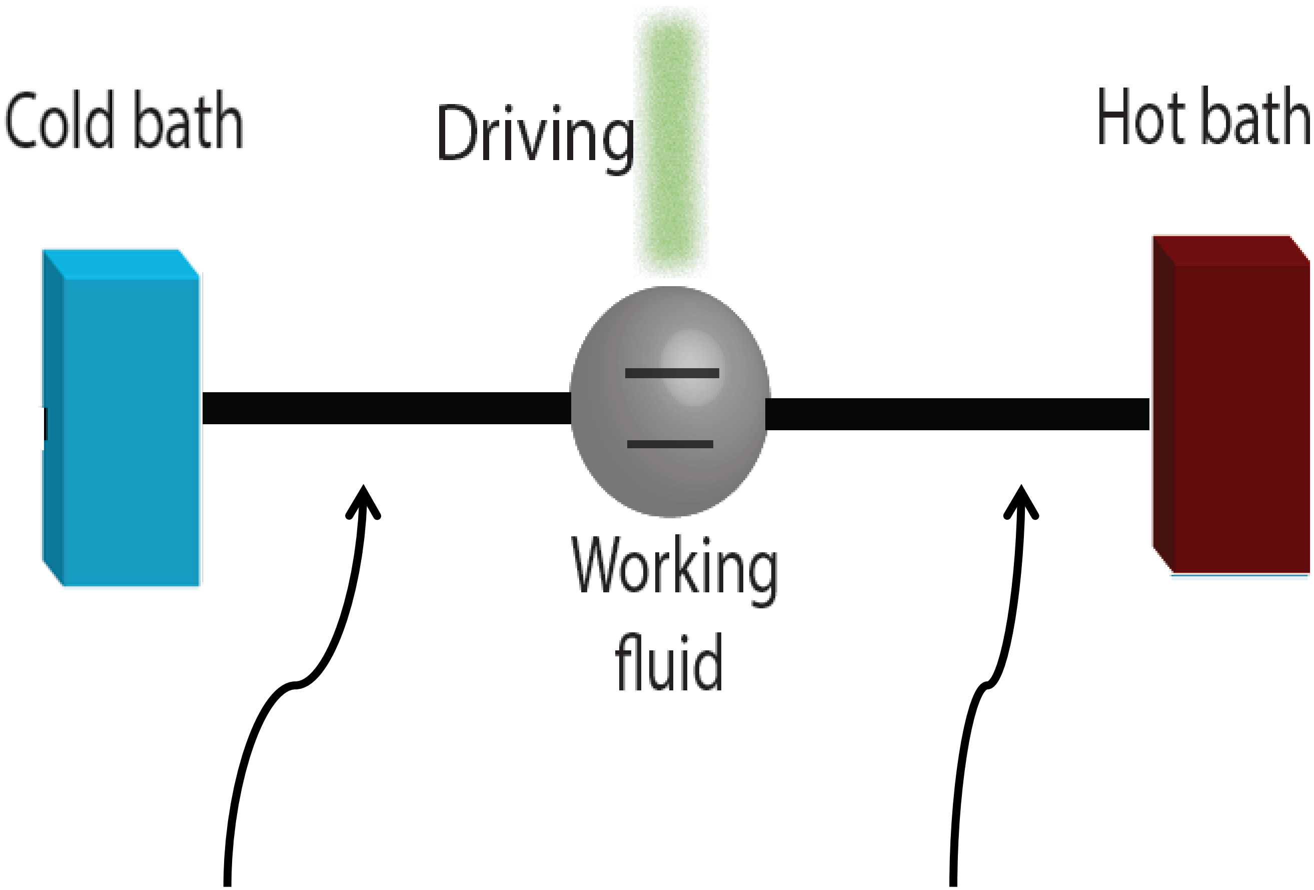
S'-B' effective weak coupling

$$\tilde{\mathcal{H}} \longrightarrow \mathcal{H} = U\tilde{\mathcal{H}}U^{\dagger}$$

$$U \ \text{Polaron transformation}$$

D. P. McCutcheon and A. Nazir, NJP 12, 113042 (2010).

Particular example



$$\sigma_z \otimes \underline{\xi_C} \sum_K \underline{g_{C,k}} (a_k^\dagger + a_k)$$

$$\sigma_x \otimes \underline{\xi_H} \sum_K \underline{g_{H,k}} (b_k^\dagger + b_k)$$

First coupling

Second coupling

Transformed Hamiltonian

$$U = e^{S} \quad S = \sigma_Z \otimes \xi_C \sum_K \frac{g_{C,k}}{\omega_{C,k}}(a_k^\dagger - a_k)$$

First coupling

$$\sigma_z \otimes \xi_C \sum_K g_{C,k}(a_k^\dagger + a_k)$$

$$\Downarrow$$

$$\sigma_+ \otimes (A_+ - A) + \ h.c$$

$$A_+ = \Pi_K e^{2\xi_C \frac{g_{C,k}}{\omega_{C,k}}(a_k^\dagger - a_k)} \qquad A = \langle A_+ \rangle$$

Transformed Hamiltonian

$$U = e^{S} \quad S = \sigma_Z \otimes \xi_C \sum_K \frac{g_{C,k}}{\omega_{C,k}}(a_k^\dagger - a_k)$$

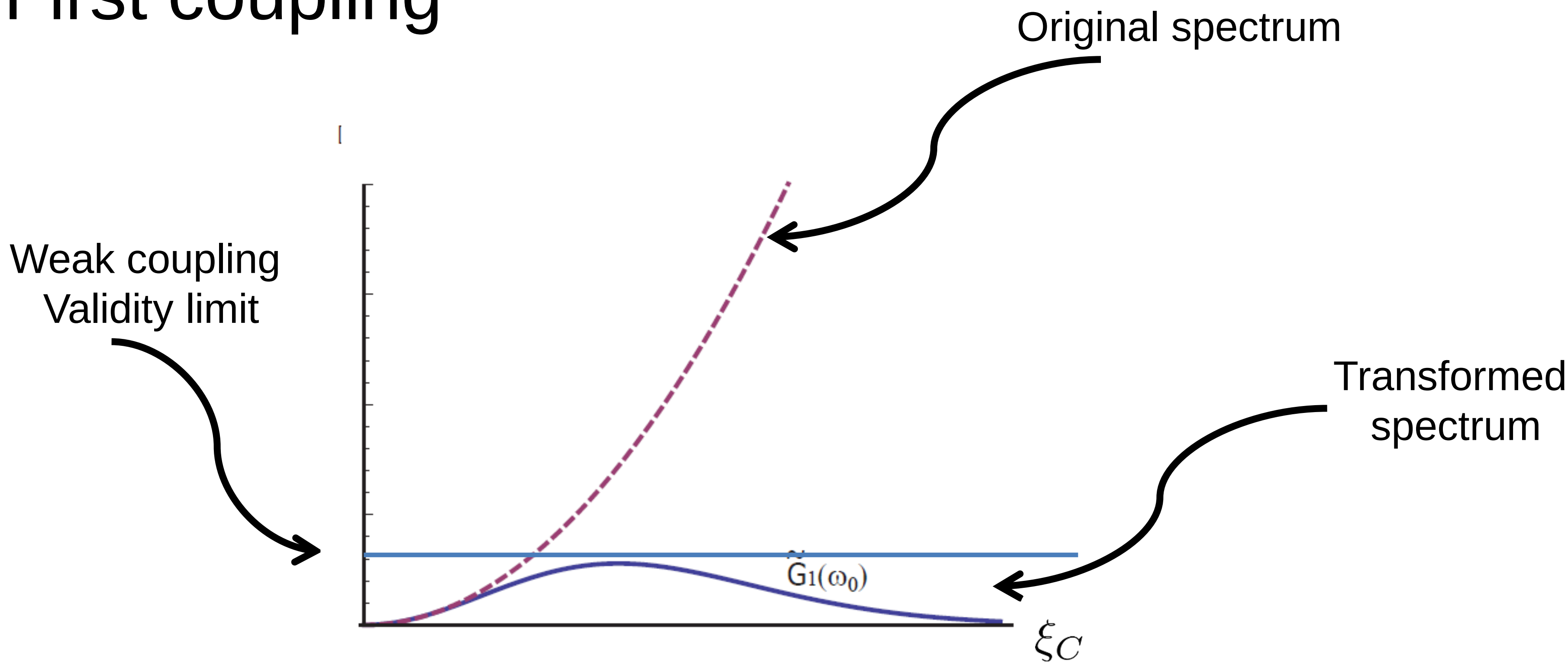
Second coupling

$$\sigma_x \otimes \xi_H \sum_K g_{H,k}(b_k^\dagger + b_k)$$

$$(\sigma_+ \otimes A_+ + \; h.c.) \otimes \xi_H \sum_K g_{H,k}(b_k^\dagger + b_k)$$

Effective weak coupling

First coupling



Second coupling

$$(\sigma_+ \otimes \underline{A_+} + h.c) \otimes \underline{\xi_H} \sum_K g_{H,k}(b_k^\dagger + b_k)$$

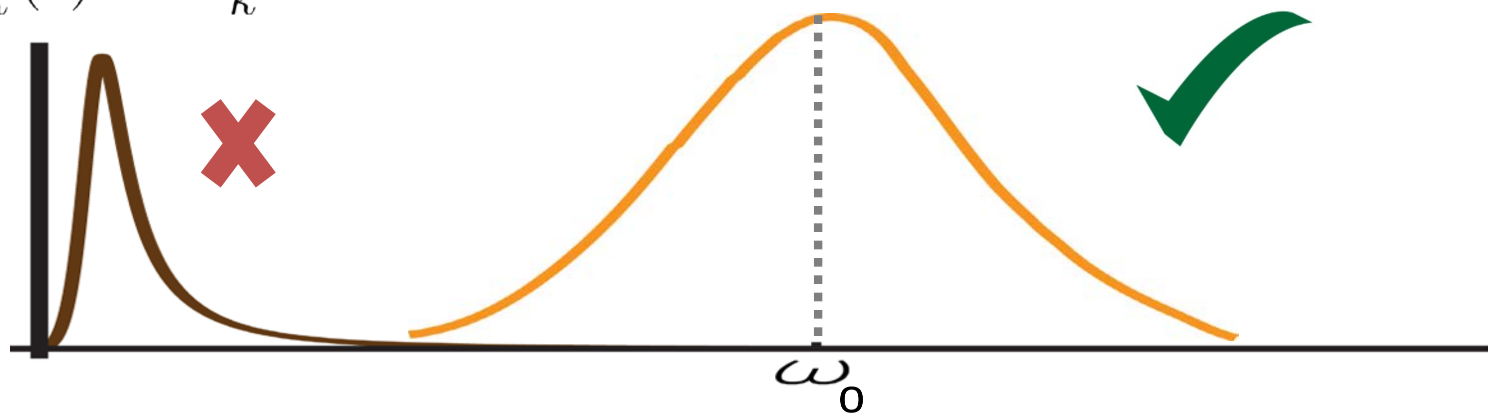
$$\text{Effectively weak if } \xi_H \sim \xi_C$$

Coupling Spectrum

WEAK COUPLING:

$$a_k^\dagger(t) = a_k^\dagger e^{\underline{i\omega_k t}}$$

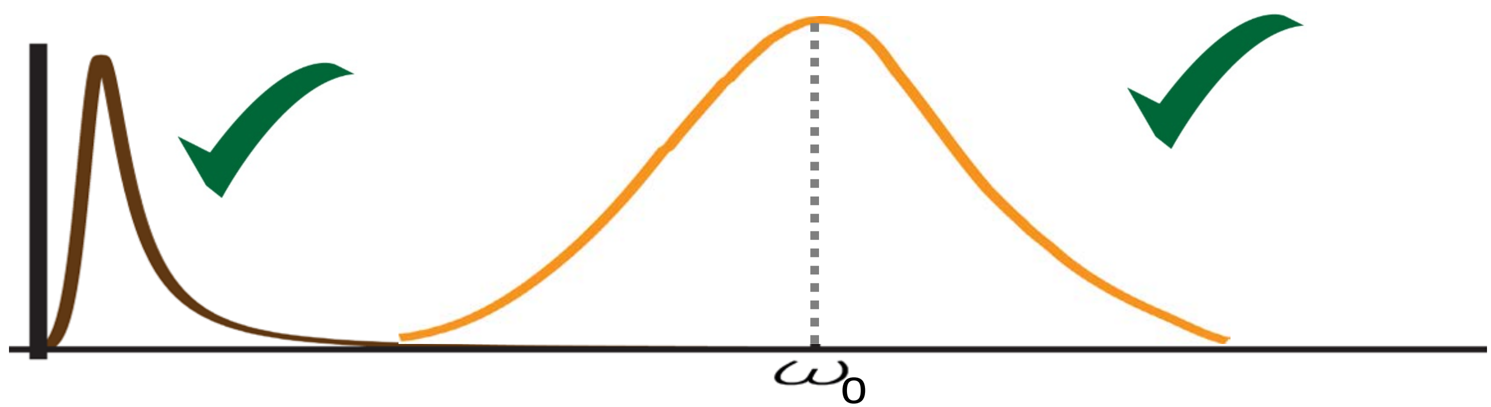
Modes should be resonant!



STRONG COUPLING:

$$A_+(t) = \Pi_K \sum_n C_{n,k} e^{\underline{itn\omega_k}}$$

Harmonic Modes also contribute!



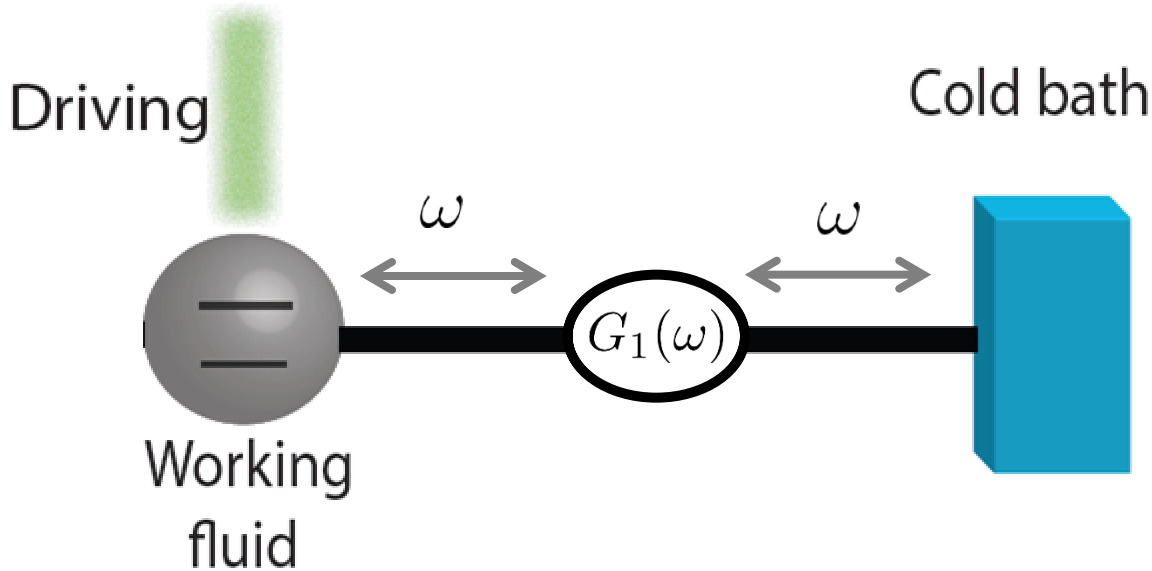
KMS

First coupling

$$F_1^{\dagger} = A_+ - A$$

Cold bath operator

$$G_1(-\omega) = e^{-\beta_C \omega} G_1(\omega)$$

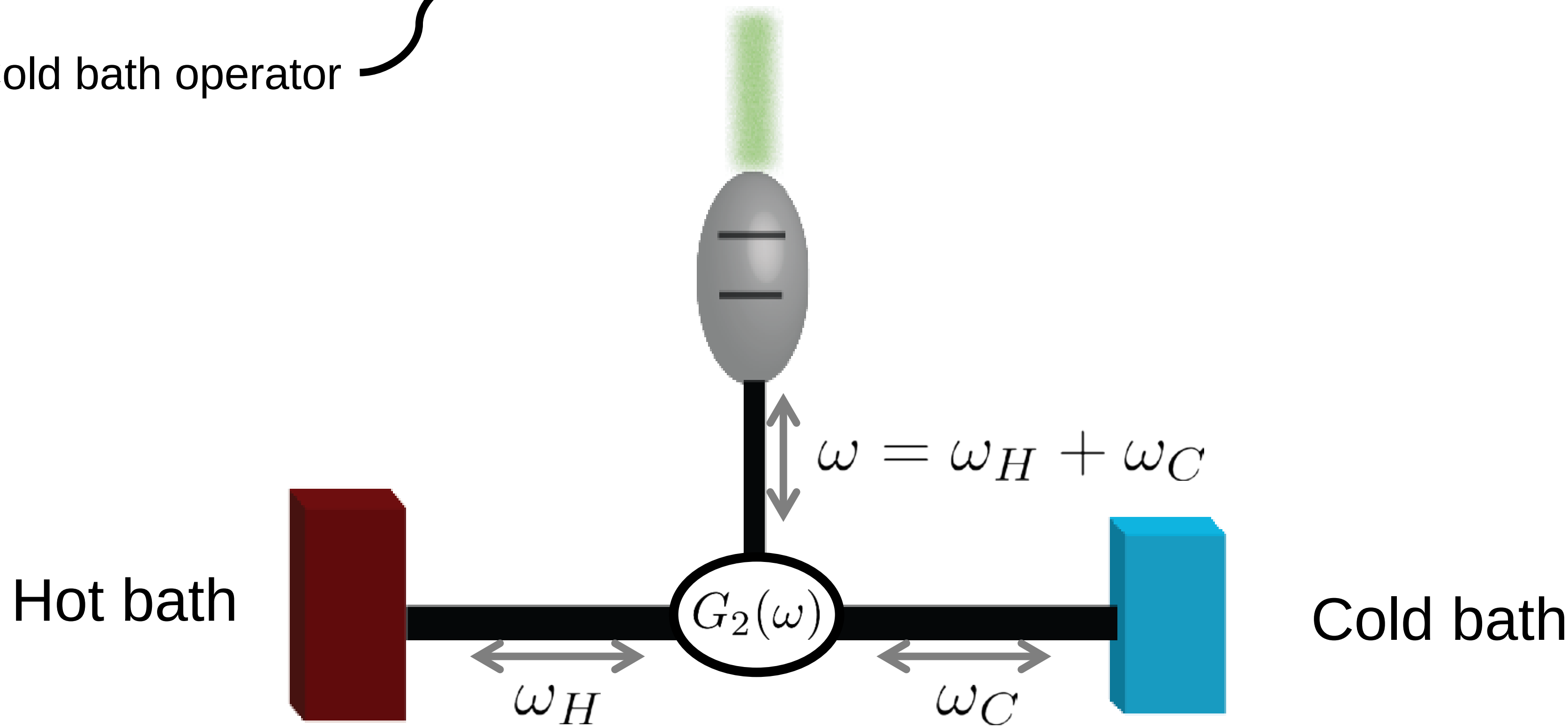


Second coupling

Hot bath operator

$$F_2^\dagger = A_+ \otimes \xi_H \sum_K g_{H,k}(b_k^\dagger + b_k)$$

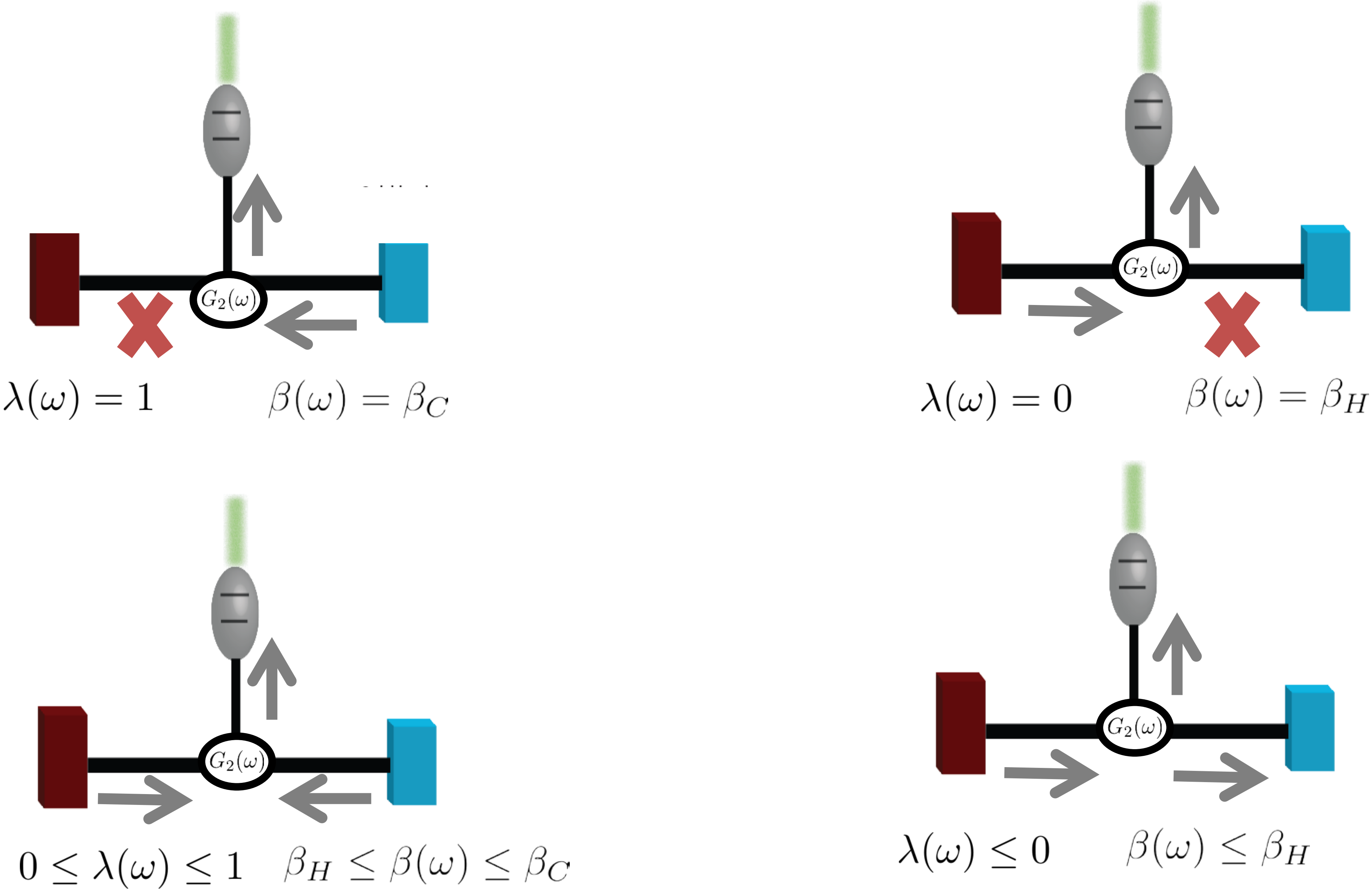
Cold bath operator



$$G_2(-\omega) = e^{-\beta(\omega)\omega} G_2(\omega) \qquad \underline{\beta(\omega)} = \underline{\lambda(\omega)}\beta_C + (1 - \lambda(\omega))\beta_H$$

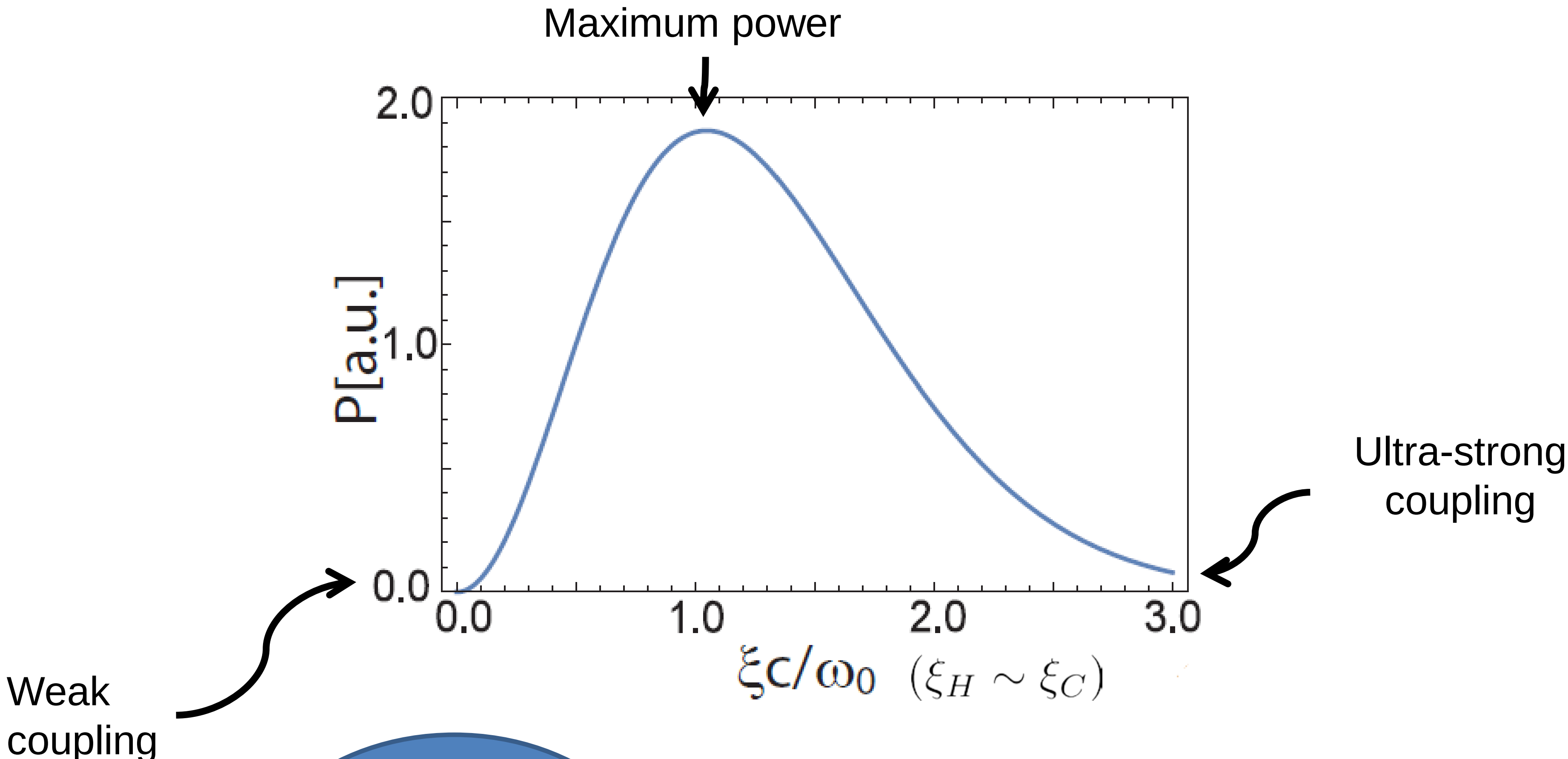
Each ω may be a different process

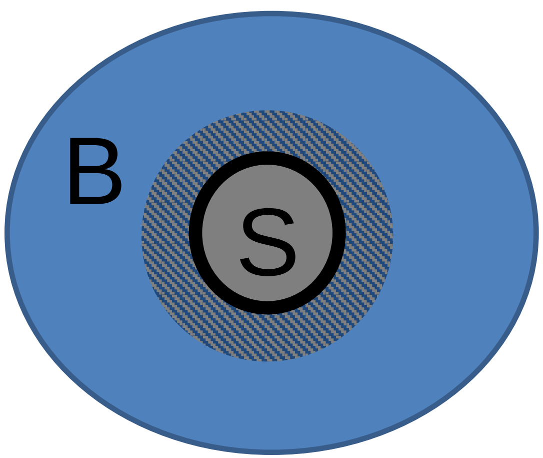
$$\beta(\omega) = \lambda(\omega)\beta_C + (1 - \lambda(\omega))\beta_H$$



$\beta(\omega)$ is not restricted to (β_H, β_C)

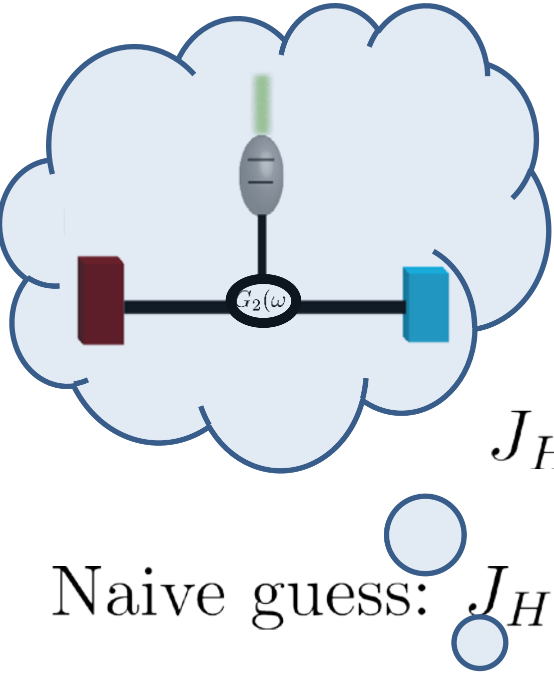
Power





Similar for cooling power!

Efficiency



$$\eta = \frac{-P}{J_H}$$

J_H?

(J_1, J_2)

Naive guess: $J_H = J_2$ $\Longrightarrow$ $\eta = 1 - \frac{\beta(\omega)}{\beta_C}$

- May be larger than Carnot ????

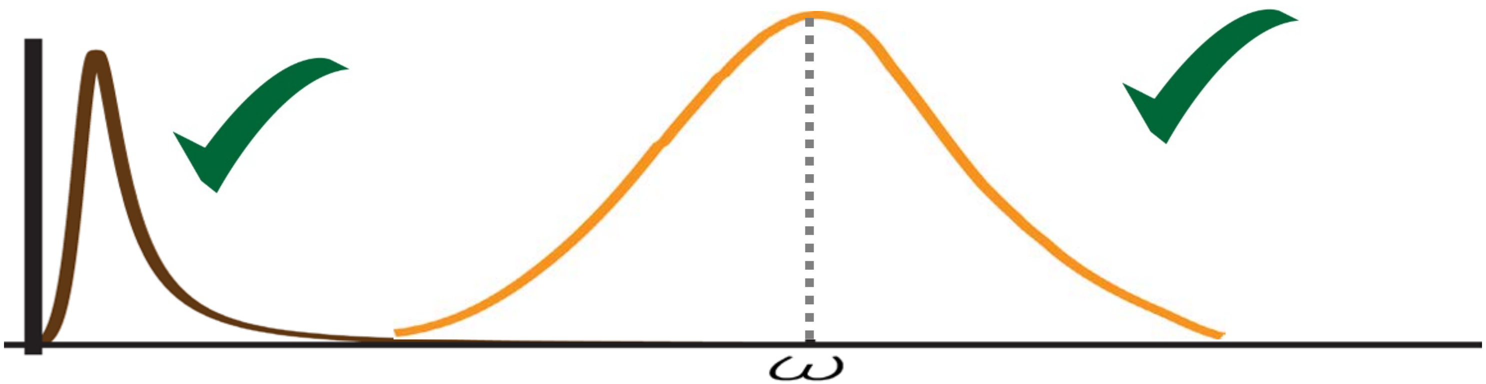
Correct heat flow: $J_H = J_2(1 - \lambda(\omega_0))$

$$\eta = \frac{-P}{J_2(1-\lambda(\omega_0))} \leq \eta_{car}$$

Similar for cooling

Conclusions

1. $$\eta = \frac{-P}{J_2(1-\lambda(\omega_0))} \leq \eta_{car}$$

2. 

3.