Quantum thermodynamics for a model of an expanding universe

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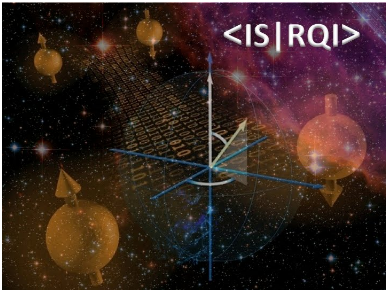
Relativistic and Quantum Physics

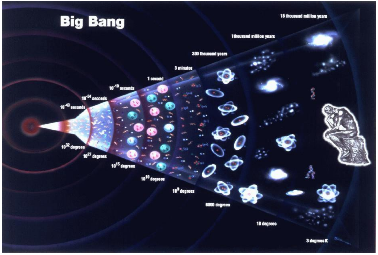


Relativity + Quantum Information = Relativistic Quantum Information

(Classical) Thermodynamics and general relativity

Important achievements in general relativistic scenarios

- Three laws of black holes;
- Cosmology and big bang;
- Firewall issues.



Outlook, aims and motivations

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- Quantum processes in the universe do not necessary involve large numbers of constituents.
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Motivations

- The Universe is relativistic and quantum system and processes can involve small numbers of constituents.
- Mainly: We cannot compute energy, entropy and work flows in relativistic quantum systems.

Simple cosmology model

Expanding universe line/metric element

$$ds^2 = \Omega^2(\tau)\left[-d\tau^2 + dx^2 + \ldots\right], \qquad g_{\mu\nu} = \Omega^2(\tau)\,(-1, 1, \ldots)$$

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Scalar quantum field

$$\phi(t, x) = \sum\!\!\!\!\!\!\int_k \left[u_k\, a_k + u_k^*\, a_k^\dagger\right], \qquad \left[a_k, a_{k'}^\dagger\right] = \delta^d(k - k')$$

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Frequency shift and choice of conformal factor

$$\omega_{\text{in/out}} = \sqrt{k^2 + m\,\Omega^2(\tau_{\mp\infty})}, \qquad \Omega(\tau) = \sqrt{1 + \epsilon\,(1 + \tanh(\sigma\,\tau))}$$

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Bogoliubov transformations and squeezing

$$a_{\mathrm{out},k} = \cosh r_k\, a_{\mathrm{in},k} + e^{i\,\theta_k}\,\sinh r_k\, a_{\mathrm{in},-k}^\dagger, \qquad \tanh r_k = \frac{\sinh\left(\pi\frac{\omega_{\mathrm{out}}-\omega_{\mathrm{in}}}{2\sigma}\right)}{\sinh\left(\pi\frac{\omega_{\mathrm{out}}+\omega_{\mathrm{in}}}{2\sigma}\right)}$$

Work and energy of two mode squeezing

All couple of modes $(k, -k)$ decouple. We can focus on a single couple and redefine $a_{\text{in},k} \equiv a_{\text{in}}$ and $a_{\text{in},-k} \equiv b_{\text{in}}$.

Initial Hamiltonian

$$H_{\text{in}} = \omega_{\text{in}} \left[a^\dagger_{\text{in}} a_{\text{in}} + b^\dagger_{\text{in}} b_{\text{in}} + \frac{1}{2} \right]$$

Final Hamiltonian

$$H_{\text{out}} = \omega_{\text{out}} \left[a^\dagger_{\text{out}} a_{\text{out}} + b^\dagger_{\text{out}} b_{\text{out}} + \frac{1}{2} \right]$$

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Start with an initial *thermal* state ρ with n_i particles. The work W **done by spacetime** is

Work performed

$$\begin{aligned} W &= \text{Tr}\left(\left(H_{\text{out}} - H_{\text{in}}\right) \rho\right) \\ &= \omega_{\text{out}}\, n_c + \left(\omega_{\text{out}} - \omega_{\text{in}}\right)\left(n_i + 1\right) \end{aligned}$$

with n_c particles that are created.

The “inner friction”

An adiabatic process would lead to a work cost W_{ad} of the form

Adiabatic work

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Our last step

We proceed to show that W_{fric} can be interpreted as an entropic quantity

Entropy and inner friction

Forward process

$$p_{\mathrm{in,out}} = |\langle n_{\mathrm{out}}|n_{\mathrm{in}}\rangle|^2 \langle n_{\mathrm{in}}|\rho|n_{\mathrm{in}}\rangle$$

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Fluctuation relation: $s_{\text{in,out}} := -\log\langle n_{\text{out}}|\rho|n_{\text{out}}\rangle + \log\langle n_{\text{in}}|\rho|n_{\text{in}}\rangle$

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$$P_{\text{in,out}}(s) = \sum_{n_{\text{in}},n_{\text{out}}} p_{\text{in,out}}\,\delta\left(s - s_{\text{in,out}}\right)$$

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Average entropy: $K\left(X\|Y\right) := -\sum_n p_x(n)\left[\log p_y(n) - \log p_x(n)\right] \geq 0$

$$s = K\left(P_{\text{in,out}}\|P_{\text{out,in}}\right)$$

Results

Initial state

$$\rho = \frac{e^{-\beta_{in} H_{\text{in}}}}{\mathcal{Z}}$$

Entropy

$$s_{\text{in,out}} = \frac{\omega_{\text{in}}}{T} n_c$$

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Final result

$$s = \frac{\omega_{\text{in}}}{T} n_c$$

This is our main result.

Extendable result

Number of created particles $n_c = \sinh^2 r$. This result can be extended to:

* Unruh effect: $\tanh r = \exp[\frac{\hbar\omega}{k_B T_U}]$;
* Schwarschild black hole: $\tanh r = \exp[\frac{\hbar\omega}{k_B T_H}]$;
* Analogue gravity models.

Conclusions and outlook

Conclusions

* We have studied applications of quantum thermodynamics to setups that appear in quantum field theory;
* Have found some entropic quantity that increases in (simple) cosmological processes;
* The results apply to different cosmological and quantum field theoretical scenarios.

Outlook

* Can teach us more about the physics at the overlap of relativity and quantum mechanics;
* Drive future theoretical efforts to uncover novel physics;
* Hopefully: provide energy balance relations in quantum field theoretical scenarios.

Thank You.